PUBLICATIONS

G.N. Ramachandran

Mathematical Philosophy Group
Indian Institute of Science
Bangalore 560 012

VOLUME - V   MR 33 - 38

BOOLEAN ALGEBRA APPLIED TO STATEMENTS OF TYPE 1 IN QUANTIFIED PREDICATE LOGIC (QPL) AND ITS CONSEQUENCES FOR CATEGORICAL SYLLOGISMS OF TRADITIONAL LOGIC

G.N. Ramachandran

Gita, 5, 10A Main Road
Malleswaram West
Bangalore 560 055

PREFACE

This report is different from the earlier reports in that

it is not a paper written on work that has been completed before

being taken for writing up. The sections 1 and 2 were completed

but when the paper was written to report the results of these

studies, several new possibilities came up, such as the discover

of the new connective "into"' (⊗) and the effect of this on the

working out of the categorical syllogism of traditional logic.

Therefore, an analysis of all possible 64 combinations was taken

and an algebraic technique for working out the valid syllogisms in

all the four figures was developed. When this was done, it was

noticed that in addition to 18 categorical syllogisms having

as conclusions

$(s \Rightarrow p)$ and $(s \Rightarrow \neg p)$, 18 more could be derived having

$(\neg s \Rightarrow p)$ and $(\neg s \Rightarrow \neg p)$. These were therefore

carefully checked and written up in the form of Table 6. There

an attempt was made to obtain algorithms for working out an

individual statement of QPL with quantified inputs and outputs

which showed that the process of working out a sequence of

Type 1 statements of QPL was not readily possible. It appeared

as if such statements occurring one after the other will have

to be all taken together and an equivalent QPL statement from

input to final output will have to be obtained by checking the

reversals of everyone of the statements. This is a process which

requires further studies and the present report was stopped at

that level.

Although this report does not explain everything, several very

interesting new results have been obtained and it gives a picture

of how the studies developed and newer results were obtained.

It is proposed to write a condensed version of this as Part III

of our series "Boolean Vector-Matrix Representation of Extended

Predicate Logic (EPL)". In view of this, this report has not been

thoroughly corrected for uniformity of presentation, and notation,

nomenclature, etc. All these changed by virtue of the needs for the

further development of the subject, and therefore the manuscript

is left as it is for this particular report.

Abstract

This report deals with the algebraic representation of QPL sentences and connectives which occur in the categorical syllogisms of traditional logic. Using only two quantifier states in statements of the type $(\forall x)\ (\underline{a}(x) \Longrightarrow \underline{b}(x))$ and $(\exists x)\ (\underline{a}(x) \Longrightarrow \underline{b}(x))$ (which have been named "Type 1 statements"), it is shown that the 64 possible moods of categorical syllogisms can be analysed and the valid ones deduced, by using only a small number of rules. These consist of (a) the equivalences between reversed statements, namely $(\forall x)\ (\underline{a} \Longrightarrow \underline{b})$ $\equiv (\forall x)\ (\neg\, \underline{b} \Longrightarrow \neg\, \underline{a})$, and $(\exists x)\ (\underline{a} \Longrightarrow \underline{b}) \equiv (\exists x)\ (\underline{b} \Longrightarrow \underline{a})$ (the latter replacing $(\exists x)\ (\underline{a}\ \&\ \underline{b})$ commonly used), (b) the effect of putting a QPL term as input "into" another quantified sentence (the newly introduced "into" operator ($\otimes$) leading to $\forall \otimes \forall = \forall$, $\forall \otimes \exists = \exists = \exists \otimes \forall$, $\exists \otimes \exists = \Delta$ (the indeterminate state of EPL), and (c) converting a $\forall$ statement, when needed, into an $\exists$ statement by the QPL implication $(\forall x)\ (\underline{a}(x) \Longrightarrow \underline{b}(x)) \supset (\exists x)\ (\underline{a}(x) \Longrightarrow \underline{b}(x))$. Eighteen of the nineteen listed moods in the four figures of traditional logic

can be proved to be valid in this way, provided the conclusion

for $s \longmapsto m$, $m \longmapsto p$ can only be $(\mathcal{Q}/x)$ $(\underline{s} \Longrightarrow \underline{p})$ or

$(\mathcal{Q}/x)$ $(\underline{s} \Longrightarrow \neg \underline{p})$, $\mathcal{Q} \equiv \forall$, or $\exists$.

Eighteen additional new moods can be shown to be valid,

which have the conclusions $(\mathcal{Q}/x)$ $(\neg \underline{s} \Longrightarrow \underline{p})$ or $(\mathcal{Q}/x)(\neg \underline{s} \Longrightarrow \neg \underline{p})$

These include one that is superior in power to the mood <u>Bamalip</u>

in the fourth figure listed in the literature. It is also proved

that the rest of the 64 syllogisms lead to only the QPL state $\triangle$,

or the SNS state D, for the conclusion, and are therefore not valid.

Finally, it is shown that a sequence of QPL statements

of Type 1 cannot be worked out step by step, using the connective

$\Longrightarrow$, but has to be taken as a whole and processed. The

consequences of this will be reported later.

Contents

1. Introduction

In the earlier Parts I and II ([1] , [2]), the basic theory

of the vector-matrix representation of Boolean algebras was

discussed and the application of BA-3 to represent the different

possible states in quantified predicate logic was considered.

It was shown that BA-3 can represent the four well-known states

of standard quantified predicate logic (QPL), snamely "for all"$(\forall)$

"for none"(Φ), "there exists"$(\exists)$, "not for all"(Λ), in the form

of Boolean 3-vectors, namely (1 0 0), (0 0 1), (1 1 0),

(0 1 1) respectively. Since BA-3 has four additional states

to make up the full complement of $2^3(=8)$ possible different

vectors, the new states were also defined and given the names

"some" (Σ) = (0 1 0), "all or none" (Θ) = (1 0 1),

"indefinite" (Δ) = (1 1 1) and "impossible" (ϕ) = (0 0 0).

It was shown that the standard form of QPL can be extended

readily to include the eight states, that has been term "Extended

Predicate Logic" (EPL) and that the different standard forms of

expressing a state in QPL by six Boolean elements (See Table 4,

Part I) can be converted to canonical forms which are representable

by 3-element BA-3 vectors.

In Part II, the 3X3 Boolean matrix connectives in QPL were

discussed and, in particular, the anologues of "and" and "or" in

classical propositional calculus and SNS logic were defined. These

matrix connectives relate one EPL state for one variable x (e.g.

$(\forall x)$ $(\underline{a}(x))$ with another EPL state in a variable y (e.g. $(\exists y)$ $(\underline{b}(y)$

Such a connective ($\mathbb{Z}$, in general) is a pure EPL connective and

there are 8X8 = 64 of the type $\mathbb{A}$ ("and") and 64 of the type $\mathbb{O}$ ("or")

An example of the application of these connectives in the form of

a practical problem was given in Section 3 of Part II. In general,

the formalism developed in Part II deals with, what may be termed,

EPL sentences of Type 2, which have the form of Eq.(1) for a

unary relation.

$$(\mathcal{Q}_1 x) (\underline{a}(x)) \overset{\mathbb{Z}}{\cancel{\mathbb{Z}}} = (\mathcal{Q}_2 y) (\underline{b}(y)) \tag{1}$$

On making further studies on the subject and trying to prove the well-known results in predicate logic, such as the different figures and moods of traditional logic, it was found that these relations had a different syntax from the ones considered in Parts I and II. They are all expressed for a single variable (x) only, and connect different statements inside the quantifier (with one another). For example,

$$(\forall x)\ (\underline{b}(x) \Longrightarrow \underline{c}(x))\ ,\ (\forall x)\ (\underline{a}(x) \Longrightarrow \underline{b}(x))$$
$$\equiv (\forall x)\ (\underline{a}(x) \Longrightarrow \underline{c}(x))\ \ (\text{Barbara}) \tag{2a}$$

$$(\forall x)\ (\underline{b}(x) \Longrightarrow \underline{c}(x))\ ,\ (\exists x)\ (\underline{a}(x)\ \&\ \underline{b}(x)$$
$$\equiv (\exists x)\ (\underline{a}(x)\ \&\ \underline{c}(x))\ \ (\text{Darii}) \tag{2b}$$

In these, $(\forall x)\ (\underline{a}(x) \Longrightarrow \underline{b}(x))$ stands for *All $\underline{a}$ are b" and $(\exists x)\ (\underline{b}(x)\ \&\ \underline{c}(x))$ stands for "Some $\underline{a}$ are $\underline{b}$" or "There exist $\underline{a}$'s that are $\underline{b}$", which employ the two quantifiers $(\forall x)$ and $(\exists x)$ respectively. The general form of this equation which represent all equations of this type may be expressed as below, in (2c) or (2d),

$$(\mathcal{Q}_1 x)\ (\underline{a}\ \mathfrak{z}_1 = \underline{b}),\ (\mathcal{Q}_2 x)\ (\underline{b}\ \mathfrak{z}_2 = \underline{c}) \equiv (\mathcal{Q}_3 x)\ (\underline{a}\ \mathfrak{z}_3 = \underline{c}) \tag{2c}$$

$\longrightarrow$

Equations of this type, in which only SNS connectives occur, and all quantifiers are for the same variable (x say) will be named "Type 1 equations" of QPL, or EPL. The rules governing the

$$(\mathcal{Q}_1 x)\ (\underline{\delta}_1),\ (\mathcal{Q}_2 x)\ (\underline{\delta}_2(x)) \equiv (\mathcal{Q}_3 x)\ (\underline{\delta}_3) \tag{2d}$$

formation of the right hand side in equations of this type were

examined and the main results that were obtained are given

below in this paper. In Sections 2 and 3, we shall first

consider generally equations of Type 1, in which z_1, z_2, z_3

are all SNS connectives (particularly implications) and the nature

of q_3 and the nature of the connective z_3 are considered

(given q_1, q_2, z_1, z_2). The relevance of these to the

well-known results in the list of equations of the type 1 in

traditional logic are then briefly considered, and some examples

are given to indicate their nature and validity.

The study was made of the Boolean algebraic rules that

have to be applied to get the output sentence which is the

resultant of two input sentences as found in categorical

syllogisms. It was possible to show that valid conclusions in 18, out of the 19 that

are listed in books, could be derived as a result amt of succession ve

implications. 18 more were obtained in which the implication

in the conclusion starts from $\neg\underline{s}$ towards $\underline{p}$ or $\neg\underline{p}$.

This is a very interesting consequence and it is discussed

particularly in Section 5.

On the other hand, quantified sentential relations

(named Type 2 QPL equations) for two variables, of the type

considered in Parts I and II — namely

$$(\mathcal{Q}_1 x)\ (\underline{a}(x))\ \mathbb{Z}\ (\mathcal{Q}_2 y)\ (\underline{b}(y))\ \equiv\ \underline{c} \tag{3}$$

lead to an entirely different algebra, as described therein.

As an illustration, the interpretation of the quantifier $(\forall x)$

as "for all (collectively)" and "for each and every

(individually)" are not exactly equivalent and may have

different consequences in the types of equations discussed in

Part II (e.g. Eq (3)), and the type of equation with one variable

throughout as shown in Eq (1). Similarly, the use of $(\exists x)$ in

connecting two QPL sentences has also different possibilities,

such as its being transferable (transitive) for Type 2 equations,

and/its leading to no consequences in the succeeding step —

[the possibility of]

i.e. to the QPL state (Δ x) — for Type 1 equations.

In the succeeding sections of this paper, several formulae,

or algorithms, are mentioned which will carry out a whole set

of logical operations in QPL and/or EPL. It is not easy to

give formal proofs for all these, because they have been arrived

at as the extracted information from the logic that can be

intuitively formulated from all the particular examples for

which they are operative. In this sense, the method of

approach is that of formulating a physical theory, where a number

of phenomena are ~~discussed~~ *dissected* and examined and the common substratum

behind all of them is formulated as a physical law.

After these studies are completed, it may be possible to give

an axiomatic basis for all the formulations that have been made

in connection with QPL and EPL. However, this is passed over

in this paper and ~~the~~ *an* attempt is to derive empirical formulae

which are simple, but which are valid over a wide range of

circumstances. This should be remembered while checking the

correctness and understanding the contents of the formulae

given here.

2. Traditional Categorical Syllogisms in our Formalism

A brief outline of the list of categorical syllogisms

of traditional logic, in various figures and moods, is given

in the article by A. Church in Encyclopedia Britannica [3], p.216.

We shall follow this presentation in our discussion, as it is

elegant and is written in a format that is immediately adaptable

to the form of a QPL statement as in Eqns (1a), (1b). On examining

the 19 syllogisms listed by Church, it is found that the only

types of QPL statements that occur are

$$(\forall x) (\underline{a} \Longrightarrow \underline{b}) \ , \quad (\forall x) (\underline{a} \Longrightarrow \neg \underline{b}) \qquad (4a, b)$$

$$(\exists x) (\underline{a} \ \& \ \underline{b}) \ , \quad (\exists x) (\underline{a} \ \& \ \neg \underline{b}) \qquad (4c, d)$$

In the QPL terms 4(a to d), and also in what follows in this

section, we shall put the variable (x) associated only with the

quantifier (as in ($\mathcal{Q}_1 x$), ($\forall x$) etc) and omit it in the SNS

sentence that is quantified. Thus, it is understood that $\underline{a}$ stands

for $\underline{a}(x)$ and $\underline{b}$ stands for $\underline{b}(x)$ etc, and the same variable

x associated with the quantifier occurs also for the SNS term.

Also, in all derivations of $\underline{b}$ from $\underline{a}$, given the truth of the

connective relation $\underline{a} \ \underline{Z} \ \underline{b}$, this is obtained from the unary form

of the logical relation $\underline{a} \ \underline{Z} = \underline{b}$. This follows from the discussion

in Part I [1] from which it is readily seen that,

$$\underline{a} \ \underline{Z} \ \underline{b} = \underline{c} \ , \quad \underline{c} = T \ \longmapsto \ \underline{a} \ \underline{Z} = \underline{b} \qquad (5)$$

Therefore, in considering syllogisms having the major premise,

the minor premise and the conclusion, we need consider only the

most general expression given by Eq (2), from which all particular

cases of Type 1 equations can be derived by giving the nature

of the QPL connectives $\mathcal{O}\!\!/$ and the SNS connectives $\underset{\sim}{Z}$.

In this section, we shall consider first the four standard

states of QPL namely, $\forall$, Φ , $\exists$, Λ and discuss the way they

combine for a general equivalence of the type of Eq (2). More

particularly given the logical states ($\mathcal{O}\!\!/_1 x$), ($\mathcal{O}\!\!/_2 x$) and $\underline{\underline{a}}$

we wish to derive the quantifier ($\mathcal{O}\!\!/_3 x$) and the truth value of $\underline{c}$.

We give below an algorithm for doing this. The proof is omitted

as the algorithm is derived as a condensation of the logical

properties of the terms and connectives employed in this equation

and it is readily verified that the algorithm works, and is in

accord with what is intuitively deduced in each case.

(a) Use only of the quantifier states $\forall$ and $\exists$ for QPL.

As shown in Table 4 of Part I, the four standard quantifier

states can all be expressed in terms of the one quantifier state

$\forall$ alone provided two "not's" ($\overline{\neg}$ and $\neg$) are used in addition,

in front of it, or after it. [see also [4].] We shall not employ $\overline{\neg}$ before the

quantifier, as it does not lead to a general elegant formula.

Then, an examination of Table 6 of Part I [1] shows that the four

quantifier states of standard QPL can be expressed in terms of

$\forall$ and $\exists$ as follows.

$$\forall = (\forall x) \, (\underline{s}(x)) \; ; \quad \exists = (\exists x) \, (\underline{s}(x)) \; ; \tag{6a}$$

$$\Phi = (\forall x) \, (\neg \underline{s}(x)) \; ; \quad \wedge = (\exists x) \, (\neg \underline{s}(x)) \tag{6b}$$

Hence we need consider only the two quantifier states $(\forall x)$ and

$(\exists x)$ and associate with them statements $(\underline{s}(x))$ and $(\neg \underline{s}(x))$

which are either affirmative or negative. We shall now work out

the expressions for $(\mathcal{Q}_3 x)$ and $\mathcal{Z}_3$ given $(\mathcal{Q}_1 x)$, $(\mathcal{Q}_2 x)$,

$\mathcal{Z}_1$ and $\mathcal{Z}_2$, in Eq (2c).

We shall do this, particularly for the type of statements

that occur in categorical syllogism, as in Eqns (4a,b,c,d),

which stand, respectively for

4a $\forall(\underline{s})$: All $\underline{a}$ are $\underline{b}$, 4b $\forall(\neg\underline{s})$: All $\underline{a}$ are not-$\underline{b}$ (7a

4c $\exists(\underline{s})$: Some $\underline{a}$ are $\underline{b}$, 4d $\exists(\neg\underline{s})$: Some $\underline{a}$ are not-$\underline{b}$ (7c

For these, if $\underline{s}(x)$ stands for "$\underline{a}(x)$ is $\underline{b}(x)$", then $\neg\underline{s}(x)$

stands for "$\underline{a}(x)$ is not-$\underline{b}(x)$". Therefore, we need only use the

small set of four entries given in Table 1 below to get the

quantifier $\mathcal{Q}_3$ given $\mathcal{Q}_1$ and $\mathcal{Q}_2$ as $\forall$ or $\exists$ in Eq.(2c) and (·d).

This is carried out by using a new type of QPL connective, for

which we have given the name "into" (see the sections below for

its different applications).

Table 1. Short table for the connective "into" ($\bigotimes$) for
use in QPL.

The explanation of the entries for $\mathcal{Q}_3$ in Table 1 is obvious.

When both $\mathcal{Q}_1$ and $\mathcal{Q}_2$ are $(\forall x)$, the sentences $\underline{s}_1$ and $\underline{s}_2$

quantified by them are valid "for all x" and therefore the

resultant sentence is also valid "for all x" and hence $\mathcal{Q}_3 = (\forall$;

On the other hand, if one of the two sentences $\underline{s}_1$ or $\underline{s}_2$ is

Table 1. Short table for the connective
"into"($\otimes$) for use in QPL[*]

$\mathcal{Q}_1$	$\mathcal{Q}_2$	$\mathcal{Q}_3$
$\forall$	$\forall$	$\forall$
$\forall$	$\exists$	$\exists$
$\exists$	$\forall$	$\exists$
$\exists$	$\exists$	$\triangle$

[*]The quantifier state $\mathcal{Q}_3$ is as in
Eq (2), when $\mathcal{Q}_1$ and $\mathcal{Q}_2$ are given.

valid for ($\forall$ x), while the other is valid only for the state

($\exists$ x), then, when the output of the first sentence, for which

it is valid for ($\forall$ x), enters as input into the second sentence,

valid only for ($\exists$ x), then it makes the resultant output of the

second valid only for ($\exists$ x). Similarly, if the first sentence

has the quantifier ($\exists$ x) and the second ($\forall$ x), the ($\exists$ x) in the

output of the first one restricts the output of the second to be

only ($\exists$ x). It is to be noted that this behaviour is quite

different from the situations considered in Part II, where the

two sentences that are combined are for different variables x and y.

On the other hand, when the first sentence gives an output

$\underline{b}$(x) for ($\exists$ x) and this is fed into the second sentence which

also has the quantifier ($\exists$ x) then there is no reason why the

value of x for which 'there exist' data in the output of the

first sentence and the values of x for which the data are taken

in the second sentence need overlap. The two states may be

disjoint, in which case the quantifier of $\underline{s}$ becomes

Φ("for none" = (0 0 1)). On the other hand, if it happens

that they are overlap, then $\underline{s}_3$ will have the quantifier $(\exists x)$

("there exists" = (1 1 0)). Since both possibilities can

happen, we get the state Δ = (1 1 1) for q_3 in the fourth

entry in Table 1. It is to be noted that in the case of two

different variables x and y (in sentences of Type 2 considered

in Part II) the quantifier of the output of the first sentence,

namely 'there exists', is the same as the quantifier of the input

in the next equation, and therefore, even if $\underline{b}(x)$ is true for one

value of x, then it can be input into next equation, under the

quantifier state $\exists$.

(b) Application of the formulae in Section 2(a) in particular
      examples.

While the general formulation given in Section 2(a) for

QPL is readily seen to be valid quite generally for all $\underset{\sim}{Z}$'s,

since this report deals essentially with traditional categorical

syllogisms, we shall illustrate a few of these and derive them

from the general formulae given here. Thus, we have already

seen two examples in Section 1 — namely Eq 1(a) and Eq 1(b) —

which refer to the moods <u>Barbara</u> and <u>Darii</u> in the **first figure**.

The first of these, namely Barbara, is immediately obvious and

corresponds to the first entry in Table 1. Similarly, the second

example, viz. Eq 1(b), corresponding to the mood <u>Darii</u>, follows

from the third row in Table 1. Writing it in our notation, we have,

$$(\forall x)\ (\underline{b}\ \underline{\underline{I}} = \underline{c})\ ,\quad (\exists x)\ (\underline{\underline{a}}\ \underline{A} = \underline{b}) \tag{8}$$

gives, in so far as the quantifier is concerned, the resultant state

$$(\exists)\ \oplus\ (\forall)\ =\ (\exists) \tag{9a}$$

For the statement inside the bracket quantified by $(\exists x)$, we have

the SNS result $\underline{\underline{a}}\ \underline{A}\ \underline{\underline{I}} = \underline{c}$. Putting in the matrices for the SNS

connectives $\underline{A}$ and $\underline{\underline{I}}$, we have,

$$|A|\,I| = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = |A| \tag{9b}$$

Hence, combining the r.h.s. of (9a) and (9b), we have the resultant

of the two quantified terms in (8) as in Eq (1b), namely

$$(\exists x)\ (\underline{\underline{a}}\ \underline{A} = \underline{c}) \tag{9c}$$

We shall give two more examples of traditional syllogisms,

and prove them from our theory. Thus, in the second figure we

have <u>Cesare</u> which has the following equations.

$$(\forall x)\,(\underline{a}(x) \Longrightarrow \underline{b}(x))\ ,\quad (\forall x)\,(\underline{c}(x) \Longrightarrow \neg\,\underline{b}(x) \qquad\qquad (10\ a,b)$$

it follows that

$$(\forall x)\,(\underline{c}(x) \Longrightarrow \neg\,\underline{a}(x)) \qquad\qquad (10c)$$

The critical step in proving this is to write,

$$(\underline{a}(x) \Longrightarrow \underline{b}(x)) \equiv (\underline{a}\ \underline{I} = \underline{b}) \qquad\qquad (11a)$$

and convert it into its equivalent contrapositive form

$$(\neg\,\underline{b})\ \underline{I} = (\neg\,\underline{a}) \qquad\qquad (11b)$$

This follows, since $(\underline{a}\ \underline{I} = \underline{b})$, $\underline{b}\ \underline{I}^{t} = \underline{a}$ and $\underline{I}^{t} = \underline{J} = \underline{N}\ \underline{I}\ \underline{N}$.

Thus, (10 a, b) are reduced to two implications as in (11c),

and the output of the first is the input of the next:

$$(\forall x)\,(\underline{c} \Longrightarrow \neg\,\underline{b})\ ,\quad (\forall x)\,(\neg\,\underline{b} \Longrightarrow \neg\,\underline{a}) \qquad\qquad (11c,\ d)$$

Hence, the net result, on combining (11c) and (11d) is (10c),

as required.

We shall now consider an example of the third figure, namely
Darapti which is given in (18), following Church's way [3] of

writing this syllogisms. Thus, given (12 a & b),

$$(\forall x)\,(\underline{a} \Longrightarrow \underline{b})\ ,\quad (\forall x)\,(\underline{a} \Longrightarrow \underline{c}) \qquad\qquad (12a,\ b)$$

it follows that
$$(\exists x)\,(\underline{a}) \longmapsto (\exists x)\,(\underline{b}\ \&\ \underline{c}) \qquad\qquad (12c)$$

The important part of this is to input the information $(\exists x)\,(\underline{a})$

in Eq (12a) and Eq (12b) and to deduce therefrom that $(\exists x)\,(\underline{b})$

$^{\backslash\backslash}$and$^{\prime\prime}$($\exists$x) (c) are true. These are derived by using the third

line of Table 1 in (12a) and (12b). Hence it follows that the

expression in (12ℰ), namely ($\exists$x) (b & c), is true.

We have followed the standard representation of the two

quantified statements, such as "All students are intelligent" and

"There exist students who are intelligent", by the symbols

($\forall$x) (a(x) $\implies$ b(x)) and ($\exists$x) (a(x) & b(x)), where a

stands for the property of being a student and b stands for

the property of being intelligent. x represents the variable

denoting that the entity considered belongs to the collection

of persons for which the above quantified statements are made.

In what has been considered above, we have followed this

terminology in our notation of QPL and SNS and worked out the

consequences for some syllagisms. It should be noted however

that a & b , in which the connective $\wedge$ is represented by

the 2x2 matrix $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$, is much stronger than a $\implies$ b for which

the connective $\underset{\sim}{I}$ is represented by the matrix $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$. If

we follow the interpretation of these matrices as we have

proved generally from the theory of relations and for SNS

logic in particular, then, in both cases, when $\underset{=}{a}$ is true $\underset{=}{b}$ is true "

and " $\underset{=}{b}$ cannot be false when $\underset{=}{a}$ is true." However, what happens

when $\underset{=}{a}$ is false is entirely different in the two cases. In the

case of $\underset{\sim}{I}$ (implies), when $\underset{=}{a}$ is F, there is no information

content for the output, and $\underset{=}{b}$ can be either true or false, or whatever it was

before the application of the information contained in the

statement involving $\underset{\sim}{I}$. On the other hand, if $\underset{=}{a} \underset{\sim}{A} = \underset{=}{b}$ is

used, then $\underset{=}{a}$ itself cannot be negative, and there is a contradiction

if $\underset{=}{a}$ is false is applied. In other words, even at the stage

of saying $\underset{=}{a}$ is true, we say that $\underset{=}{a}$ cannot be false (the full

implication of this consequence has to be considered in great

detail) and the application of $\underset{\sim}{A}$ to statements of this type

cannot be readily substantiated. On the other hand, the

objection to writing $\underset{=}{a} \Longrightarrow \underset{=}{b}$ under the extential quantifier $(\exists x)$

that is commonly made (see [5]) — namely that a false statement

can imply any statement whatsoever — is not relevant in our

formalism, since a false statement implies no statement in our

formalism (and not any statement whatsoever).

In view of this, we shall consider the reformulation of all

standard syllogisms in traditional logic in terms of the following

symbolic statements in QPL, using only the implication symbol $\Rightarrow$:

$$(\forall x) \ (\underline{a} \Rightarrow \underline{b}) \ \equiv \ \text{"All } \underline{a} \text{ are } \underline{b}\text{"} \qquad (13a)$$

and

$$(\exists x) \ (\underline{a} \Rightarrow \underline{b}) \ \equiv \ \text{"Some } \underline{a} \text{ are } \underline{b}\text{"} \qquad (13b)$$

The negations of these implications are

$$(\forall x) \ (\underline{a} \Rightarrow \neg \underline{b}) \ \equiv \ \text{"All } \underline{a} \text{ are not-}\underline{b}\text{"} \qquad (13c)$$

$$(\exists x) \ (\underline{a} \Rightarrow \neg \underline{b}) \ \equiv \ \text{"Some } \underline{a} \text{ are not-}\underline{b}\text{"} \qquad (13d)$$

The reverses of these, from $\underline{b}$ to $\underline{a}$, are readily verified

from Venn-diagrams and are as follows:

$$(\forall x)\,(\underline{a} \Rightarrow \underline{b}) \equiv (\forall x)\,(\neg\,\underline{b} \Rightarrow \neg\,\underline{a})\;;$$

$$(\forall x)\,(\underline{a} \Rightarrow \neg\,\underline{b}) \equiv (\forall x)\,(\underline{b} \Rightarrow \neg\,\underline{a}) \tag{13 e, f}$$

$$(\exists x)\,(\underline{a} \Rightarrow \underline{b}) \equiv (\exists x)\,(\underline{b} \Rightarrow \underline{a})\;;$$

$$(\exists x)\,(\underline{a} \Rightarrow \neg\,\underline{b}) \equiv (\exists x)\,(\neg\,\underline{b} \Rightarrow \underline{a}) \tag{13 , h}$$

The second set (13g, h) is particularly to be noted — namely

that its nature is different from the former two (13e, f). This

has profound consequences for the structure of predicate logic,

made of sentences of type 1.

(c) Nature of the connective "into" for all pairs of quantifiers
    in LPL

We shall illustrate the nature of the connective "into"

in QPL, by taking a simpler example than Eq (2). Consider the

following

$$\text{Input : } (\,\mathcal{Q}\!/_{1}x)\,(\underline{a}') \tag{14a}$$

$$\text{Into equation : } (\,\mathcal{Q}\!/_{2}x)\,(\underline{a}\;\underline{Z} = \underline{b}) \tag{14b}$$

$$\text{Output : } (\,\mathcal{Q}\!/_{3}x)\,(\underline{b}'),\ \text{where } \underline{a}'\;\underline{Z}' = \underline{b}' \tag{14c}$$

Then, on inputting a different quantifier $(\mathcal{Q}_2 x)$ for $\underline{a}(x)$

into (13a), we obtain (using the same considerations as in

Section 2(b) the output quantifier to be

$$(\mathcal{Q}_1 x) \; \bigotimes \; (\mathcal{Q}_2 x) \; = \; (\mathcal{Q}_3 x) \tag{14d}$$

Since (14) is a very general rule for $\mathcal{Q}_3$, for equations

as in (13), we shall give a general table for the effect of the

operator "into" on $\mathcal{Q}_1$ when introduced into $\mathcal{Q}_2$ (Table 2).

Table 2. Multiplication table for the connective "into" ($\bigotimes$)
when $\mathcal{Q}_1$ and $\mathcal{Q}_2$ are given.

The extension of this table to the cases when either $\mathcal{Q}_1$, or

$\mathcal{Q}_2$, or both, are one of the four non-standard states of EPL,

namely Σ, Θ, Δ, $\emptyset$, is not very obvious and we shall discuss

this in the next Section 3. We give below, in Table 2(b), a

listing of the sixteen possible values of $\mathcal{Q}_1 \bigotimes \mathcal{Q}_2$ when

$\mathcal{Q}_1$ and $\mathcal{Q}_2$ take the four possible QPL states $\forall$, $\exists$, Φ, Λ, the

Table 2: Multiplication table for the connective "into" (⊗) when Q_1 and Q_2 are given

(a) Q_3 and Z', given Q_1, Q_2, Z, when Q_1 and Q_2 are both one of the four standard quantifiers in QPL

Q_1 \ Q_2	∀ Q_3	∀ Z'	∃ Q_3	∃ Z'	Φ Q_3	Φ Z'	Λ Q_3	Λ Z'
∀	∀	E	∃	E	Φ	N	Λ	N
∃	∃	E	Δ	E*	Λ	N	Δ	N+
Φ	∀	N	∃	N	Φ	E	Λ	E
Λ	∃	N	Δ	N+	Λ	E	Δ	E*

*May also be N   ;   +May also be E .

(b) Q_3 in the canonical form when Q_1 and Q_2 are also given in the canonical form for the same four states of QPL as in (a) above.*

Q_1 \ Q_2	∀	∃	Φ	Λ
∀	∀	∃	∀	∃
∃	∃	Δ	∃	Δ
Φ	Φ	Λ	Φ	Λ
Λ	Λ	Δ	Λ	Δ

*Z' is omitted as it is E for all entries.

The portion marked out in the box is the essential minimum, on which this table is based, and which is given in Table 1.

values of $\mathcal{V}_3$ being also given in the QPL form. This leads

to a great convenience for it turns out that the operator $\underset{\sim}{Z}{}'$

in Table 2(a) can always be made equal to $\underline{E}$, and consequently the

statement that is quantified, namely $(\underline{a} \underset{\sim}{Z} = \underline{b})$ is unaltered.

This Table 2(b) will therefore be very useful in working out

problems in QPL. It is to be noted, however, that sometimes the

output for $\mathcal{V}_3$ could be one of the non-QPL states, namely $\triangle$ of

EPL. In such a case, the resultant statement has no informational

value and the corresponding syllogism will not have the third

component and is therefore considered to be invalid. It is very

interesting that traditional logic has fully comprehended the

complete vagueness of the SNS statement when an input with $\mathcal{V}_1 = \mathbf{\exists}$,

is introduced into a $\mathcal{V}_2 = \mathbf{\exists}$. Consequently there is no example

of this type in the list of 19 syllogisms found in traditional

logic (see $[6]$).

3. Theory of the new Connective "into" ($\otimes$) in EPL

(a) Effect of $\otimes$ applied between two basic states.

Tables 1 and 2 cover all cases that would be met with,

in standard QPL, of the connective "into". The extension of

those to the 8X8 = 64 possibilities of $\mathcal{V}_1 \otimes \mathcal{V}_2$, in EPL, is

not difficult. We first find out the resultant, $\mathcal{V}_3$, of

$\mathcal{V}_1 \otimes \mathcal{V}_2$, for the nine cases which involve the three basic

states, each of $\mathcal{V}_1$ and $\mathcal{V}_2$, and then employ these for the

remaining cases by using the distributive laws

$$ \mathcal{V}_i \otimes (\mathcal{V}_j \oplus \mathcal{V}_k) = (\mathcal{V}_i \otimes \mathcal{V}_j) \oplus (\mathcal{V}_i \otimes \mathcal{V}_k) \quad (15a) $$

and

$$ (\mathcal{V}_j \oplus \mathcal{V}_k) \otimes \mathcal{V}_\ell = (\mathcal{V}_j \otimes \mathcal{V}_\ell) \oplus (\mathcal{V}_k \otimes \mathcal{V}_\ell) \quad (15b) $$

The results of the operation $\otimes$ from $\mathcal{V}_1$ into $\mathcal{V}_2$ can

be listed as follows:

$$ \forall \otimes \forall = \forall , \quad \Phi \otimes \Phi = \Phi , \quad \Sigma \otimes \Sigma = \triangle \quad (16\ a,b,c) $$

$$ \forall \otimes \Phi = \forall , \quad \Phi \otimes \forall = \Phi , \quad (16\ d.e) $$

$$ \Sigma \otimes \forall = \forall \otimes \Sigma = \Sigma , \quad \Sigma \otimes \Phi = \Phi \otimes \Sigma = \Sigma \quad (16\ f,g,h,k) $$

Of these, (16a), (16b), (16d) and (16e) are taken from Table 2(b),

where they have been listed from intuition. The derivation of

(16 f to k) is obtained also from this table as follows. For

$\Sigma \otimes \forall$, we start from $\exists \otimes \forall = \exists$ in Table 2(b). Writing $\exists$

as $(\forall \oplus \Sigma)$, we have

$$(\forall \oplus \Sigma) \otimes \forall = (\forall \otimes \forall) + (\Sigma \otimes \forall) = \forall \oplus \Sigma \qquad (17)$$

Since $\forall \otimes \forall = \forall$ from (16a), we obtain (16f) from (17). The

products in (16 g,h,k) similarly follow.

We are now left with (16c), namely $\Sigma \otimes \Sigma = \overset{\Delta}{\Phi}$. For this,

we use $\exists \otimes \exists = \Delta$ of Table 2(b). Expanding both sides in

terms of the basic states $\forall$ and Σ, we have

$$\exists \otimes \exists = (\forall \oplus \Sigma) \otimes (\forall \oplus \Sigma)$$

$$= (\forall \otimes \forall) \oplus (\forall \otimes \Sigma) \oplus (\Sigma \otimes \forall) \oplus (\Sigma \otimes \Sigma)$$

$$= \forall \oplus \Sigma \oplus \Sigma + (\Sigma \otimes \Sigma) = \Delta = \forall + \Sigma + \Phi \qquad (18)$$

This shows that Φ is one of the components of $(\Sigma \otimes \Sigma)$.

Similarly, by expanding $\Lambda \otimes \Lambda = \Delta$, we find that $\Sigma \otimes \Sigma$

contains $\forall$. Obviously, "some" "into" "some" can be "some"

itself, so that $\Sigma \otimes \Sigma$ contains Σ also. Hence,

$$\Sigma \otimes \Sigma = \forall + \Phi + \Sigma = \Delta = (1 \quad 1 \quad 1) \tag{19}$$

$\big[$ As a check, we can work back $\dashv \otimes \dashv$, and the three related

products, and we find that they are all equal to Δ , as listed

in Table 2(b) $\big]$.

For ready reference, Table 3 gives the products obtained

via the "into" operator ~~for~~ *from one of* (the three basic EPL states ~~with one~~ *to*

another.

Table 3:   Multiplication table of $q_1 \otimes q_2$ for the three
basic states.

(b) Effect of "into" applied to all eight *(quantifier)* states of EPL

Table 4 giving this was obtained by expanding each term

as the Boolean sum ($\oplus$) of basic states, and using (15a and b)

and Table 3. *All entries are in the canonical form.*

Table 3: Multiplication table of $q_1 \oplus q_2$ for the three basic states

q_1 \ q_2	∀	Σ	Φ
∀	∀	Σ	∀
Σ	Σ	Δ	Σ
Φ	Φ	Σ	Φ

Table 4: 8 X 8 multiplication table for the connective "into"

q_1 \ q_2	∀	∃	Φ	Λ	Σ	Θ	Δ	φ
∀	∀	∃	∀	∃	Σ	∀	Δ	φ
∃	∃	Δ	∃	Δ	Δ	Δ	Δ	φ
Φ	Φ	Λ	Φ	Λ	Σ	Φ	Δ	φ
Λ	Λ	Δ	Λ	Δ	Δ	Δ	Δ	φ
Σ	Σ	Δ	Σ	Δ	Δ	Σ	Δ	φ
Θ	Θ	Δ	Θ	Δ	Σ	Θ	Δ	φ
Δ	Δ	Δ	Δ	Δ	Δ	Δ	Δ	φ
φ	φ	φ	φ	φ	φ	φ	φ	φ

(c) Examples of syllogisms in Traditional Logic

(i) Baroco: We shall illustrate the use of Table 2(b) in deriving the syllogisms, by first considering the mood Baroco of the second figure in Traditional Logic. The syllogism reads as follows (as per Ref [3]) (see also Section 2 above for the notation in our formalism which we use).

$$(\forall x)\ (\underline{p}(x) \Rightarrow \underline{m}(x))\ ,\quad (\exists x)\ (\underline{s}(x) \Rightarrow \neg\,\underline{m}(x)) \qquad (20a,\ b)$$

Hence
$$(\exists x)\ (\underline{s}(x) \Rightarrow \neg\,\underline{p}(x)) \qquad (20c)$$

Firstly, from SNS, using $(\underline{a} \Rightarrow \underline{b}) \equiv (\neg\,\underline{b} \Rightarrow \neg\,\underline{a})$, we can rewrite (20a) as

$$(\forall x)\ (\neg\,\underline{m}(x) \Rightarrow \neg\,\underline{p}(x)) \qquad (20d)$$

Then, the SNS output of (20b) is the input in (20d). Also, the quantifier of the output of (20b), namely $\exists$, going "into" the quantifier $\forall$ of (20d), yields $\exists \otimes \forall = \exists$. Hence, (20b) and (20d) together yield a statement having the quantifier $\exists$, and the SNS statement $\underline{s} \Rightarrow \neg\,\underline{p}$, which yields the required result (20c).

(ii) Darii: The example _Darii_, of the first figure, rewritten

in our formalism, is as follows:

$$(\forall x) \ (\underline{m}(x) \Longrightarrow \underline{p}(x)) \ \& \ (\exists x) \ (\underline{s}(x) \Longrightarrow \underline{m}(x) \qquad (21 \ a,b)$$

$$\equiv (\exists x) \ (\underline{s}(x) \Longrightarrow \underline{p}(x)) \qquad\qquad (21c)$$

The proof follows in a straightforward manner by applying

(21b) into (21a), when the resultant quantifier is $\exists \oplus \forall = \exists$

and the resultant SNS connective is $\underset{\sim}{I} \ \underset{\sim}{I} = \underset{\sim}{I}$, thus yielding

(21c).

(iii) Darapti: In the example _Darapti_, considered in Section 2(b),

the conclusion is given as $(\exists x) \ (\underline{b} \ \& \ \underline{c})$. However, this

implies $(\exists x) \ (\underline{b} \Longrightarrow \underline{c})$, so that the syllogism can also be

written, in our formalism, in the form

$$(\forall x) \ (\underline{m}(x) \Longrightarrow \underline{p}(x)) \ , \ (\forall x) \ (\underline{m}(x) \Longrightarrow \underline{s}(x)), \ (\exists x) \ (\underline{m}(x)) \ (22a,b,c]$$

$$\equiv (\exists x) \ (\underline{s}(x) \Longrightarrow \underline{p}(x)) \qquad\qquad (22d)$$

In fact, we can go further, and write, as the consequence of

(22a, b, c), the result

$$(\exists x)\ (\underline{p}(x) \implies \underline{s}(x)), \text{ or } (\exists x)\ (\underline{s}(x) \impliedby \underline{p}(x)) \tag{22e}$$

and, it also follows, from (22d) and (22e), that (22a, b) lead to

$$(\exists x)\ (\underline{p}(x) \iff \underline{s}(x)) \tag{22f}$$

It appears that (22f) is the most powerful conclusion that can

be derived from (22a,b,c) (see section 4(b) and Eq (28)).

(iv) Bamalip: This example, of the fourth figure, is interesting

since the connective "into" is applied twice. It reads as

follows (as per Ref [3]):

$$(\forall x)\ (\underline{p}(x) \implies \underline{m}(x)),\ (\forall x)\ (\underline{m}(x) \implies \underline{s}(x)),\ (\exists x)\ (\underline{p}(x)) \tag{23a,b,c}$$

lead to

$$(\exists x)\ (\underline{p}(x) \implies \underline{s}(x)) \tag{23d}$$

Taking (23c) and applying its quantifier "into" that of (23a),

we get the output $(\exists x)\ (\underline{m}(x)$. Applying this once again "into"

(23b), we get the output $(\exists x)\ (\underline{s}(x))$. Thus, we obtain effectively

the statement (23d). The resultant (23d) is not the most powerful

one to be obtained from (23a, b) (see section 5).

It has been possible to check that, with the notation adopted by us, all the examples of the classical syllogisms given in Ref[3] could be proved simply, as indicated for the small number of examples shown above. On the other hand, since each syllogism has three members in it, each of which can have four forms, and two of them are inputs, there can be 16 possible syllogisms for each of the four figures, leading to 64 possible examples, as mentioned in Ref [6] page 20, 21. Of these, only 19 represent valid arguments according to the classical Aristotelian framework. It would be worthwhile to consider why most of them are invalid. It can arise from one of the two possibilities.

(a) The connection of the quantifiers in the first two members might lead, via the connective "into" ($\otimes$), to the "indefinite" state Δ.

or (b) The combination of the implications may be such that

the output of the first statement is the negation of the

input of the second statement, so that the resultant of the SNS

implication of the two statements will together lead to the

"doubtful" state, D, of SNS. In either case, the resulting

statement in the conclusion has no informational value and

hence, the first two statements do not lead to any useful

consequence and the syllogism is said to be invalid. The check

with these two criteria on the 64 examples of syllogism is being

made and will be reported below. It appears that this test

should reveal any other possible cause for an indefinite or

doubtful output that could be present in the syllogistic argument

composed of three quantified statements. (Added later: one was

found namely that the property $(\forall x)\ (\underline{a} \Rightarrow \underline{b}) \supset (\exists x)\ (\underline{a} \Rightarrow \underline{b})$

has to be used sometimes, see Section 4(a)).

(Note: Since this is the first draft and the further studies

are being made as the report is being written, we will not

revise any of the earlier sections according to what will be

revealed in these further studies. On the other hand, the new

results will be described in full, and their consequences to

any of the previous sections will be mentioned and clarified

as we go along.)

4. Analysis of all possible 64 combinations in categorical syllogisms

The categorical syllogism has essentially the structure

of Eq (2a) with the second term being the major premise and

the first term being the minor premise, and these two together

leading to the conclusion on the right hand side of Eq (2a).

Considering the nature of the quantifier $\mathcal{Q}_1$, $\mathcal{Q}_2$, $\mathcal{Q}_3$, as

well as of the SNS operators z_1, z_2, z_3 , we restrict ourselves

to $\mathcal{Q}_1$, $\mathcal{Q}_2$, $\mathcal{Q}_3$ being either $\forall$ or $\exists$. In the affirmative

form of the statements, e.g. "All men are mortal", we take z

to be the SNS connective I (as in $a \Longrightarrow b$) and the negational

sentence, e.g. "All men are not-four-legged" will be represented

by using the SNS connective $I\,N$ (as in $a \Longrightarrow \neg b$). This is

in the spirit in which the syllogisms have been symbolized by

Church in Ref. [3] . Thus, we do not use the states in which the

SNS sentence quantified by $\forall$ or $\exists$ is negated. Instead, the

negated form of "a imply b" is taken to be "a imply $\neg\ b$".

We have used this notation in all the previous sections, only

mentioning briefly that the negation of a categorical statement

puts the negation on the right hand side of the implication.

Since the four types of categorical statements do not correspond

to the canonical quantifiers $\forall$, $\cancel{\exists}$, $\exists$, $\wedge$, we shall use the

symbolism employed in the literature for these four types of

statements, namely $\underline{A}$, $\underline{E}$, $\underline{I}$, $\underline{O}$. They stand for the following:

$$\underline{A}: \quad \text{All } \underline{a} \text{ is } \underline{b} \equiv (\forall x)\ (\underline{a} \Rightarrow \underline{b}) \tag{24a}$$

$$\underline{E}: \quad \text{All } \underline{a} \text{ is not } \underline{b} \equiv (\forall x)\ (\underline{a} \Rightarrow \neg\ \underline{b}) \tag{24b}$$

$$\underline{I}: \quad \text{Some } \underline{a} \text{ is } \underline{b} \equiv (\exists x)\ (\underline{a} \Rightarrow \underline{b}) \tag{24c}$$

$$\underline{O}: \quad \text{Some } \underline{a} \text{ is not } \underline{b} \equiv (\exists x)\ (\underline{a} \Rightarrow \neg\ \underline{b}) \tag{24d}$$

In combining these statements in the four figures to form

various types of categorical syllogisms, when the first and second

statements are taken together the quantified output of the first

sentence is taken as the quantified input of the second statement. When doing this, we have to apply the connective "into" defined earlier and remember that the combination

$\rfloor \otimes \rfloor$ leads to the indefinite state Δ. It is readily verified that, because of this, the combinations II, IO, OI, OO will all lead to informationless conclusions in each of the four figures. Consequently, 16 of the 64 examples are inapplicable in a practic. way. In the same way, we can show that the invalidity from combining the SNS connectives occurs only when $\underset{\sim}{I}\,\underset{\sim}{N}$ is fed to $\underset{\sim}{I}$ or $\underset{\sim}{I}\,\underset{\sim}{N}$. This is seen from Eq (25) where the four possible products between pairs of I or I N are shown.

$$\underset{\sim}{I}\,\underset{\sim}{I} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} = \underset{\sim}{I} \qquad (25a)$$

$$\underset{\sim}{I}\,(\underset{\sim}{I}\,\underset{\sim}{N}) = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} = \underset{\sim}{I}\,\underset{\sim}{N} \qquad (25b)$$

$$(\underset{\sim}{I}\,\underset{\sim}{N})\,\underset{\sim}{I} = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = D \qquad (25c)$$

$$(\underset{\sim}{I}\underset{\sim}{N})\,(\underset{\sim}{I}\,\underset{\sim}{N}) = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = D \qquad (25d)$$

This is equivalent to saying that two successive implications

will not lead to D only if the middle term is affirmative

in both of them, or negational in both.

Ineffectiveness arising from the EPL state Δ , or the

SNS state D, in the conclusion is obviously a general rule

for not listing the particular syllogism, because it is of no

value. We shall now apply these to the 16 items in each of the

four figures. The scheme of implications for each is given in

Table 5 below, for ready reference, although they are mentioned

in most books on logic e.g. Ref. [6].

Table 5. Four figures of Traditional Logic

(a) Sixteen possible combinations in the first figure

Since both the major premise as well as the minor premise

have four possible forms A , E , I , O there are 16 combinations

Table 5: Four figures of Traditional Logic

Statement	Figure 1	Figure 2	Figure 3	Figure 4
Major premise	m $\longmapsto$ p	p $\longmapsto$ m	m $\longmapsto$ p	p $\longmapsto$ m
Minor premise	s $\longmapsto$ m	s $\longmapsto$ m	m $\longmapsto$ s	m $\longmapsto$ s
Conclusion	s $\longmapsto$ p	s $\longmapsto$ p	s $\longmapsto$ p	s $\longmapsto$ p

of these that can be considered in the first figure so as to

lead to a conclusion going from $\underline{s}$ to $\underline{p}$. On examining these,

it is found that the combinations $\underline{EA}$, $\underline{EE}$, $\underline{EI}$, $\underline{EO}$, $\underline{OA}$, $\underline{OE}$, $\underline{OI}$, $\underline{OO}$

are useless since the SIS output is always the doubtful state

(1   1), as per Eqns (25c, d). Thus eight more become invalid,

and, together with the four mentioned above, for which the

quantifier state is Δ, ten syllogisms within the sixteen possible

in the first figure all become of no use (two of these eight,

namely $\underline{OI}$ and $\underline{OO}$, also occur in the list of four for Δ). This

gives only six remaining ones and of these four are found to be

listed in books dealing with traditional categorical syllogisms,

namely $\underline{AA}$, $\underline{AI}$, $\underline{EA}$, $\underline{EI}$.

We shall consider why two of the six, namely $\underline{IE}$ and $\underline{OA}$,

are invalid a little later in this subsection after considering

the four moods valid in figure 1. Their Latin names are also

given in the Eqns (26 a to d) below.

AAA-1 (Barbara):

$$(\forall x)\ (\underline{m} \Rightarrow \underline{p}),\ (\forall x)\ (\underline{s} \Rightarrow \underline{m})\ ;\ (\forall x)\ (\underline{s} \Rightarrow \underline{p}) \qquad (26a)$$

EAE-1 (Celarent)

$$(\forall x)\ (\underline{m} \Rightarrow \neg\,\underline{p}),\ (\forall x)\ (\underline{s} \Rightarrow \underline{m})\ ;\ (\forall x)\ (\underline{s} \Rightarrow \neg\,\underline{p}) \qquad (26b)$$

AII-1 (Darii)

$$(\forall x)\ (\underline{m} \Rightarrow \underline{p}),\ (\exists x)\ (\underline{s} \Rightarrow \underline{m})\ ;\ (\exists x)\ (\underline{s} \Rightarrow \underline{p}) \qquad (26c)$$

EIO-1 (Ferio)

$$(\forall x)\ (\underline{m} \Rightarrow \neg\,\underline{p}),\ (\exists x)\ (\underline{s} \Rightarrow \underline{m})\ ;\ (\exists x)\ (\underline{s} \Rightarrow \neg\,\underline{p}) \qquad (26d)$$

Of these, the first two only differ in that the major premise

is an assertion in (26a) and a denial in (26b). The first two

(26a, b) differ from the second pair (26c, d) in that, the minor

premise is $(\forall x)$ in the former and $(\exists x)$ in the latter. It

will be noticed that, in the way in which the two of them are

applied one "into" the other, $(\exists x)$ precedes $(\forall x)$ and we have

used the result of Section 2 for this — namely $\exists \otimes \exists = \exists$.

The question may be asked whether we cannot have the corresponding equations in which a statement with $(\forall x)$ precedes one having the quantifier $(\exists x)$. Actually the two possibilities of this type, with a positive and a negative statement for the major premise, are absent in the above list of four valid syllogisms. This is so, because the output of $(\forall x)$ $(\underline{a} \implies \underline{b})$ is $(\exists x)$ $(\underline{b})$ and it is this that is input "into" the next statement $(\exists x)$ $(\underline{b} \implies \underline{c})$. Hence it leads to $\triangle$ and the corresponding syllogism is valueless.

Thus, the statement $(\forall x)$ $(\underline{a}(x) \implies \underline{b}(x))$, gives, with the input $(\forall x)$ $(\underline{a}(x))$, only an output $(\exists x)$ $(\underline{b}(x))$, and not $(\forall x)$ $(\underline{b}(x))$. The best way of appreciating this is to take a simple example, namely "All men are mammals", in which men is $\underline{a}$ and mammal is $\underline{b}$. In fact, if we put the input "all men" $((\forall x)$ $(\underline{a}(x)))$ into it, we will get the corresponding mammals, but it need not exhaust the domain of all mammals. We in fact

have to write the above statement more precisely as "All men

are some mammals". Hence the output of $((\forall x) (\underline{a}(x) \Rightarrow \underline{b}(x)))$

is in the state $\exists$. Consequently, if the next statement,

into which this output goes, has the quantifier $\exists$, then we get

as input to that the "into" product $\exists \otimes \exists \cong \Delta$, resulting in

the indefinite quantifier state for the major and minor premises

put together.

If the argument of the above para is taken into account,

the two examples $\underline{AE}$ and $\underline{AO}$ go out and we are left only with

four out of the 16 possibilities under Figure 1, namely those

listed in Eqns (26 a to d). Even in these, it is readily seen

that (26b) can be derived from (26a) by putting $\neg \underline{p}$ instead

of $\underline{p}$ and similarly (26d) can be derived from (26c).

Consequently there are really two moods in Figure 1 that are

independent — namely $\underline{AAA}$-1 ($\underline{Barbara}$) and $\underline{AII}$-1 ($\underline{Darii}$).

(b) Discussion of the second figure and reversal of QPL statements.

The second figure has the form

$$p \longmapsto m \text{ (Major)}, \quad s \longmapsto m \text{ (Minor)}, \quad s \longmapsto p \text{ (Conclusion)}$$

In this, it will be seen that the $m \longmapsto p$ (Major) of the

first figure has been converted to the form $p \longmapsto m$. Thus,

there is a need to work out the consequence of reversing the

direction of an implication statement in QPL and the equivalences

of the forward and reverse statements. (A slight repetition

may be excused, as the ideas obtained at this stage have been

briefly introduced earlier in the text, to make the argument

clearer).

Considering the statement "All a are b", it is clear that

the reversal of this is "All not-b are not-a". Thus for the

universal quantifier, we have the following equivalences under

reversal

$$(\forall x) \; (a \Rightarrow b) \equiv (\forall x) \; (\neg b \Rightarrow \neg a) \tag{27a}$$

$$(\forall x) \; (a \Rightarrow \neg b) \equiv (\forall x) \; (b \Rightarrow \neg a) \tag{27b}$$

problem is however not so simple for the existential

ntifier $(\exists x)$. In this case, using the standard notation

x) ($\underline{a}$ & $\underline{b}$) for "some $\underline{a}$ are $\underline{b}$", it is clear that the statement

it is quantified is symmetric in $\underline{a}$ and $\underline{b}$. This can be readily

rified from a Venn diagram, which shows that "some $\underline{a}$ are $\underline{b}$"

so means that "some $\underline{b}$ are $\underline{a}$". At the same time, from the

nn diagram, it can be seen that when "some $\underline{a}$ are $\underline{b}$", $\neg\underline{a}$

n correspond to either $\underline{b}$ or $\neg\underline{b}$, so that it gives the state

. Similarly starting from $\neg\underline{b}$ we arrive at the state a_D.

is is a new type of relationship between $\underline{a}$ and $\underline{b}$ which is

t met with in either classical logic (propositional calculus),

· SNS logic. However, these properties of $(\exists x)$ ($\underline{a}$ & $\underline{b}$)

.11 have to be borne in mind while working out the consequences

f reversed statements with the quantifier $(\exists x)$, going from

to $\underline{a}$, when it has been defined for $\underline{a}$ to $\underline{b}$. In view of this,

e shall use the symbology $(\exists x)$ ($\underline{a} \Rightarrow \underline{b}$) and $(\exists x)$ ($\underline{b} \Rightarrow \underline{a}$)

o denote the forward and backward relations of this type,

which have the properties mentioned above of an implication.
In the light of this, we shall write the equivalences under
reversal of a CL statement which is quantified by the
existential quantifier $(\exists x)$ as below.

$$(\exists x) \; (\underline{a} \Rightarrow \underline{b}) \equiv (\exists x) \; (\underline{b} \Leftarrow \underline{a}) \tag{28a}$$

$$(\exists x) \; (\underline{a} \Rightarrow \neg \underline{b}) \equiv (\exists x) \; (\neg \underline{b} \Leftarrow \underline{a}) \tag{28b}$$

(It is to be noted that for (28a, b), while a_T leads to b_T or
b_F as the case may be, a_F will only lead to the state b_D, as
mentioned above.) Although both (28a) and (28b) are symmetric
relations, we shall use the notation $(\exists x) \; (\underline{a} \Rightarrow \underline{b})$, or
$(\exists x) \; (\underline{b} \Rightarrow \underline{a})$, as the case may be, to indicate the direction
in which it is applied.

In addition to these, we have the property mentioned in
the last sub-section for the first figure, namely $(\forall x) \; (\underline{a} \Rightarrow \underline{b})$,
followed by $(\exists x) \; (\underline{b} \Rightarrow \underline{c})$, gives only $(\exists x) \; (c_D)$, and no
useful QPL relation from $\underline{a} \longmapsto \underline{c}$ follows.

In other words, two categorical statements, successively
having the quantifiers $(\forall x)$ and $(\exists x)$ will lead to the quantifier
state Δ , even if the SNS output of the first (say $\underline{b}$) is the
same as the input into the second statement, and no D state occurs.

Using the above results for the reversed statement and

conditions of validity, we find that, in the second figure,

most of the syllogisms lead either to the state Δ for the

quantifier, or D for the SNS term, and are of no value. Only

four of them remain, and we shall mention these below.

While working these out one more rule had to be used.

Suppose, we use $(\forall x)$ $(\underline{b} \implies \underline{a})$ in the form of $(\forall x)$ $(\neg \underline{a} \implies \neg \underline{b})$

and combine it with another statement, which has the form

$(\forall x)$ $(\underline{b} \implies \underline{c})$, with the SNS term $\underline{b}$ as input, then because

the term put in is $\neg \underline{b}$ and the implication is from $\underline{b}$, we

will obtain a net SNS term c_D in the D state. However, under

the same conditions, we can use the fact that

$$(\forall x)\ (\underline{b} \implies \underline{a})\ \supset\ (\exists x)\ (\underline{b} \implies \underline{a})\ , \tag{29}$$

which, on reversal, gives $(\exists x)$ $(\underline{a} \implies \underline{b})$. The output $(\exists x)$ $(\underline{b})$

of this can be the input in $(\forall x)$ $(\underline{b} \implies \underline{c})$, and therefore,

there can be a useful syllogism arising from the partial

information contained by $(\exists x)$ $(\underline{b} \implies \underline{a})$ in the form of

$(\exists x)$ $(\underline{b} \implies \underline{a})$. We will give examples of these as we consider

the second, third and fourth figures.

(c) Details of the applications for the second figure.

Considering the pair of statements p ⟼ m and s ⟼ m

in this second figure, we can reverse the direction of the major

statement p ⟼ m to get one from m ⟼ p , and also reverse

the order of the two statements.to get an implication from s to p.

Taking the first combination **AA**, this yields (30a, b):

$$(\forall x) \ (\underline{p} \Rightarrow \underline{m}) \ , \ (\forall x) \ (\underline{s} \Rightarrow \underline{m}) \tag{30a}$$

$$\equiv (\forall x) \ (\underline{s} \Rightarrow \underline{m}), \ (\forall x) \ (\neg \underline{m} \Rightarrow \neg \underline{p}) \longmapsto \underline{p} = D \tag{30b}$$

yielding no useful conclusion. However, we can use the reversal

of the quantifier $(\exists x) \ (\underline{p} \Rightarrow \underline{m})$ contained in the first QPL

term of (30a), to obtain $(\forall x) \ (\underline{p} \Rightarrow \underline{m}) \supset (\exists x) \ (\underline{p} \Rightarrow \underline{m})$

$\equiv (\exists x) \ (\underline{m} \Rightarrow \underline{p})$. However, on putting into it the output

of the statement $(\forall x) \ (\underline{s} \Rightarrow \underline{m})$, namely $(\exists x) \ (\underline{m})$, we have

the case of its output $(\exists x)$ going "into" another $(\exists x)$, which,

as we have already seen in Section 3, will give only the result

$(\triangle x) \ (\underline{p})$ as the final output. Hence, in both ways of working

out this combination **AA** in the second figure, we get either the

SNS state D or the QPL state $\triangle$, both of which are

indeterminate. Hence $\underline{AA}$ is of no use in figure 2.

The combination $\underline{AE}$, on the other hand, gives on reversal

of $(\forall x)$ $(\underline{p} \Longrightarrow \underline{m})$ what is shown (31), which gives a useful

syllogism $\underline{AEE}$-2. This is the mood $\underline{Camestres}$.

$$(\forall x) (\underline{s} \Longrightarrow \neg \underline{m}), (\forall x) (\neg \underline{m} \Longrightarrow \neg \underline{p}) ;$$
$$(\forall x) (\underline{s} \Longrightarrow \neg \underline{p}) \ (\underline{AEE}\text{-}2) \tag{31}$$

In the same way, we find that $\underline{AI}$ is of no use in figure 2,

while $\underline{AO}$ gives the output $\underline{O}$ leading to the mood, $\underline{AOO}$-2, namely

$\underline{Baroco}$.

Similarly, for $(\forall x)$ $(\underline{p} \Longrightarrow \neg \underline{m})$, if we combine it, after

reversal, with statements of the form $\underline{A}$, $\underline{E}$, $\underline{I}$, $\underline{O}$ of s $\longmapsto$ m ,

two of them lead to indefinite conclusions and two of them lead

to useful syllogisms, namely $\underline{EAE}$-2 (Cesare) and $\underline{EIO}$-2 (Festino).

For the remaining eight combinations in figure 2, either

D or Δ results and no more useful syllogisms are obtained.

Thus, the four syllogisms mentioned under categorical syllogisms

of traditional logic corresponding to the figure 2 are derivable

by our method and we can also show that no others are possible,

of the type $\underline{\underline{s}} \Rightarrow \underline{p}$ or $\underline{\underline{s}} \Rightarrow \neg \underline{\underline{p}}$, quantified by $\forall$ or $\exists$.

(d) Application to syllogisms in the third figure

In this figure, the statements are of the type

m $\longmapsto$ p, m $\longmapsto$ s, and we can obtain a conclusion of the type

s $\longmapsto$ p, by reversing the second of these. Considering the

first mood, AA, it is found that one of the statements, on

reversal, has $\underline{\underline{m}}$, or $\neg \underline{\underline{m}}$, as output, while the other has $\neg \underline{\underline{m}}$,

or $\underline{\underline{m}}$, respectively as input, leading to the SNS state D for

the conclusion. On the other hand,

$$(\forall x) \ (\underline{\underline{m}} \Rightarrow \underline{\underline{p}}) \supset (\exists x) \ (\underline{\underline{m}} \Rightarrow \underline{\underline{p}}) \equiv (\exists x) \ (\underline{\underline{p}} \Rightarrow \underline{\underline{m}}) \quad (32a)$$

and, on combining with $(\forall x) \ (\underline{\underline{m}} \Rightarrow \underline{\underline{s}})$, we obtain the conclusion

as in (32b)

$$(\exists x) \ (\underline{\underline{p}} \Rightarrow \underline{\underline{m}}), \ (\forall x) \ (\underline{\underline{m}} \Rightarrow \overset{\Delta}{\underline{\underline{p}}}) = (\exists x) \ (\underline{\underline{p}} \Rightarrow \underline{\underline{s}}) \equiv (\exists x)(\underline{\underline{s}} \Rightarrow \underline{\underline{p}}) \quad (32b)$$

in the I form, and we obtain the mood AAI-3 (Darapti).

In the same way, EA leads to a conclusion $\exists(\underline{s} \Longrightarrow \neg \underline{p})$

in the form Q, giving the valid mood EAO-3 (Felopton).

On the other hand, if we do the same procedure of deriving

an $\exists$ from a $\forall$, and combine it suitably with another $\forall$ ∧ we _for A E and E E_

get the following (_strange results._) For AE, we have,

$$(\forall x) (\underline{m} \Longrightarrow \underline{p}) \supset (\exists x) (\underline{m} \Longrightarrow \underline{p}) \equiv (\exists x) (\underline{p} \Longrightarrow \underline{m}) \qquad (33a)$$

and, on combining with E, we obtain

$$(\exists x) (\underline{p} \Longrightarrow \underline{m}), (\forall x) (\underline{m} \Longrightarrow \neg \underline{s}) = (\exists x) (\underline{p} \Longrightarrow \neg \underline{s})$$
$$\equiv (\exists x) (\neg \underline{s} \Longrightarrow \underline{p}) \qquad (33b)$$

Similarly, for EE, we have, for the second E,

$$(\forall x) (\underline{m} \Longrightarrow \neg \underline{s}) \supset (\exists x) (\underline{m} \Longrightarrow \neg \underline{s})$$
$$\equiv (\exists x) (\neg \underline{s} \Longrightarrow \underline{m}) \qquad (34a)$$

On combining with the first E, we obtain

$$(\exists x) (\neg \underline{s} \Longrightarrow \underline{m}), (\forall x) (\underline{m} \Longrightarrow \neg \underline{p}) = (\exists x) (\neg \underline{s} \Longrightarrow \neg \underline{p}) \qquad (34b)$$

In (33b) and (34b), we obtain conclusions for $(\exists x)$, of

implications inside the bracket <u>from</u> $\neg \underline{s}$ to $\underline{p}$ or to $\neg \underline{p}$.

Such conclusions like "Not-$\underline{s}$ is $\underline{p}$" and "Not-$\underline{s}$ is not-$\underline{p}$" have not

been considered in traditional logic, and these statements are

not derivable from $(\exists x)$ $(\underline{s} \Rightarrow \underline{p})$ and $(\exists x)$ $(\underline{s} \Rightarrow \neg \underline{p})$.

Hence, we shall designate forms like these by dashed symbols

as follows:

$$(\forall x) \ (\neg \underline{s} \Rightarrow \underline{p}) \equiv \underline{A}' \ , \ (\forall x) \ (\neg \underline{s} \Rightarrow \neg \underline{p}) \equiv \underline{E}' \quad (35a)$$

$$(\exists x) \ (\neg \underline{s} \Rightarrow \underline{p}) \equiv \underline{I}' \ , \ (\exists x) \ (\neg \underline{s}) \Rightarrow \neg \underline{p}) \equiv \underline{O}' \quad (35b)$$

Then the moods of (33) and (34) are <u>AEI</u>'-3 and <u>EEO</u>'-3.

We shall list all the possibilities having conclusions of

the above types $\underline{A}'$, $\underline{E}'$, $\underline{I}'$, $\underline{O}'$ in the next section. However,

we shall continue to consider the traditional ones with conclusions

$\underline{A}, \underline{E}, \underline{I}, \underline{O}$ in the figures 3 and 4 in this section. First we

shall consider the third figure which is the subject matter of

this subsection. Thus, <u>AI</u> and <u>AO</u> lead to valid conclusions as below:

$$(\forall x)\ (\underline{m} \Rightarrow \underline{p}),\ (\exists x)\ (\underline{m} \Rightarrow \underline{s}) \equiv (\exists x)\ (\underline{s} \Rightarrow \underline{m}),\ (\forall x)\ (\underline{m} \Rightarrow \underline{p})$$

$$\equiv \exists (\underline{s} \Rightarrow \underline{p}) \quad \underline{AII\text{-}3} \ (\underline{Datisti}) \tag{36a}$$

$$(\forall x)(\underline{m} \Rightarrow \neg \underline{p}),\ (\exists x)\ \underline{m} \Rightarrow \underline{s}) \equiv (\exists x)(\underline{s} \Rightarrow \underline{m}),\ (\forall x)(\underline{m} \Rightarrow \neg \underline{p})$$

$$\equiv \exists (\underline{s} \Rightarrow \neg \underline{p}) \quad \underline{EIO\text{-}3} \ (\underline{Feriso}) \tag{36b}$$

Two more are obtained from $\underline{IA}$ and $\underline{OA}$, namely $\underline{IAI\text{-}3}$ ($\underline{Disamis}$)

and $\underline{OAO\text{-}3}$ ($\underline{Bocardo}$). Thus, all six valid moods of the third

figure are obtainable by using our technique.

In fact, six more come out to be valid, with implications

starting from not-$\underline{s}$ as conclusions, namely in the forms

$\underline{A}'$, $\underline{E}'$, $\underline{I}'$ and $\underline{O}'$. As mentioned earlier, we shall list these

.n the next section. Thus, in the third figure all 12 valid

)ossibilities of combining pairs of the forms $\underline{A}$, $\underline{E}$, $\underline{I}$, $\underline{O}$ in

he third figure have valid conclusions. The four that are not

alid have the combinations $(\exists, \exists)$, which lead to the state

$\exists \exists = \triangle$ for the quantifier and are therefore indefinite.

(e) Application to the syllogisms in the fourth figure

The fourth figure has the first two terms of the type

$p \longmapsto m$, $m \longmapsto s$, leading to the conclusion $s \longmapsto p$. The

first one to be considered is $\underline{AA}$ for which the books on this

subject write the mood to be of the form $\underline{AAI}$-4 (Bamalip) [3],[6].

The form I represents the statement $(\exists x) (\underline{s} \Longrightarrow \underline{p})$, and it is

obtained from the partial information $(\exists x) (\underline{m} \Longrightarrow \underline{s})$ contained

in $(\forall x) (\underline{m} \Longrightarrow \underline{s})$. However, it is possible to derive from $\underline{AA}$

in figure 4 a more powerful conclusion with the quantifier $(\forall x)$

throughout, as follows:

$$(\forall x) (\underline{p} \Longrightarrow \underline{m}), (\forall x) (\underline{m} \Longrightarrow \underline{s}) = (\forall x) (\underline{p} \Longrightarrow \underline{s})$$
$$\equiv (\forall x) (\neg \underline{s} \Longrightarrow \neg \underline{p}) (E'), \quad \underline{AAE'}\text{-}4 \tag{37}$$

Thus, not only can the forms $\underline{A}'$, $\underline{E}'$, $\underline{I}'$, $\underline{O}'$ occur as conclusions,

but in this instance, such a conclusion is the proper one to be

taken since it is superior in power to that listed commonly.

Thus, for all x, if "not-$\underline{s}$ implies not-$\underline{p}$", then

obviously there exists "some x for which $\underline{s} \Longrightarrow \underline{p}$" ; but

there can also be "some x for which $\underline{s} \Longrightarrow \neg p$".

To illustrate this, we shall take the following example. Thus,

"for all" "In Bangalore $\Rightarrow$ In Karnataka", and "for all"

"In Karnataka $\Rightarrow$ In India" together imply, "for all" "In

Bangalore $\Rightarrow$ In India". The last statement is well-known to be

equivalent to "for all" "Not in India $\Rightarrow$ Not in Bangalore".

This is much more powerful than saying merely that "Some Indians

are in Bangalore". Therefore we have a very strong case for

examining all the 64 possibilities of combining the four forms

with one another in the four figures for obtaining the new forms

of conclusion, namely $\underline{A}'$, $\underline{E}'$, $\underline{I}'$, $\underline{O}'$. We have done this in the

next section and the essential results are summarised. We do not

give the detailed derivations for the fourth figure for the

traditional ~~words~~ moods, but merely state that the four listed examples

other than Bamalip, namely AEE (Calemes), EAO (Fesapo), EIO

(Fresison), IAI (Dimatis) come out all right by our analysis

of all possibilities in the fourth figure.

5. New type of syllogisms leading to conclusions of the form $\neg \underline{s} \Rightarrow \underline{p}$ and $\neg \underline{s} \Rightarrow \neg \underline{p}$.

(a) Outline of the algebraic technique for working out valid syllogisms

We have so far considered the standard type of categorical

syllogisms in traditional logic. While doing this, we have

developed an algebraic technique of finding out which combinations

of two QPL statements in the various figures will give rise to a

conclusion of the type $(\forall x) \; (\underline{s} \Rightarrow \underline{p}, \text{ or } \neg \underline{p})$ and $(\exists x)$

$(\underline{s} \Rightarrow \underline{p}, \text{ or } \neg \underline{p})$. The inputs also are QPL statements of the

form quantified by $(\forall x)$ or $(\exists x)$. In this algebraic technique,

we use reversals of these statements to obtain effectively a

set of two statements in succession in which the output of the

first statement is the input of the second statement. Consequently,

the two quantified statements, each of which is an implication,

can be combined together to form a single implication from the

first term to the third term. We shall call this way of writing

the two QPL statements in succession as "a chain of two implications"

or a "derivation". In effect, we have only two types of

derivations which are used, namely,

$$(\forall x) \, (\underline{a} \Rightarrow \underline{b}), \; (\forall x) \, (\underline{b} \Rightarrow \underline{c}) \; = \; (\forall x) \, (\underline{a} \Rightarrow \underline{c}) \qquad (38a)$$

and

$$(\exists x) \, (\underline{a} \Rightarrow \underline{b}), \; (\forall x) \, (\underline{b} \Rightarrow \underline{c}) \; = \; (\exists x) \, (\underline{a} \Rightarrow \underline{c}) \qquad (38b)$$

In order to convert the two statements in each mood of a

figure into a chain of two such implications, we use reversals

of one, or both, of the statements (as in 39a, b) and also use

the fact (where needed) that the implication $(\forall x) \, (\underline{a} \Rightarrow \underline{b})$

implies the existential statement $(\exists x) \, (\underline{a} \Rightarrow \underline{b})$.

$$(\forall x) \, (\underline{a} \Rightarrow \underline{b}) \; \equiv \; (\forall x) \, (\neg \underline{b} \Rightarrow \neg \underline{a}) \qquad (39a)$$

$$(\exists x) \, (\underline{a} \Rightarrow \underline{b}) \; \equiv \; (\exists x) \, (\underline{b} \Rightarrow \underline{a}) \qquad (39b)$$

Further, we use the results from sections 2 and 3 that $\forall \otimes \forall = \forall$,

$\forall \otimes \exists = \exists \otimes \forall = \exists$, and $\exists \otimes \exists = \Delta$, and also the SNS-type

of result, that $(a \Rightarrow b), \; (\neg \underline{b} \Rightarrow \underline{c})$ in succession will lead

to the state D for the term c irrespective of what a is,

so that the resultant syllogism leads to no conclusion of

value, and becomes "invalid".

(b) Proof of uniqueness of the technique

As in the previous section 4, we have worked out various

examples in which the above formulae (38) and (39), and the

associated results mentioned above, were employed, and the

18 syllogisms of traditional logic were derived in this way.

The nineteenth one, namely Bamalip, was shown to be replaceble

by a superior syllogism$ of the new type with an implication

starting from $\neg s$. It might be wondered whether this method

of working out valid syllogisms, in which various reversals are

used, and $(\exists x)$ is employed instead of $(\forall x)$ for a number of cases,

cannot lead to two ambiguous results. We have examined this, and

it is possible to show very simply that no ambiguity will come

in any case of a pair of statements, according to any of the

four figures, and that, either a uniquely possible conclusion

can be derived, or it can be shown that the pair of statements

will lead to an indefinite statement having either the QFL

state $\triangle$, or the SNS state D. We shall give an outline of the

proof below and it is not very difficult. However, the details

of the proof are omitted and only the main steps are given.

While combining two statements having the quantifiers

$(\forall x)$ and/or $(\exists x)$, we should first note that the combination

of one statement with $(\exists x)$ followed by a second statement with

$(\exists x)$ will not lead to any conclusion since $\exists \otimes \exists = \triangle$, the

indefinite state. Hence we need consider only the combinations

given below.

(i) If both statements are $(\forall_1 x)$ and $(\forall_2 x)$, then either

they do not lead to any conclusion because the output of the

first statement is the negation of the input to the second

statement, or there is a valid conclusion having the quantifier

$(\forall x)$, in which case reversing both of them will lead once again

to a conclusion which is equivalent to the earlier one. For example,

$$(\forall x)\ (a \Rightarrow b),\ (\forall x)\ (b \Rightarrow c) = (\forall x)\ (a \Rightarrow c) \tag{40a}$$

$$(\forall x)\ (\neg c \Rightarrow \neg b),\ (\forall x)\ (\neg b \Rightarrow \neg a)$$
$$= (\forall x)\ (\neg c \Rightarrow \neg a) \tag{40b}$$

It is obvious that the right hand sides of (40a) and (40b) are

equivalent and no ambiguity arises.

(ii) If we have the combination $(\forall, \exists)$ or $(\exists, \forall)$, then

one of these produces invalidity, because the sequence of

statements are of the type (41), leading to an indefinite conclusion,

having the QPL state Δ .

$$(\forall x)\ (a \Rightarrow b),\ (\exists x)(b \text{ or } \neg b \Rightarrow c) = \Delta\ (a \Rightarrow c, \text{ or } a \mathrel{\underset{\sim}{D}} c) \tag{41}$$

In such a case, only the other combination with $(\exists x)$ preceeding

$(\forall x)$ need be considered, and this has the quantifier $(\exists x)$, but

the SNS operator may be $\Rightarrow$ (I), or D. In the former case, there is a unique valid syllogism, and in the latter case, no valid syllogism is possible for this combination.

(iii) In the case (i), although the combination ($\forall_1 x$) and ($\forall_2 x$) do not lead to any valid syllogism, we can obtain the state ($\exists x$) by implication from one of them, and then, we can have four possible combinations. Thus, the implication being taken from either ($\forall_1 x$) or ($\forall_2 x$) leads to two possibilities ($\forall_1 x$), ($\exists_2 x$) and ($\exists_1 x$), ($\forall_2 x$), and each pair can be taken as it is, or in the reversed sense, leading to four cases. By experience, when all the 64 examples were worked out, not a single duplication, or ambiguity, was noticed. Therefore, a proof for this has been worked out and it is quite simple to see. It is best verified by saying that if there is a valid syllogism, then on reversing it and on interchanging the ($\exists x$) and the ($\forall x$), no other valid result comes out. Thus, suppose

$$(\exists x) \ (\underline{a} \Rightarrow \underline{b}), \ (\forall x) \ (\underline{b} \Rightarrow \underline{c}) = (\exists x) \ (\underline{a} \Rightarrow \underline{c}) \qquad (42a)$$

In this $\underline{a}$ can be either $\underline{s}$ or $\neg \underline{s}$, and $\underline{c}$ can be either

$\underline{p}$ or $\neg \underline{p}$. If now, we reverse the two statements on the left

hand side, we obtain

$$(\forall x) \; (\neg \underline{c} \Longrightarrow \neg \underline{b}), \; (\exists x) \; (\underline{b} \Longrightarrow \underline{a}) = \forall \otimes \exists = \triangle \text{ and } D \qquad (42b)$$

If, on the other hand, we interchange the $(\forall x)$ and $(\exists x)$ in (42a)

then we obtain the QPL indefinite state $\triangle$:

$$(\forall x) \; (\underline{a} \Longrightarrow \underline{b}), \; (\exists x) \; (\underline{b} \Longrightarrow \underline{c}) = \forall \otimes \exists = (\triangle \; x) \; (\underline{a} \Longrightarrow \underline{c}) \qquad (42c)$$

On reversing this, we obtain,

$$(\exists x) \; (\underline{c} \Longrightarrow \underline{b}), \; (\forall x) \; (\neg \underline{b} \Longrightarrow \neg \underline{a}) = (\exists x) \; (\underline{c} \; D \; \underline{a}) \qquad (42d)$$

leading to the SNS doubtful state D. Thus, in the three cases

(42b), (42c) and (42d), either the QPL state is $\triangle$, or the SNS

state is D, or both of them occur, and they are indefinite and

useless as conclusions of a syllogism. Thus, we have shown quite

generally that the procedure mentioned earlier in this section

of working out a chain of two implications is a very general one

and can lead to no ambiguities for a pair of QPL statements of

Type 1.

(c) List of the eighteen new syllogisms

Since there can only be one conclusion for a pair of statements in any of the four figures, we can omit the 18 examples which are well-known in traditional logic from consideration for the purpose of deriving the set of new syllogisms. We can also similarly omit 16 examples which have the sequence $(\exists, \exists)$ because they lead to the indefinite state Δ, by combination via the operation "into". Thus, we are left with 30 examples and it is interesting to note that 18 of them come out to be valid syllogisms of the new type in which the implication within the bracket is of the type $\neg \underline{s} \Rightarrow \underline{p}$ or $\neg \underline{s} \Rightarrow \neg \underline{p}$. We have listed these, along with the 18 syllogisms of the old type, in Table 6.

Table 6. Summary of categorical syllogisms in traditional logic, including the new possibilities

---

Table 7 gives the names of the valid moods of the old and new types. The first three vowels in the names give the symbols, $\underline{A}$, $\underline{E}$, $\underline{I}$ or $\underline{O}$ of the three statements in the syllogisms. The presence of a fourth vowel signifies that it belongs to the new type.

Table 7. Symbols and names of valid moods of the old and new types.

To distinguish the new ones from the old ones, the letter symbolism

Table 6

Summary of Categorical Syllogisms in Traditional Logic, including the new Possibilities*

Figure+	Moods with conclusions $\underline{s} \Rightarrow \underline{p}$ and $\underline{s} \Rightarrow \neg \underline{p}$	Moods with conclusions $\neg \underline{s} \Rightarrow \underline{p}$ and $\neg \underline{s} \Rightarrow \neg \underline{p}$
First	AAA, AII, EAE, EIO	AEI', EEO', IEI', OEO'
Second	AEE, EAE, AOO, EIO	AAO', EEO', IEI', OAI'
Third	AAI, EAO, AII, EIO, IAI, OAO	AEI', EEO', AOI', EOO', IEI', OEO'
Fourth	AEE, EAO, EIO, IAI	AAE', EEO', EOO', IEI'

*Forms: $\underline{A}$ = All $\underline{a}$ is $\underline{b}$, $\underline{E}$ = All $\underline{a}$ is not-$\underline{b}$,
$\underline{I}$ = Some $\underline{a}$ is $\underline{b}$, $\underline{O}$ = Some $\underline{a}$ is not $\underline{b}$;
$\underline{A}$'= All not-$\underline{a}$ is $\underline{b}$, $\underline{E}$' = All not-$\underline{a}$ is not-$\underline{b}$,
$\underline{I}$'= Some not-$\underline{a}$ is $\underline{b}$, $\underline{O}$' = Some not-$\underline{a}$ is not-$\underline{b}$.

+Figures: 1 :   m ⟼ p,   s ⟼ m,   s ⟼ p

2 :   p ⟼ m,   s ⟼ m,   s ⟼ p

3 :   m ⟼ p,   m ⟼ s,   s ⟼ p

4 :   p ⟼ m,   m ⟼ s,   s ⟼ p

Table 7: Symbols and Names of Valid Moods of the
Old and New Types

Old Type		New Type	
AAA-1 ,	Barbara	AEI'-1 ,	Kaveriar
AII-1 ,	Darii	EEO'-1 ,	Electronic
EAE-1 ,	Celarent	IEI'-1 ,	Dimension
EIO-1 ,	Ferio	OEO'-1 ,	Ponderous
AEE-2 ,	Camestres	AAO'-1 ,	Bangalore
AOO-2 ,	Baroco	EEO'-1 ,	Neelotpal
EAE-2 ,	Cesare	IEI'-1 ,	Nivedita
EIO-2 ,	Festino	OAI'-1 ,	Moravia
AAI-3 ,	Darapti	AEI'-1 ,	Magnesia
AII-3 ,	Datisti	AOI'-1 ,	Sarojini
EAO-3 ,	Felapton	EEO'-1.,	Development
EIO-3 ,	Feriso	EOO'-1 ,	Mesozoic
IAI-3 ,	Disamis	IEI'-1 ,	Indefinite
OAO-3 ,	Bocardo	OEO'-1 ,	Lodestone
AEE-4 ,	Calemes	AAE -1 ,	Management
EAO-4 ,	Fesapo	EEO -1 ,	Generous
EIO-4 ,	Fresison	EOO -1 ,	Leonora
IAI-4 ,	Dimatis	IEI -1 ,	Silesia

the conclusion is given an extra prime as in $\underline{A}'$, $\underline{E}'$, $\underline{I}'$ and $\underline{O}'$.

We shall not give the derivations of all of these examples, but

take only a few of them to indicate how the rules in (39a and b)

and the associated results are applied for this purpose. It will

be noticed that, while doing this, we are applying no new result,

or technique, that has not been used in deriving the well-known

18 categorical syllogisms of traditional logic. The same rules

are applied, with only the extra provision that a conclusion

starting from $\neg \underline{s}$ can also be listed. It is obvious, on examining

the Table 6, that the new syllogisms are not duplications, or

straightforward equivalences, of the old ones. In fact, we use

only statements starting from an affirmation in the first two

statements on the left hand side of the syllogisms and only 64

of these are used. However, for the concluding statement we now

allow the SIS statements $\neg \underline{s} \Rightarrow \underline{p}$ and $\neg \underline{s} \Rightarrow \neg \underline{p}$, starting

from $\neg \underline{s}$ also, in addition to the two implications starting

from $\underline{s}$ which are listed in the left half of Table 6. It can be

shown that the case of negations for the first two statements

are derivable in a straightforward manner from the 36 syllogisms

listed in Table 6. (See Appendix 1 for a short proof of this).

(d) New syllogisms belonging to the first figure

In the first figure, the very first combination $\underline{AA}$ leads to the well-known syllogism $\underline{AAA}$-1 (Barbara) of the old type. Taking the next one $\underline{AE}$ which is not in the traditional list, it consists of $(\forall x) (\underline{m} \Rightarrow \underline{p})$, $(\forall x) (\underline{s} \Rightarrow \neg \underline{m})$. From this, we obtain

$$(\forall x) (\underline{m} \Rightarrow \underline{p}) \supset (\exists x) (\underline{m} \Rightarrow \underline{p}) \equiv (\exists x) (\underline{p} \Rightarrow \underline{m}) \tag{43a}$$

$$(\forall x) (\underline{s} \Rightarrow \neg \underline{m}) \equiv (\forall x) (\underline{m} \Rightarrow \neg \underline{s}) \tag{43b}$$

$$(\exists x) (\underline{p} \Rightarrow \underline{m}), (\forall x) (\underline{m} \Rightarrow \neg \underline{s}) = (\exists x) (\underline{p} \Rightarrow \neg \underline{s})$$

$$\equiv (\exists x) (\neg \underline{s} \Rightarrow \underline{p}) - I' (\underline{AEI}'\text{-}1) \tag{43c}$$

The conclusion is of the form I', leading to the form $\underline{AEI}'$— 1, for which we have given the name $\underline{Kaveriar}$. This is based on the name $(\underline{Kaveri})$ of one of the important rivers of Karnataka state in which Bangalore is located. The termination 'ar' means 'river' in Tamil.

The next combination $\underline{AI}$ leads to $\underline{AII}$-1, which is $\underline{Darii}$ of

the traditional list. The one after this, namely $\underline{AO}$, leads

to no useful conclusion, but only to either D or $\triangle$, as in

(44a) and (44b). Thus,

$$(\forall x) (\underline{m} \Rightarrow \underline{p}), (\exists x) (\underline{s} \Rightarrow \neg \underline{m}) \text{ leads to D} \qquad (44a)$$

Reversing both, we obtain the following inputs, leading to a $\triangle$.

$$(\forall x)(\neg \underline{p} \Rightarrow \neg \underline{m}), (\exists x)(\neg \underline{m} \Rightarrow \underline{s}) = (\triangle x)(\neg \underline{p} \Rightarrow \underline{s}) \ (44b)$$

Just like $\underline{AI}$, the combination $\underline{EE}$ also leads only to an

$(\exists x)$ conclusion, producing $\underline{EEO}'$-1, and, just like $\underline{AO}$, $\underline{EO}$ also

leads only to D or $\triangle$. Similarly $\underline{IA}$ and $\underline{OA}$ lead to only D or

$\triangle$, while $\underline{IE}$ and $\underline{OE}$ lead to the valid syllogisms of the new

type $\underline{IEI}'$-1 and $\underline{OEO}'$-1.

As already mentioned, the quantifier combinations in

$\underline{II}$, $\underline{IO}$, $\underline{OI}$ and $\underline{OO}$ all lead to $\exists \otimes \exists = \triangle$, and they cannot lead

to valid syllogisms. Hence, just as with the old examples, the

first figure contains only four of the new type of syllogism.

(e) New syllogisms in the second, third and fourth figures

Table 6 lists these new results. Since their derivation,

from a pair of QPL statements, converted to form a chain of two

implications, with the sequence $\forall$, $\forall$ or $\exists$, $\forall$, does not involve

any technique not already described and used for the traditional

examples, we do not work them out in detail.

6. Manipulations with Statements of Type 1 in QPL

Although the 64 binary combinations of the statements

involved in categorical syllogisms are all capable of being

represented by another statement of this type, and these have

been listed in the previous section, it would be worthwhile

that are to be
mentioning the general rules/employed in working out the outputs

of statements of this type, for all possible inputs. Suppose

we have the QPL term $(\forall x)$ ($\underline{a} \Rightarrow \underline{b}$) or $(\exists x)$ ($\underline{a} \Rightarrow \underline{b}$), then

its outputs, corresponding to the inputs $(\forall x)$ ($\underline{a}$, or $\neg \underline{a}$) and

$(\exists x)$ $(\underline{a}/\text{or } \neg\underline{a})$, are as given in Table 8. (The proofs of

these are implicit in the discussion of previous sections.)

Table 8. Algorithms for obtaining the outputs of Type 1
statements of QPL

In compiling Table 8, formulae in SNS, and the "into"

connective in QPL, described in this report, have been employed.

The very small number of three valid deductions (Sl.Nos 1, 3 and 5

of Table 8) can be readily computerized. While testing this

procedure for routine application for a pair of statements, it

was found that it is necessary to have also the "implicate"

operator ($\Longleftarrow$) in addition to the connective operator "imply"

($\Longrightarrow$). Consequently, the three cases where this connective $\underset{\sim}{J}$

can give an output that is not D or Δ are also included in Table.8.

(Their full capabilities and application have not been worked out.)

However, initial trials show that a ~~series of~~ set of two statements having

Table 8. Algorithms for obtaining the outputs of Type 1 statements of QPL

Sl. No.	Input	Statement	Output
		Implies ($\Rightarrow$)	
1.	$(\forall x)\ (\underline{a}(x))$	$(\forall x)\ (\underline{a} \Rightarrow \underline{b})$	$(\exists x)\ (\underline{b}(x))$
2.	$(\forall x)\ (\neg\ \underline{a}(x))$	$(\forall x)\ (\underline{a} \Rightarrow \underline{b})$	D
3.	$(\exists x)\ (\underline{a}(x))$	$(\forall x)\ (\underline{a} \Rightarrow \underline{b})$	$(\exists x)\ (\underline{b}(x))$
4.	$(\exists x)\ (\neg\ \underline{a}(x))$	$(\forall x)\ (\underline{a} \Rightarrow \underline{b})$	D
5.	$(\forall x)\ (\underline{a}(x))$	$(\exists x)\ (\underline{a}(x) \Rightarrow \underline{b}(x))$	$(\exists x)\ (\underline{b}(x))$
6.	$(\forall x)\ (\neg\underline{a}(x))$	$(\exists x)\ (\underline{a}(x) \Rightarrow \underline{b}(x))$	D
7.	$(\exists x)\ (\underline{a}(x))$	$(\exists x)\ (\underline{a}(x) \Rightarrow \underline{b}(x))$	$\triangle$
8.	$(\exists x)\ (\neg\ \underline{a}(x))$	$(\exists x)\ (\underline{a}(x) \Rightarrow \underline{b}(x))$	$\triangle$ and D
		Implicates ($\Leftarrow$)	
9.	$(\forall x)\ (\underline{a}(x))$	$(\forall x)\ (\underline{b}(x) \Leftarrow \underline{a}(x))$	$(\exists x)\ (\underline{b}(x))$
10.	$(\exists x)\ (\underline{a}(x))$	$(\forall x)\ (\underline{b}(x) \Leftarrow \underline{a}(x))$	$(\exists x)\ (\underline{b}(x))$
11.	$(\forall x)\ (\underline{a}(x))$	$(\exists x)\ (\underline{b}(x) \Leftarrow \underline{a}(x))$	$(\exists x)\ (\underline{b}(x))$

Rules for reversal

$$(\forall x)\ (\underline{a} \Rightarrow \underline{b}) \equiv (\forall x)\ (\neg\ \underline{b} \Rightarrow \neg\ \underline{a}); \quad (\exists x)\ (\underline{a} \Rightarrow \underline{b}) \equiv (\exists x)(\underline{b} \Rightarrow \underline{a})$$

$$(\forall x)\ (\underline{b} \Leftarrow \underline{a}) \equiv (\forall x)\ (\neg\ \underline{a} \Leftarrow \neg\ \underline{b}); \quad (\exists x)\ (\underline{b} \Leftarrow \underline{a}) \equiv (\exists x)(\underline{a} \Leftarrow \underline{b})$$

implications produce some difficulties in implementing them

in the sequence in which they are written.

We shall illustrate this below by one example of the pair

of statements taken from one of the 64 moods of categorical

syllogisms, namely $\underline{OAI}$'-2. These are $(\exists x) (\underline{p} \Rightarrow \neg \underline{m})$,

$(\forall x) (\underline{s} \Rightarrow \underline{m})$. We wish to find out the output state of $\underline{p}$,

when the input states of $\underline{s}$ are T and F. Thus the input

$(\forall x) (s_T)$ put into $(\forall x) (\underline{s} \Rightarrow \underline{m})$, gives $(\exists x) (m_T)$. This,

going into the reversed first term, namely $(\exists x) (\neg \underline{m} \Rightarrow \underline{p})$

gives $(\Delta x) (p_D)$, i.e a Δ quantifier and a D state. Now,

if we input $(\forall x) (s_F)$ into $(\forall x) (\underline{s} \Rightarrow \underline{m})$, the output is m_D

and this, going into $(\exists x) (\neg \underline{m} \Rightarrow \underline{p})$, gives again p_D.

Thus, we get the wrong result that both $(\forall x) (s_F)$ and $(\forall x) (s_T)$

leads to the output of $\underline{p}$ in the D state, showing that the two

given QPL statements, in the sequence given, lead to an invalid

syllogism. But, our Table 6 gives that the two together are

valid and yield the statement $(\exists x) (\neg \underline{s} \Rightarrow \underline{p})$, so that,

for the input $(\forall x) (s_F)$, the output is $(\exists x) (p_T)$.

It may be mentioned that this difficulty does not arise

in SI.S, where the forward ($\Longrightarrow$) and reverse ($\Longleftarrow$) implications

are equivalent, and both will give the same result, and no

difficulty arises. Thus,

$$(\underline{a} \Longrightarrow \underline{b}), \ (\underline{b} \Longrightarrow \underline{c}) \ = \ (\underline{a} \Longrightarrow \underline{c}) \tag{45}$$

Also,

$$(\underline{a} \Longrightarrow \underline{b}) \equiv (\neg \underline{b} \Longleftarrow \neg \underline{a}) \tag{46a}$$

$$(\underline{b} \Longrightarrow \underline{c}) \equiv (\neg \underline{c} \Longleftarrow \neg \underline{b}) \tag{46b}$$

and we get

$$(\neg \underline{c} \Longleftarrow \neg \underline{b}), \ (\neg \underline{b} \Longleftarrow \neg \underline{a}) = (\neg \underline{c} \Longleftarrow \neg \underline{a})$$
$$\equiv (\underline{a} \Longrightarrow \underline{c}) \tag{47}$$

On the other hand, the special situation, as mentioned above,

arises in QPL for statements ot Type 1, because $(\forall x) (\underline{a} \Longrightarrow \underline{b})$

and $(\exists x) (\underline{a} \Longrightarrow \underline{b})$, on reversal, behave differently (see Table 8).

Thus, in an argument composed of statements of Type 1 in QPL,

it appears as if the input term cannot be applied to the first

st tement in order to obtain its output, which it is then applied

into the second statement and its output obtained, and so on —

a process which we have developed in great detail in connection

with the arguments in [*propositional calculus, or*] SNS logic. It looks as if a sequence of

statements will have to be manipulated as such in many situations,

and not one by one as we would imagine. This is the lesson, that

the studies made on the 64 categorical syllogisms that are

possible, has given us.

It will be apparent, from all these, that algorithmic rules

will have to be carefully formulated so that the truth value

and quantifier value of a term can be uniquely worked out for a

sequence of Type 1 statements in QPL. The full examination of

this requires extended study and is being reserved for a later

report. However, one result seems to be clear. If we have a

chain of $\underline{n}$ QPL implications, then either the resultant implication

is $\underline{always}$ $\triangle$ or/and D; or there is just one combination of these

that leads to single QPL statement of type 1, and all the remaining

$(2^n - 1)$ possible equivalent ways of writing the chain lead to the

net resultant having either a Δ or/and a D. Hence, given

a QPL input, an argument composed of a finite number of

quantified categorical statements can always be worked out to

yield a unique output. These very brief comments will be clarified

and extended in the next report, but an outline of the proof

is given in Appendix 2. _____

References

1. Ramachandran, G.N. Current Science (1983) 52, 293-302

2. Ramachandran, G.N. Current Science (1983) 52, 335-347

3. Church, A. Article on "Logic" in Encyclopaedia Britannica
     1966 Edn., Vol. 14, p. 216, Chicago.

4. Stolle, R.R. "Set Theory and Logic", (1962), Freeman,
     San Francisco

5. Suppes, P. "Introduction to Logic" (1957), Van Nostrand,
     New York.

6. Delong, H. "A Profile of Mathematical Logic", (1971),
     Addison-Wesley, Reading.

Appendix 1

General algorithm for finding the resultant of two categorical statements.

It is possible to work out   complete general rules for

combining two categorical statements (of Type 1) to obtain a

resultant statement of the same type. The treatment here deals

with any categorical statement, either of the types $\underline{A}$, $\underline{E}$, $\underline{I}$, $\underline{O}$

or of the types $\underline{A}'$, $\underline{E}'$, $\underline{I}'$, $\underline{O}'$. Thus, each of) the statements combined

can have either an affirmation or a negation as input, and

similarly either a negation or an affirmation as output. We show

that two statements with the quantifiers ($\forall$x) or ($\exists$x) for

either of them, on combining together, lead to a third statement

of the same category and that the only formulae for combination

that we need employ are what is contained in the categorical

syllogisms $\underline{AAA-1}$ ($\underline{Barbara}$) or $\underline{AII-1}$ ($\underline{Darii}$).

(a) One quantifier $\exists$ and the other $\forall$

We have already seen that two categorical statements, in

which one has the existential quantifier ($\exists$ x) and the other the

universal quantifier ($\forall$x), can combine in two different ways,

with the common middle term ($\underline{m}$) being the output of the first

statement and the input of the second statement. However,

depending upon the sequence ($\exists$, $\forall$) or ($\forall$, $\exists$), only one of them

(in this case, the former) can lead to a non-Δ quantifier. In either

case, we verify that the middle term $\underline{m}$ appears with an

affirmation in both of them, or a negation in both of them, so

that it can go in from the first statement to the second statement.

If this happens for a resultant non-Δ quantifier, then we have

the output based on $\underline{AAA}$-I ($\underline{Darii}$), namely

$$(\exists x)\ (\underline{a} \Longrightarrow \underline{m}),\ (\forall x)\ (\underline{m} \Longrightarrow \underline{b})\ =\ (\exists x)\ (\underline{a} \Longrightarrow \underline{b}) \qquad (A1)$$

We shall illustrate this by a random example, ($\forall$x) ($\neg \underline{s} \Longrightarrow \underline{m}$)

($\exists$ x) ($\neg \underline{m} \Longrightarrow \underline{p}$). In this case, the sequence in which the

two are given leads to a state (Δ x) for the resultant. So

we reverse each of them and obtain,

$$(\exists x)\ (\underline{p} \Longrightarrow \neg \underline{m}),\ (\forall x)\ (\neg \underline{m} \Longrightarrow \underline{s})\ =\ (\exists x)\ (\underline{p} \Longrightarrow \underline{s}) \qquad (A2)$$

For doing this, we associate $\underline{p}$ with $\underline{a}$ of (A1), $\neg\underline{m}$ of (A2)

with $\underline{m}$ of (A1), and $\underline{s}$ with $\underline{b}$ of (A1). It is readily verified

that this technique can be applied to any combination of one $\exists$ with

one $\forall$, in the two statements that are under consideration.

One of the two possibilities only can at most form a valid

non-($\triangle$ x) conclusion, in which case it is unique. Both of them

may also lead to indefinite conclusions, having either $\triangle$ or D,

in which case the pair of statements started with has no valid

combination except the indefinite state.

(b) Both quantifiers $\forall$

If the two statements have $\forall$, then, as they are, we need

consider only any combination of them in which the middle term

is output of the first and the input to the next. If the two

middle terms have the same sign (affirmation or negation) then

it has a valid resultant, for example,

$$(\forall_1 x)\,(\underline{s} \Longrightarrow \neg\,\underline{m}),\ (\forall_2 x)\,(\neg\,\underline{m} \Longrightarrow \underline{p})\ =\ (\forall x)\,(\underline{s} \Longrightarrow \underline{p})\quad (A3)$$

For obtaining this, we use the standard syllogism AAA-1 (Barbara).

which we shall write as

$$(\forall x)\ (\underline{a} \Rightarrow \underline{m}),\ (\forall x)\ (\underline{m} \Rightarrow \underline{b}) = (\forall x)\ (\underline{a} \Rightarrow \underline{b}) \qquad (A4)$$

In these, the relationship between (A3) and (A4) is that we

associate $\underline{s}$ with $\underline{a}$, $\neg\underline{m}$ with $\underline{m}$ and $\underline{p}$ with $\underline{b}$ of Barbara (A4).

It can quite happen that the two sentences, both with the

universal quantifier $(\forall x)$, lead only to the doubtful state D

for the resultant, e.g.

$$(\forall_1 x)\ (\neg\underline{s} \Rightarrow \underline{m}),\ (\forall_2 x)\ (\neg\underline{m} \Rightarrow \neg\underline{p}) = D \qquad (A5)$$

In such a case, we can look for a way of combining the two to

obtain a conclusion of lesser power. We do this by converting

either one of the two statements into an existential statement

by implication. We have already shown, in Section 5(b), that

this process can lead only to one of the four ways of combining

a pair of $\exists$ and $\forall$ statements, leading to a valid conclusion,

and that there is no duplication or ambiguity. Hence we only

illustrate the technique below, with the example of (A5).

Converting first ($\forall_1 x$) into ($\exists x$), we obtain the pair of

statements

$$(\exists x) (\neg \underline{\underline{s}} \Rightarrow \underline{\underline{m}}), (\forall x) (\neg \underline{\underline{m}} \Rightarrow \neg \underline{\underline{p}}) = D \qquad (A6a)$$

Reversing the two statements on the l.h.s. of (A6a), we obtain

$$(\forall x) (\underline{\underline{p}} \Rightarrow \underline{\underline{m}}), (\exists x) (\underline{\underline{m}} \Rightarrow \neg \underline{\underline{s}}) = \triangle \qquad (A6b)$$

This leads, however, to $\triangle$. Therefore, we convert ($\forall_2 x$) alone

into ($\exists x$), and obtain (A6c),

$$(\forall x) (\neg \underline{\underline{s}} \Rightarrow \underline{\underline{m}}), (\exists x) (\neg \underline{\underline{m}} \Rightarrow \neg \underline{\underline{p}}) = \triangle \text{ and } D \qquad (A6c)$$

and, on reversing the two statements on the l.h.s. of this, we

get a valid conclusion, as in (A6d):

$$(\exists x)(\neg \underline{\underline{p}} \Rightarrow \neg \underline{\underline{m}}), (\forall x)(\neg \underline{\underline{m}} \Rightarrow \underline{\underline{s}}) = (\exists x) (\neg \underline{\underline{p}} \Rightarrow \underline{\underline{s}}) \quad (A6d)$$

Thus, with ($\forall_1$, $\forall_2$) as the first two statements of a

syllogism, we obtain as conclusion either a conclusion with

($\forall x$), using AAA-1, or a conclusion with ($\exists x$) as quantifier,

if the former is not possible. In the former case, the conclusion

is straightforward, and in the latter case, it is one of the four

possibilities thus obtained, using AII-1, but is unique.

(c) Both quantifiers $\exists$.

In this case, since $\exists \otimes \exists = \Delta$, no valid conclusion

can be obtained, and such a pair of statements leads to no

definite conclusion.

(d) Reversal of the resultant

It is to be noted that conclusions such as the right hand

sides of (A4) and (A6d) are not the only ones that are obtained

on combining the two statements on the left hand sides of such

equations. Their reversals are equally valid as conclusions.

Thus, we have also

$$(A4) \ \equiv \ (\forall x) \ (\neg \underline{b} \Rightarrow \neg \underline{a})$$

and

$$(A6d) \ \equiv \ (\exists x) \ (\underline{s} \Rightarrow \neg \underline{p})$$

A case of special interest where such a reversal is made

after applying <u>Darii</u> is the derivation of <u>OAI'</u>-2 (<u>Moravia</u>).

This syllogism reads as follows:

$$(\exists x) \ (\underline{p} \Rightarrow \neg \underline{m}), \ (\forall x) \ (\underline{s} \Rightarrow \underline{m}) = (\exists x) \ (\neg \underline{s} \Rightarrow \underline{p}) \qquad (A7a)$$

The first step is to reverse the term $(\forall x)\ (\underline{s} \Rightarrow \underline{m})$ to get

its equivalent $(\forall x)\ (\neg\underline{m} \Rightarrow \neg\underline{s})$. Then we obtain the

following, having the pattern of $\underline{Darii}$, whose conclusion is given

in (A7b) below.

$$(\exists x)\ (\underline{p} \Rightarrow \neg\underline{m}),\ (\forall x)\ (\neg\underline{m} \Rightarrow \neg\underline{s}) = (\exists x)\ (\underline{p} \Rightarrow \neg\underline{s}) \qquad (A7$$

On reversing the conclusion in (A7b), we obtain the right hand

side of (A7a), thus proving the syllogism $\underline{Moravia}$, using $\underline{Darii}$.

The pattern of $\underline{Moravia}$ is of great interest in relation to the

working out of the conclusion of n statements, under certain

conditions (see Appendix 2).

Thus, we can test if any pair of categorical statements

have a valid conclusion. The conclusion can also be derived in

a straightforward manner by the procedure described here. We

need, for this, only the two syllogisms $\underline{AAA}$-1 ($\underline{Barbara}$) and

$\underline{AII}$-1 ($\underline{Darii}$). It is obvious that this technique can be readily

converted into an algorithm and computerized, but this is not

presented here.

Appendix 2

Procedure for finding the resultant of a chain of n categorical
statements

Suppose that we have a set of $(n + 1)$ terms $\underline{s}$, m_1, m_2,...,

m_j,..., m_{n-1}, p and n relations connecting them made up

of statements of Type 1. Each such statement will be of one

of the forms $\underline{A}$, $\underline{E}$, $\underline{I}$, $\underline{O}$, $\underline{A}'$, $\underline{E}'$, $\underline{I}'$, $\underline{O}'$, and therefore can

have the quantifier $\forall$ or $\exists$, and can have either affirmative

or negative inputs and outputs inside the bracket that is

quantified. Then, if every middle term occurs as an input in

one statement and an output in a different statement, then

n statements connecting the above $(n + 1)$ terms can be written

in the form of a chain as follows:

$$\mathcal{B}_1 \equiv (\mathcal{Q}_1 x) (\underline{s} \Rightarrow \underline{b}_1) \tag{A8a}$$

$$\mathcal{B}_2 \equiv (\mathcal{Q}_2 x) (a_2 \Rightarrow b_2) \tag{A8b}$$

$$\mathcal{B}_j \equiv (\mathcal{Q}_j x) (\underline{a}_j \Rightarrow \underline{b}_j) \tag{A8c}$$

$$\mathcal{B}_n \equiv (\mathcal{Q}_n x) (a_n \Rightarrow \underline{p}) \tag{A8d}$$

The condition for the chain is that the output of $\mathcal{B}_1$ is the

input to $\mathcal{B}_2$ and in general, the output of $\mathcal{B}_j$ is the input

to $\mathcal{B}_{j+1}$. We shall give below the conditions under which

such a chain of implications has a valid conclusion and the

formulae for the conclusions when they are valid.

(a) First quantifier is $\exists$ and all others $\forall$

The quantifiers $(\mathcal{Q}_1 x), \ldots, (\mathcal{Q}_n x)$ can all be only $(\forall x)$

or $(\exists x)$. We also know further that irrespective of the input

to a categorical statement in the form of a universal or

existential quantifier, the output is always $(\exists x)$ (see Table $\mathcal{B}$).

Hence no middle QPL term in (A8) can have an $(\exists x)$, since

$\exists \otimes \exists = \triangle$, and this generates the $\triangle$ state, which is then

carried through to the end, so that the conclusion is also an

indefinite state. On the other hand, the first statement can be

an $(\exists x)$ and all the rest can be $(\forall x)$ in which case, under

suitable conditions, a valid conclusion can be arrived at. For

obtaining such a valid conclusion an additional set of equations

(which we may call as the "transitivity" conditions), as in

(A9), are required:

$$\underline{b}_1 = \underline{a}_2, \quad \underline{b}_2 = \underline{a}_3, \ldots, \underline{b}_j = \underline{a}_{j+1}, \ldots, \underline{b}_{n-1} = \underline{a}_n \qquad (A9)$$

When these are satisfied, we call the chain as a "connected chain".

If the chain is not connected, and any one of the equations in

(A9) is not satisfied, and some $\underline{b}_j = \neg \underline{a}_{j+1}$, then the input to

β_{j+1} is F, so that the SNS state of the output from the implication

is D, which is then carried over right upto the end and the final

output is in the doubtful state.

Suppose, we have both the conditions mentioned above satisfied,

then the statements are implementable in serial order from

β_1 to β_n and the resultant is

$$\Upsilon = (\exists x) (\underline{s} \Rightarrow \underline{p}) \qquad A10)$$

(b) All quantifiers are $\forall$.

If all the quantifiers are $\forall$, and the transitivity conditions

(A9) are all satisfied, then the conclusion is

$$\mathcal{Y}' = (\forall x)\,(\underline{s} \Rightarrow \underline{p}) \tag{A11}$$

Under these conditions, all the equations in (A8) are reversible

and we can write $\mathcal{B}_n$ to $\mathcal{B}_1$ in serial order as follows:

$$\mathcal{B}_n \equiv (\forall x)\,(\neg \underline{p} \Rightarrow \neg \underline{a}_n) \tag{A12a}$$

$$\mathcal{B}_{n-1} \equiv (\forall x)\,(\neg \underline{b}_{n-1} \Rightarrow \neg \underline{a}_{n-1}) \tag{A12b}$$

$$\cdot \ \cdot \ \cdot \ \cdot \ \cdot \ \cdot \ \cdot \ \cdot \ \cdot \ \cdot \ \cdot \ \cdot$$

$$\mathcal{B}_j \equiv (\forall x)\,(\neg \underline{b}_j \Rightarrow \neg \underline{a}_j) \tag{A12c}$$

$$\cdot \ \cdot \ \cdot \ \cdot \ \cdot \ \cdot \ \cdot \ \cdot \ \cdot \ \cdot \ \cdot$$

$$\mathcal{B}_1 \equiv (\forall x)\,(\neg \underline{b}_1 \Rightarrow \neg \underline{s}) \tag{A12d}$$

In this case, the equations in (A12) can be implemented in serial

order as written, and the resultant becomes

$$\mathcal{Y}'' = (\forall x)\,(\neg \underline{p} \Rightarrow \neg \underline{s}) \tag{A13}$$

(A11) and (A13) are equivalent by reversal and are not two

different conclusions. Hence a set of statements of Type 1,

all with $\forall$, which form a connected chain, has a unique conclusion,

in which the first term in the sequence ($\underline{s}$) implies the last term

in the sequence ($\underline{p}$) under the universal quantifier ($\forall x$).

(c) Last statement alone has $\exists$ and all others $\forall$.

If this happens, we reverse all the APL sentences in (A8)

and we obtain a sequence of the same type as (A12), but in

which the first statement (A12a) alone becomes (A14), and all

the others are unaltered.

$$\beta_n \equiv (\exists x)\ (\underline{p} \Rightarrow \underline{a}_n) \tag{A14}$$

Therefore, we need new set of transitivity conditions (A15)

which is the same as (A9), except that the nth one has a

negation in addition, as in (A15) below.

$$\underline{a}_n = \neg \underline{b}_n\ ,\ \underline{a}_{n-1} = \underline{b}_{n-1},\ \cdots,\ \underline{a}_1 = \underline{b}_1 \tag{A15}$$

They are listed in the reverse sequence from n to 1 in (A15),

because they are applied that way. Then the set of statements

β_n to β_1 can be implemented in the reverse order and we get

the resultant,

$$\gamma''' = (\exists x)\ (\underline{p} \Rightarrow \neg \underline{s}) \equiv (\exists x)\ (\neg \underline{s} \Rightarrow \underline{p}) \tag{A16}$$

(d) Summary of this appendix

Summarising what we have said above, a set of n categorical

statements have a valid conclusion only under the following

three conditions:

1) The conclusion is $(\forall x)$ $(\underline{s} \Rightarrow \underline{p})$ if all the quantifiers

are $(\forall x)$ and the sequence of statements form a connected chain

by satisfying (A9). This is the extension of AAA-1 (Barbara),

from 2 to n statements, by adding any number of them, with $(\forall x)$,

to the right; and the conclusion is the same as in Barbara.

2) The conclusion is $(\exists x)$ $(\underline{s} \Rightarrow \underline{p})$ if the first statement

is $(\exists x)$ and all the rest are $(\forall x)$, and they form a connected

chain satisfying (A9). This is obtained by extending AII-1

(Darii), as above to n statements by adding any number of them

with $(\forall x)$ to the right, and the conclusion is the same as in

Darii.

3) The conclusion is $(\exists x)$ $(\neg \underline{s} \Rightarrow \underline{p})$, if the last statement

is an $(\exists x)$ and all the preceding ones are $(\forall x)$ and the connected

chain is formed in the reverse sense from the n th statement

to the first one via a sequence of implications, and the

transitivity equations in (A15) are satisfied. Effectively,

this is also an extension of Darii, but using reverse implications,

($\Leftarrow$), whose properties are indicated in Table 8. It can also

be considered as an extension of OAI'-2 (Moravia), from 2 to n

statements, by adding any number of them with ($\forall$x), but to the

left. However, the validity of Moravia itself rests on that of

Darii, so that we need only Barbara and Darii, for proving the

validity of any chain of Type 1 statements.

It appears as if no valid conclusion is possible under any

other conditions, namely (i) if the chain is not connected by

the transitivity conditions, or (ii) if there is an ($\exists$x) in

any statement other than in the first, or the last, position in

the chain.

The discussion in this appendix has not been thoroughly

rechecked, and it is given here to provoke discussion, rather than

as definitely conclusive results, as they appear to be novel.

Vector-Matrix Representation of Boolean Algebras and Application to Extended Predicate Logic (EPL)

to Extended Predicate Logic (EPL)

Part III - Algebfaic Theory for Categorical Statements in Quantified Predicate Logic (QPL)[*]

G.N. Ramachandran

Gita, 5, 10A Main Road
Malleswaram West
Bangalore 560 055

Vector-Matrix Representation of Boolean Algebras and Application to Extended Predicate Logic (EPL)

Part III — Algebraic Theory for Categorical Statements in Quantified Predicate Logic (QPL)[*]

G.N. Ramachandran

Gita, 5, 10A Main Road
Malleswaram West
Bangalore 560055

Abstract

The paper deals with the application of QPL to categorical syllogisms of traditional logic. It is pointed out that the so-called "doubtful" state (D) of SNS logic and the "indefinite" state ($\triangle$) of extended predicate logic occur naturally in the manipulation of QPL sentences having inputs with the quantifiers $\forall$ and $\exists$. When these are taken into account, and the reversal equivalences $(\forall x)\ (\underline{a} \Rightarrow \underline{b}) \equiv (\forall x)\ (\neg \underline{b} \Rightarrow \neg \underline{a})$ and $(\exists x)\ (\underline{a} \Rightarrow \underline{b}) \equiv (\exists x)\ (\underline{b} \Rightarrow \underline{a})$ are also taken into account, along with the inclusion condition $(\forall x)\ (\underline{a} \Rightarrow \underline{b}) \supset (\exists x)\ (\underline{a} \Rightarrow \underline{b})$, all the 18 categorical syllogisms found in standard treatments can be algebraically derived, by reversing either, or both, of the statements in the two input premises, and checking all the four possible rearrangements for a definite conclusion. It is proved that at most one of the four leads to a definite conclusion and the other/three lead either to $\triangle$ for the quantifier or D for the state of the SNS relation that is quantified. These considerations

are summarized in a set of four algorithmic procedures, using

which the validity or non-validity of any mood out of the 64

that are possible (by using the standard forms $\underline{A}$, $\underline{E}$, $\underline{I}$, $\underline{O}$ with

$\underline{s} \Rightarrow \underline{p}$ or $\underline{s} \Rightarrow \neg\underline{p}$) in the four figures can be established

and the conclusion derived (as one of the forms $\underline{A}$, $\underline{E}$, $\underline{I}$, $\underline{O}$)

when it is valid. Surprisingly, not only was it possible to

derive the 18 syllogisms listed traditionally, but in addition,

18 more valid new ones could be derived which, however, have

conclusions in one of the four forms $\underline{A}'$, $\underline{E}'$, $\underline{I}'$, $\underline{O}'$ (newly defined,

having $\neg\underline{s} \Rightarrow \underline{p}$ and $\neg\underline{s} \Rightarrow \neg\underline{p}$ inside the bracket that is

quantified). These have been listed and mnemonic names coined

for them. A very general theorem is obtained, namely that any

pair of categorical statements, with $\forall$ or $\exists$ as quantifier, has

no valid conclusion if both are $\exists$, $\exists$; has always a valid

conclusion if both are $\forall$, $\forall$; but may be valid or invalid if one

has the quantifier $\forall$ and the other $\exists$. In proving this, it is

shown that every pair of categorical statements can be brought

into isomorphism with the pair in one of the two classical

syllogisms _Barbara_ or _Darii_, which are the only independent

patterns (moods) possible for the pair. This has also been

extended to the case of $\underline{n}$ categorical statements in sequence,

and it is shown that there are only three independent patterns

possible for them to have a valid conclusion — one as an extension

of _Barbara_ with all statements quantified by $\forall$, and two as

extensions of _Darii_, with one $\exists$ and all the rest being $\forall$.

————

CONTENTS

Part III — Algebraic Theory for Categorical Statements in Quantified Predicate Logic (QPL)

1. Introduction

Studies on the representation of logical connectives by operators based on Boolean vectors and matrices were started some five years ago with special reference to the computerization of logic. The earlier studies were written up in the form of reports from the Mathematical Philosophy Group of the Indian Institute of Science (Ramachandran, 1979, 1980; Ramachandran and Thanaraj, 1980a, 1980b, 1981; H. Ramachandran and Thanaraj, 1981). From 1981 onwards, a very general Boolean vector-matrix algebra was developed for sentential logic which extended the standard propositional calculus with two truth values T and F, to a formalism isomorphic with the Boolean algebra of genus 2 having four states — which in logic are T(true), F(false), D(doubtful) and X(impossible). An electronic analog machine was also constructed (Ramachandran, Johnson and Thanaraj, 1980), using standard logic

gates to represent all the operators of this extended propositional

calculus, for which the name Syād-Nyāya-System logic was given

(syād = may be, nyāya = logic, in Sanskrit). The electronic

circuits in this machine closely represent the algebraic formalism

that was developed for this purpose. A brief summary of all these

was written up as a paper for Current Science (Ramachandran, 1982)

and it summarized the properties of the Boolean algebraic

representation of both the logical states and the logical connectives

of propositional calculus, in its extended form of SNS logic.

Thereafter, the Boolean algebraic technique was extended to

predicate logic, and two papers (Part I and Part II with the same

title as the present paper) were published also in Current Science

(Ramachandran 1983 a,b).

In the papers dealing with predicate logic, it was shown that

the boolean algebra of genus 3 and, in particular, its vector-matrix

representation by three-element Boolean vectors and 3X3 Boolean

matrices, leads to the extension of the four well-known states

in quantified predicate logic (QPL), (namely "for all", "there

exists", "not for all" and "for none"), by adding four more

states ("for some only", "for all or none", "indefinite" and

"impossible"). Also the nature of the QPL connectives between

such quantified SNS terms, as in $(\forall x)\ (\underline{a}(x)) \implies (\exists y)\ (\underline{b}(y))$

or $(\forall x)\ (\underline{a}(x))\ \&\ (\exists x)\ (\underline{b}(y)) = \underline{c}$ was discussed in great

detail in Ramachandran (1983 a,b). (We have given the name

"Extended Predicate Logic" (EPL) for this extended form of QPL).

It was shown that logical connectives of EPL are, in general,

representable by 3X3 matrices, or operators containing Boolean

sums ($\oplus$) or products ($\otimes$). In fact, there are $(8 \times 8) = 64$

possible matrices having the nature of an "and" and 64 having the

nature of an "or". We shall not go into the details of these

here except to point out that they all deal with QPL sentences

in which the connective is also a QPL operator. We shall call

such sentences as "Sentences of Type-2". The formalism needed to

deal with single sentences of Type-2 of all types was developed in

Parts I and II of this series and, on examining the literature,

it was found that this type of connective has not been stated

and investigated so extensively before.

However, it was brought to our attention that these

operators, or connectives, do not deal with sentences having

logical connectives of another type in QPL — such as, "Given

$(\forall x)$ $(\underline{a}(x) \implies \underline{b}(x))$ and the QPL input $(\exists x)$ (a_T), what is

the QPL output containing $\underline{b}(x)$?" As we will see later, it is

$(\exists x)$ (b_T). Sentences of this type, in which the same variable x

occurs throughout the sentence and both the input and the output

have the same scope for its domain, may be called as "Sentences

of Type-1". The purpose of this paper is to examine and discuss

the properties of such sentences, pairs of them and a connected

chain of them and to give the conclusion in each case for any

given input, or inputs. In particular, we confine ourselves to

the so-called "categorical" statements which occur in the classical

syllogisms of traditional logic (Church, 1966). We shall discuss

in extenso; but we are able to design a very general procedure whereby the conclusion of any pair or statements from out of the eight forms A, E, I, O, A', E', I', O' can be obtained by using only two standard results — namely those which correspond to the classical syllogisms Barbara and Darii. Thus, it is not necessary to remember the symbols of all valid syllogism, but instead work out the conclusion of any pair of categorical statements by a straightforward derivational process by converting them into a form which is isomorphous with Barbara or Darii.

This treatment has been extended to n categorical statements in the form of a connected sequence (see Section 4 for the definition of connectedness), and it is found that, by applying the same two standard syllogisms Barbara and Darii, the conclusion, when it is valid, of an n-statement chain can be worked out, and that this occurs only for three types of sequences, whose nature can be specified. This is described in Section 5.

2. Outline of SNS logic with special reference to the connective for implication.

(a) Description of the states and the unary connective "implies"

As mentioned above, the main purpose of this paper is to derive the conditions governing a pair of (and sequences of n) categorical statements, for them to lead to a valid conclusion, and to work out formulae for deriving the conclusion in such a case. We found that the principles based on the Boolean algebraic formalism that were developed for SNS and EPL, are required for doing these. A long report (Ramachandran 1983 c) on these studies has been prepared for private circulation, and the present paper is a condensation of that, written such that only the notation and ideas commonly used in propositional calculus (PC) and QPL are employed for the proofs. Therefore, the essential ideas connected with SNS and EPL are briefly presented in this section and the next section — particularly with special reference to those aspects that find application in the theory of this paper.

In particular, the Boolean algebraic representation of the logical

connectives lead to some new features regarding "implication"

in Propositional Calculus and Predicate Logic. These new ideas

play an important part in the derivations worked out here and

their basic principles are described in Sections 2 and 3.

Table 1. Truth Table of "a implies b" and "a and b".

We shall start with the truth table of $a \Rightarrow b$, which is

shown in Table 1(a), in which $T \longmapsto 1$ ($\longmapsto$ = "corresponding to",

or "leads to") and $F \longmapsto 0$. The nature of the truth value of

b when $a = F$ leads to some very interesting new ideas. Thus

from Table 1a, we have

$$a_T \longmapsto b_T, \text{ and } a_F \longmapsto b_T \text{ or } b_F \ (b \text{ "may be" T or F}) \tag{3}$$

Note the slight change in nomenclature about the repercussions

of a being false, namely that b "may be" T or F. In standard

literature on logic, it is said that a_F implies both b_T and b_F.

What we mean is essentially the same idea, namely that the false

Table 1. Truth Table of "a implies b"   and   "a and b"

(a) a ⟹ b   (b) a & b

a \ b	T	F
T	1	0
F	1	1

a \ b	T	F
T	1	0
F	0	0

truth value for $\underline{a}$ cannot distinguish between b_T and b_F and

cannot give a unique T or F truth value as the answer; but the

situation is better understood by saying that, when $\underline{a}$ is false,

$\underline{b}$ could have the truth value T, or it could have the truth value

F, and there is no way of saying which is correct. In other words,

the information content of what the implication $\underline{a} \Longrightarrow \underline{b}$ gives,

when $\underline{a}$ is F, is nil. Thereafter, $\underline{b}$ cannot be the input of

another logical statement, connecting it, say, to $\underline{c}$ — to give

useful results about $\underline{c}$.

The situation when "$\underline{a}$ & $\underline{b}$" is true, and $\underline{a}$ is given, can

be deduced from the truth Table 1(b). In this case also,

$$a_T \longmapsto b_T, \text{ but } a_F \longmapsto \neg b_T \text{ and } \neg b_F \text{ (i.e. } \underline{b} \text{ can}$$
$$\text{neither be true, nor false)} \tag{4}$$

In this case, when $\underline{a}$ is false, the statements "$\underline{a}$ & $\underline{b}$" itself

becomes meaningless for $\underline{a}$ itself must be true for "$\underline{a}$ & $\underline{b}$"

to be true. In such a situation, to ask for the state of $\underline{b}$ is

meaningless and it is an impossibility for $\underline{b}$ to be conceived

of, if $\underline{a}$ is F, and $\underline{a} \,\&\, \underline{b}$ is true. We denote this by the

symbol X standing for the "impossible" state.

A very simple representation of the "doubtful" state D and

the "impossible" state X can be given by using two-element

Boolean vectors $(a_\alpha \quad a_\beta)$ to represent the truth value of a term

$\underline{a}$. We take the value 1 or 0 of a_α to indicate whether T exists

or does not exist and similarly the value of a_β to indicate

whether F exists or does not exist. Then, we have

$$T = (1 \quad 0), \quad F = (0 \quad 1), \quad D = (1 \quad 1), \quad X = (0 \quad 0) \tag{5}$$

The representation $(1 \quad 0)$ for T is obvious, since truth exists

and falsehood is absent, and similarly $(0 \quad 1)$ for F. On the

other hand, in the doubtful state, the term concerned can have

the state T, or the state F, and we are unable to say whether it is

always one or the other. Hence, the representation by $(1 \quad 1)$

is reasonable. On the other hand, if we take (0   0), both

truth is absent and the falsehood is absent and the term is

an impossible one. In general, the state X comes as the

consequence of a contradiction, and though we have worked out

how the consequence of a contradiction can be turned back and

useful results obtained in logical arguments (as in a <u>reductio</u>

<u>ad absurdum proof</u>), we shall not discuss these here, since the

present paper deals essentially with implications. It may be

mentioned that the output of an implication as such is never

the impossible state X, but only one of T, F, or D.

We shall now focus our attention on the implication relation

and its use as a unary relation between $\underline{a}$ and $\underline{b}$, when $\underline{a} \implies \underline{b}$

is true. This is expressed by using a logical operator

$\underset{\sim}{I}$ (for "implies") connecting $\underline{a}$ and $\underline{b}$ in the form of an equation

$\underline{a} \underset{\sim}{I} = \underline{b}$. We have shown (see e.g. Ramachandran, 1982) that

$\underset{\sim}{I}$ can be represented by a 2X2 Boolean matrix $|I|$, which has

for its elements the same ones as in its truth table (Table 1(a)),

and then it is readily verified that

$$(1 \quad 0) \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} = (1 \quad 0); \quad (0 \quad 1) \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} = (1 \quad 1) \qquad (6a)$$

corresponding to

$$a_T \underset{\sim}{I} = b_T \quad ; \quad a_F \underset{\sim}{I} = b_F \qquad (6b)$$

The full ramifications of the Boolean vector-matrix formalism

will not be commented upon here, except for one main idea, namely

that of reversing a logical equation. Thus, if $\underset{=}{a} \underset{\sim}{I} = \underset{=}{b}$ is valid,

then in the reverse sense from $\underset{=}{b}$ to $\underset{=}{a}$, there is a logical

equation. If we denote it by $\underset{=}{b} \underset{\sim}{J} = \underset{=}{a}$, then the matrix $|J|$ for

it is the transpose of the matrix $|I|$ for the forward relation.

This comes out of the general theory of relations (see Ramachandran,

1983a). On testing the effect of this $|J|$, we obtain, analogous

to (6), the results

$$(1 \quad 0) \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = (1 \quad 1) ; \quad (0 \quad 1) \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = (0 \quad 1) \qquad (7a)$$

which yield the logical correspondences

$$b_T \, \underset{\sim}{J} = a_D \; ; \quad b_F \, \underset{\sim}{J} = a_F \, , \text{ or } \quad \neg \, \underline{b} \Longrightarrow \neg \, \underline{a} \tag{7b}$$

Thus, we get the consequence that an implication from $\underline{a}$ to $\underline{b}$

has an equivalent logical relation from $\underline{b}$ to $\underline{a}$, namely

$$(\underline{a} \Longrightarrow \underline{b}) \equiv (\neg \, \underline{b} \Longrightarrow \neg \underline{a}) \tag{7c}$$

This equivalence (or tautology) is taken as an axiom in standard

treatments of PC, but we obtain it $_\wedge^{here}$ as a consequence of the vector-

matrix formalism, based on the definition of a logical operator

by its truth table. In this paper, we shall take the equivalence

(7) as a well established theorem and the generalization of this

to predicate calculus plays a fundamental role in our approach

to categorical syllogisms.

(b) Two implications in succession in PC

Consider the equivalence for$_\wedge$pair of implications, usually

taken as an axiom:

$$(\underline{a} \Longrightarrow \underline{b} \, , \, \underline{b} \Longrightarrow \underline{c}) \equiv (\underline{a} \Longrightarrow \underline{c}) \tag{8a}$$

However, this can be derived in our formulation by the fact that,

for the matrix $|I| = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ representing the connective $\underset{\sim}{I}$,

the equation $|I|I| = |I|$ holds, so that

$$(\underline{a}\,\underset{\sim}{I} = \underline{b}\, , \,\underline{b}\,\underset{\sim}{I} = \underline{c}) \equiv \left(\underline{a}\,\underset{\sim}{I}\,\underset{\sim}{I} \equiv \underline{a}\,\underset{\sim}{I} = \underline{c}\right) \tag{8b}$$

However, the proof is immaterial, and we take (8) as a theorem

for our treatment in this paper.

we can reverse the l.h.s. of (8) and obtain

$$(\neg \underline{c} \Rightarrow \neg \underline{b},\ \neg \underline{b} \Rightarrow \neg \underline{a}) \equiv (\neg \underline{c} \Rightarrow \neg \underline{a}) \tag{9'}$$

But, the right hand side of (9') is the reverse of the r.h.s.

of (8a) and is equivalent to it. Hence (8a) and (9) express the

same result, in two different ways, and they are equivalent to

one another.

However, if in (8a) the second implication is from $\neg \underline{b}$ to $\underline{c}$,

we obtain,

$$(\underline{a} \Rightarrow \underline{b}\, , \,\neg \underline{b} \Rightarrow \underline{c}) = \underline{a}\,)(\,\underline{c} = \underset{\sim}{D} \tag{10}$$

The statement $\underline{a}$ $)($ $\underline{c}$ means "$\underline{a}$ is disconnected with $\underline{c}$",

or both $\underline{a} = T$, and $\underline{a} = F$, put as inputs to the l.h.s. of (10),

lead/to $\underline{c}$ being D. *only* (If $\underline{a} = T$, $\underline{b} = T$, $\neg \underline{b} = F$, leading to

$\underline{c} = D$; and if $\underline{a} = F$, $\underline{b} = D$, $\neg \underline{b} = D$ and $\underline{c} = D$). Independent

of our proof, (10) is an obvious result in PC. We shall use the

symbol $\underline{D}$, as in r.h.s. of (10), to indicate that the l.h.s. of

the equation leads to no definite consequence.

(c) A number (n) of implications in sequence

Consider

$$\underline{a}_1 \Longrightarrow \underline{b}_1, \quad \underline{a}_2 \Longrightarrow \underline{b}_2, \ldots, \underline{a}_n \Longrightarrow \underline{b}_n \tag{11}$$

It is obvious that

$$\text{if } b_1 = a_2, \quad b_2 = a_3, \ldots, b_n = a_n \tag{12a}$$

$$\text{then } (11) \equiv (\underline{a}_1 \Longrightarrow \underline{b}_n) \tag{12b}$$

On the other hand, between the equations in (11),

$$\text{if } b_j = \neg a_{j+1}, \text{ for some } j = 1, 2, \ldots, n-1 \tag{13a}$$

$$\text{then } (11) \equiv (\underline{a}_1)(\underline{b}_n) = \underline{D} \tag{13b}$$

However, this can be derived in our formulation by the fact that,

for the matrix $|I| = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ representing the connective $\underset{\sim}{I}$,

the equation $|I||I| = |I|$ holds, so that

$$\left(\underline{\underline{a}} \, \underset{\sim}{I} = \underline{\underline{b}} \, , \, \underline{\underline{b}} \, \underset{\sim}{I} = \underline{\underline{c}}\right) \equiv \left(\underline{\underline{a}} \, \underset{\sim}{I} \, \underset{\sim}{I} \equiv \underline{\underline{a}} \, \underset{\sim}{I} = \underline{\underline{c}}\right) \tag{8b}$$

However, the proof is immaterial, and we take (8) as a theorem

for our treatment in this paper.

We can reverse the l.h.s. of (8) and obtain

$$\left(\neg \underline{\underline{c}} \Rightarrow \neg \underline{\underline{b}}, \neg \underline{\underline{b}} \Rightarrow \neg \underline{\underline{a}}\right) \equiv \left(\neg \underline{\underline{c}} \Rightarrow \neg \underline{\underline{a}}\right) \tag{9$\cdot$}$$

But, the right hand side of (9·) is the reverse of the r.h.s.

of (8a) and is equivalent to it. Hence (8a) and (9) express the

same result, in two different ways, and they are equivalent to

one another.

However, if in (8a) the second implication is from $\neg \underline{\underline{b}}$ to $\underline{\underline{c}}$,

we obtain,

$$\left(\underline{\underline{a}} \Rightarrow \underline{\underline{b}} \, , \, \neg \underline{\underline{b}} \Rightarrow \underline{\underline{c}}\right) = \underline{\underline{a}} \,)(\, \underline{\underline{c}} = \underset{\sim}{\underline{\underline{D}}} \tag{10}$$

The statement $\underline{a})(\underline{c}$ means "$\underline{a}$ is disconnected with $\underline{c}$",

or both $\underline{a} = T$, and $\underline{a} = F$, put as inputs to the l.h.s. of (10),

lead only to $\underline{c}$ being D. (If $\underline{a} = T$, $\underline{b} = T$, $\neg \underline{b} = F$, leading to

$\underline{c} = D$; and if $\underline{a} = F$, $\underline{b} = D$, $\neg \underline{b} = D$ and $\underline{c} = D$). Independent

of our proof, (10) is an obvious result in PC. We shall use the

symbol $\underset{\sim}{D}$, as in r.h.s. of (10), to indicate that the l.h.s. of

the equation leads to no definite consequence.

(c) A number (n) of implications in sequence

Consider

$$\underline{a}_1 \Rightarrow \underline{b}_1, \quad \underline{a}_2 \Rightarrow \underline{b}_2, \ldots, \underline{a}_n \Rightarrow \underline{b}_n \tag{11}$$

It is obvious that

$$\text{if } b_1 = a_2, \quad b_2 = a_3, \ldots, b_n = a_n \tag{12a}$$

$$\text{then } (11) \equiv (\underline{a}_1 \Rightarrow \underline{b}_n) \tag{12b}$$

On the other hand, between the equations in (11),

$$\text{if } b_j = \neg a_{j+1}, \text{ for some } j = 1, 2, \ldots, n-1 \tag{13a}$$

$$\text{then } (11) \equiv (\underline{a}_1)(\underline{b}_n) = \underset{\sim}{D} \tag{13b}$$

We shall not indicate the proof of this in any detail, but

it readily follows from (10).

The same conditions (12a) are sufficient for the equations

in (11) to be manipulated in the reverse sense. Thus, we obtain

for the resultant

$$\neg \underline{b}_n \Rightarrow \neg \underline{a}_n, \ldots, \neg \underline{b}_1 \Rightarrow \neg \underline{a}_1$$

$$\equiv (\neg \underline{b}_n \Rightarrow \neg \underline{a}_1) \tag{14}$$

It is to be noted that the r.h.s. of (14) is identical with (12b)

Also, if (13a) holds, and

$$\neg b_{j+1} = a_j \text{, for some } j = 1, 2, \ldots, n-1 \tag{15a}$$

then $\underset{\wedge}{\overset{(l.h.s \, of)}{\diagup}}$ (14) $\equiv (\neg b_n))((\neg a_1)$ \hfill (15b)

which, again, is identical with (13b).

Hence, for n implications in a sequence in PC, the net

resultant is the same, irrespective of the direction in which

the implications are applied — forward or reverse. This is not

the case in QPL, as we shall see later.

Thus, we have proved the condition for a pair of implications

to lead to a valid resultant implication, and extended it for

n implications. These results are obvious, and well understood,

but the way in which they are presented here have an important

application to the theory of our paper, applied to implications

in statements of Type-1 in QPL.

3. Essentials of QPL needed for our derivations
(a) Sentences and states in EPL

In the four types of categorical statements $\underline{A}$, $\underline{E}$, $\underline{I}$, $\underline{O}$

the standard QPL states $\forall$ and $\exists$ are used, with the implication

being affirmative or negative. Thus, we may symbolically express

the four forms of categorical statements by (16 a, b, c, d).

$$\underline{A} \equiv (\forall x)\,(\underline{a}(x) \Longrightarrow \underline{b}(x)) \ , \quad \underline{E} \equiv (\forall x)\,(\underline{a}(x) \Longrightarrow \neg\,\underline{b}(x)) \qquad (16a,\ b)$$

$$\underline{I} \equiv (\exists x)\,(\underline{a}(x) \Longrightarrow \underline{b}(x)) \ , \quad \underline{O} \equiv (\exists x)\,(\underline{a}(x) \Longrightarrow \neg\,\underline{b}(x)) \qquad (16c,\ d)$$

In these, the first two for $\underline{A}$ and $\underline{E}$ follow the standard practice

adopted traditionally for QPL. On the other hand, for $\underline{I}$ and $\underline{O}$

we have made a small change, namely writing the PC statements

within the brackets that is quantified as $\underline{a}(x) \implies \underline{b}(x)$, instead

of $\underline{a}(x) \,\&\, \underline{b}(x)$ which is commonly used. The use of the logical

$\&$ is very significant in that the relation between $\underline{a}$ and $\underline{b}$

under the existential quantifier $\exists$ is a symmetric one. Thus,

"Some $\underline{a}$ are $\underline{b}$" can also mean "Some $\underline{b}$ are $\underline{a}$". This is particularl

evident from the Venn diagram in Fig. 1(b) below. Therefore,

if we wish to emphasize the direction in which the implication

is applied, namely from $\underline{a}$ to $\underline{b}$ or from $\underline{b}$ to $\underline{a}$ we write the

same statement as

$$(\exists x)\ (\underline{a} \implies \underline{b}) \equiv (\exists x)\ (\underline{b} \implies \underline{a}) \tag{17}$$

In Eqn (17) we have omitted the variable x for $\underline{a}$ and $\underline{b}$ and

put it only for the quantifier $(\exists x)$. This practice will be

done throughout hereafter since all statements of Type-1 have the

same variable x for the terms inside the bracket that is

quantified. The significant thing to be noticed about Eqn (17)

is that we have in effect reversed the relation "Some $\underline{a}$ are $\underline{b}$"

into "Some $\underline{b}$ are $\underline{a}$" and shown that the two are equivalent in the

same sense as we have used for propositional calculus.

However, the reversal under the universal quantifier $(\forall x)$

is clearly different. As is very well understood, the PC sentence

that is under the universal quantifier obeys all the rules of PC

and therefore on reversing (16a), we will get the equation,

$$(\forall x)\ (\underline{a} \Rightarrow \underline{b}) \equiv (\forall x)\ (\neg \underline{b} \Rightarrow \neg \underline{a}) \tag{18}$$

We shall proove both (17) and (18) below when discussing the

properties of the implication under quantification. It is clearly

seen from (17) and (18) that the reversal of the PC sentence

is different according as whether it is quantified by $(\forall x)$ or

$(\exists x)$. As a consequence, in a sequence or two or more quantified

implication, the reversed [*sequence*] can produce a different result than

the forward sequence. We will have many occasions to see this

as we go along.

Before explaining the properties of the implication statements

$(\forall x)$ $(\underline{a} \Longrightarrow \underline{b})$ and $(\exists x)$ $(\underline{a} \Longrightarrow \underline{b})$, we shall briefly mention

the properties of the QPL statements in general. In attempting

to obtain the Boolean algebraic representation of statements

like those mentioned above, we found the need to have three

basic states similar to the two basic states $(T = (1 \quad 0)$,

$F = (0 \quad 1))$ for SNS logic. (See Ramachandran 1983 a, b).

Denoting a QPL vector by $(\gamma \quad \delta \quad \epsilon)$, the three basic states are

$(1 \quad 0 \quad 0) =$ "for all" , $(0 \quad 0 \quad 1) =$ "for none" ,

$(0 \quad 1 \quad 0) =$ "for some only and not for all or for none"

(19a,b,c)

Putting the Boolean values 1 and 0 for each of $(\gamma \quad \delta \quad \epsilon)$,

we obtain the total of 2X2X2 = 8 different Boolean vectors that

are possible in this algebra. On examination, the other five

states are found to have the logical equivalences

$$(1 \quad 1 \quad 0) = \text{"for all or some"} \equiv \text{"there exists"}, \qquad (20a)$$

$$(0 \quad 1 \quad 1) = \text{"for none or some"} \equiv \text{"not for all"} \qquad (20b)$$

$$(1 \quad 0 \quad 1) = \text{"for all or none"}, \quad (1 \quad 1 \quad 1) = \text{"indefinite"},$$

$$(0 \quad 0 \quad 0) = \text{"impossible"} \qquad (20c, \ d, \ e)$$

The general theory of the eight states in what may be called

extended predicate logic (EPL) is given in (Ramachandran 1983 a,b)

and applied to QPL sentences containing statements of the Type-2.

Here we will use only the two states $(\forall x)$ and $(\exists x)$ and will

obtain the other two states Φ (for none), and Λ (not for all)

by having negated statements **under** the quantifier. We shall not

also use the Boolean algebraic notation as ~~which~~ given by (19)

and (10) which was extensively employed for Type-2 statements.

Instead, we shall use the conventional logical formulae. But, as

we need the indefinite state (Δ x) very often, and since it occurs

frequently on combining two or more categorical statements, we shall

briefly discuss its properties here.

Just like the state D, = (1   1) of SNS, which means

"either true or false", the state Δ , = (1 1 1) of QPL

also has no information content and cannot be carried over in

the succeeding steps of$\overset{a}{\wedge}$connected series of statements. This

is best seen by writing it as,

(1 1 1) = (1 1 0) or (0 0 1) = Either "there exists"

$$\text{or "there does not exist"} \tag{21}$$

We see that there is complete doubt about the quantifier, namely

whether it is $\overset{\text{"there exist"}}{/}$, or the negation of it, namely "there does not

exist". We can also $\overset{obtain}{\underset{\wedge\wedge\wedge}{\cdot\cdot\cdot}}$ a (1 1 1) as the superposition of all

the three basic states in (19), but this interpretation is not

used in the present paper. As we shall illustrate by examples

later, the output of many pairs of statements of Type 1 containing

an implication leads to (Δ x) when certain quantified inputs

are put in. Just as we $\overset{do\ with}{\underset{\wedge\ \wedge\ \sim}{}}$ D, we shall simply write the symbol

Δ at the conclusion to indicate that the conclusion$\overset{does\ not\ have}{\underset{\wedge}{}}\overset{}{\underset{\wedge}{}}$ any

informational value (instead of writing in full (Δ x)($\underline{a}$(x) $\Longrightarrow$ $\underline{b}$(x)).

(b) Outputs of single implication statements of Type-1 in QPL.

A summary of the eight independent input-output relations

for implication of the form found in (16a) and (16c) are

summarized in Table 2.

Table 2. Algorithms for obtaining the outputs of Type-1
statements of QPL

The proofs of these can be readily derived by drawing Venn

diagrams of the corresponding sets which have the relevant relations

between them. We shall only indicate the proof for Sl. Nos.

1 to 4 and 7, as they cover all essential ideas needed for the

proof also for the remaining cases.

When the first one with Sl.No. 1 is translated into set

theory, it says in effect — "In the universe of discourse if

any member has the property $\underline{a}$, then it has the property $\underline{b}$", and

this is indicated in Fig.1(a). As will be seen from the figure, when all

Table 2. Algorithms for obtaining the outputs of Type-1 statements of QPL

Sl. No.	Input	Statement	Output
		Implies ($\Rightarrow$)	
1	$(\forall x)\ (\underline{a})$	$(\forall x)\ (\underline{a} \Rightarrow \underline{b})$	$(\exists x)\ (\underline{b})$
2	$(\forall x)\ (\neg \underline{a})$	$(\forall x)\ (\underline{a} \Rightarrow \underline{b})$	D
3	$(\exists x)\ (\underline{a})$	$(\forall x)\ (\underline{a} \Rightarrow \underline{b})$	$(\exists x)\ (\underline{b})$
4	$(\exists x)\ (\neg \underline{a})$	$(\forall x)\ (\underline{a} \Rightarrow \underline{b})$	D
5	$(\forall x)\ (\underline{a})$	$(\exists x)\ (\underline{a} \Rightarrow \underline{b})$	$(\exists x)\ (\underline{b})$
6	$(\forall x)\ (\neg \underline{a})$	$(\exists x)\ (\underline{a} \Rightarrow \underline{b})$	D
7	$(\exists x)\ (\underline{a})$	$(\exists x)\ (\underline{a} \Rightarrow \underline{b})$	Δ
8	$(\exists x)\ (\neg \underline{a})$	$(\exists x)\ (\underline{a} \Rightarrow \underline{b})$	Δ and D
		Reverse implies ($\Leftarrow$)	
9	$(\forall x)\ (\underline{a})$	$(\forall x)\ (\underline{b} \Leftarrow \underline{a})$	$(\exists x)\ (\underline{b})$
10	$(\exists x)\ (\underline{a})$	$(\forall x)\ (\underline{b} \Leftarrow \underline{a})$	$(\exists x)\ (\underline{b})$
11	$(\forall x)\ (\underline{a})$	$(\exists x)\ (\underline{b} \Leftarrow \underline{a})$	$(\exists x)\ (\underline{b})$

Rules for reversal

$(\forall x)\ (\underline{a} \Rightarrow \underline{b}) \equiv (\forall x)\ (\neg \underline{b} \Rightarrow \neg \underline{a});\quad (\exists x)\ (\underline{a} \Rightarrow \underline{b}) \equiv (\exists x)(\underline{b} \Rightarrow \underline{a})$

$(\forall x)\ (\underline{b} \Leftarrow \underline{a}) \equiv (\forall x)\ (\neg \underline{a} \Leftarrow \neg \underline{b});\quad (\exists x)\ (\underline{b} \Leftarrow \underline{a}) \equiv (\exists x)(\underline{a} \Leftarrow \underline{b})$

the members of the set $\underline{a}$ is exhausted, we cover only some (or all)

of set $\underline{b}$, so that the output is only $(\exists x)$ $(\underline{b})$. Obviously,

$(\exists x)$ can also take care of the case when the set $\underline{a}$ = set $\underline{b}$.

The above property that $(\forall x)$ $(\underline{a})$, when fed into $(\forall x)$ $(\underline{a} \Longrightarrow \underline{b})$,

gives as output only $(\exists x)$ $(\underline{b})$ (and not $(\forall x)$ $(\underline{b})$) is a very

important result and is vital for all the succeeding arguments.

As regards Sl. No. 2, we note from Fig 1(a) that points

inside the square outside the shaded region, marked $\underline{a}$, can be

either in the set $\underline{b}$ or the set $\neg \underline{b}$. Therefore, $\neg \underline{a}$ can correspond

to either $\underline{b}$ or $\neg \underline{b}$, when $(\forall x)$ $(\underline{a} \Longrightarrow \underline{b})$, so that the output $\underline{b}$

has the SNS state D, as indicated.

Considering Sl. No.3, if $(\exists x)$ $(\underline{a})$ is the input, then we

will have some part of the shaded area of set $\underline{a}$ in Fig. 1(a)

occupied. Clearly this corresponds to $(\exists x)$ $(\underline{b})$ (as given in

column 3 for this case), for the limiting case, when set $\underline{a}$ = set $\underline{b}$

and all of set a is included in the input, is also contained

under the quantifier $(\exists x)$.

As regards Sl.No. 4, it can be verified from Fig. 1(a) that

the input $(\exists x)$ $(\neg a)$ can correspond to regions demarcated for

set b or set $\neg b$, so that the output b will be in the SNS state

D, and the quantifier state $(\exists x)$. We denote this by the symbol

D in the third column, indicating that b cannot be carried over

to further steps in any sequence of statements.

The outputs in Sl.Nos. 5 and 6 are immediately obvious from

Fig. 2(a), following arguments like the above, and they are not

discussed in detail.

The result, (Δx) $(b(x))$ of Sl.No. 7 is understandable from

Figs. 2(b) and 2(c). The sets a and b, with the same intersection

a & b as in Fig. 2(a), are shown in Figs 2(b) and (c), with two

different possibilities for the region of the diagram covered by

$(\exists x)$ $(\underline{a})$. In Fig. 2(b), where the region $(\exists x)$ $(\underline{a})$ is included

in $\underline{a}$ & $\underline{b}$, the output is $(\exists x)$ $(\underline{b})$; but if it is as in Fig. 2(c),

then the region covered by the input $(\exists x)$ $(\underline{a})$ is not included

in $\underline{a}$ & $\underline{b}$, and the output is $\neg(\exists x)$ $(\underline{b}) \equiv (\cancel{\exists}\,' x)$ $(\underline{b})$ (see Part II

for the meaning of the symbol $\neg$). Hence, for the full output

$\underline{b}$, the quantifier is "$(\exists x)$ or $(\cancel{\exists})$" = (1 1 1), as per Eqn. (21),

which leads to the "indefinite" state (Δx).

The result that the QPL output becomes (Δx) $(\underline{b})$ when __both__

the implication and the input are in the $(\exists x)$ states, is very

important for the discussions in the succeeding sections. The

case of Sl.No. 8 is a combination of the conditions in Sl.Nos. 4

and 7, and hence we write the output as Δ and D , and it has

no information value, for succeeding steps, if any, containing $\underline{b}$.

Fig.1 Venn diagrams for implications with $(\forall x)$, illustrating
       (a) $(\forall x)$ $(\underline{a} \Rightarrow \underline{b})$, and (b) $(\forall x)$ $(\neg\underline{b} \Rightarrow \neg\underline{a})$.

Fig.2 Venn diagrams for the implications $(\exists x)$ $(\underline{a} \Rightarrow \underline{b}) \equiv$
       $(\exists x)$ $(\underline{b} \Rightarrow \underline{a})$. (a) Existence of $\underline{a}$ & $\underline{b}$,
       (b) $(\exists x)$ $(\underline{a}) \subset \underline{a}$ & $\underline{b}$, (c) $(\exists x)$ $(\underline{a}) \not\subset \underline{a}$ & $\underline{b}$.
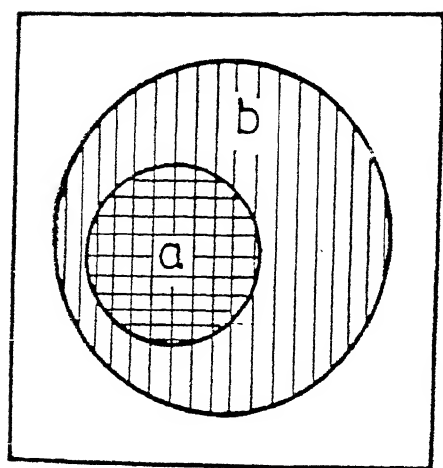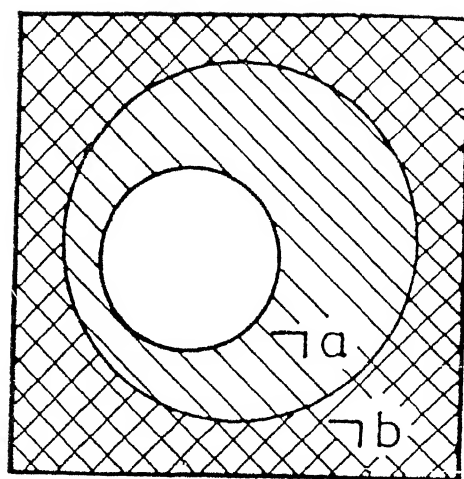
Fig.1
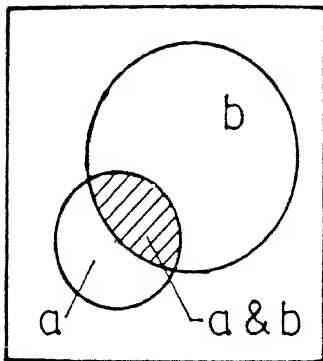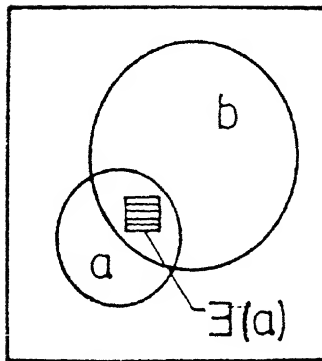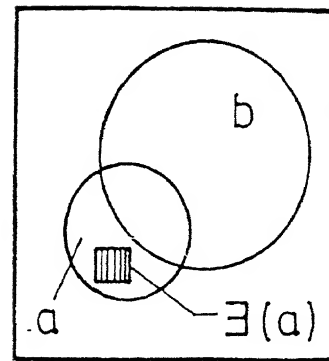
(a) (b) (c)

Fig. 2

The last three lines in Table 2 corresponding to Sl.Nos.

9, 10 and 11 require no comment. They correspond to the reverse

operator of implication ($\Leftarrow$), and only the three cases where

a useful conclusion is obtained are listed, since all the rest

lead either to D or $\triangle$ or both. The implication is in the reverse

direction and it corresponds to the mathematical condition

"necessary", since the forward implies ($\Rightarrow$) corresponds to the

mathematical condition "sufficient".

(c) Reversal of quantified implications

Now, we shall examine the property of reversing an implication

from $\underline{\underline{b}}$ to $\underline{\underline{a}}$ in the case of Sl.No. 1. The Venn diagram in

Fig. 1(b) clearly gives the result $(\forall x)$ ($\neg \underline{\underline{b}} \Rightarrow \neg \underline{\underline{a}}$), thus

confirming Eqn. (18) written earlier for the reversal of an

implication quantified by $(\forall x)$. This equivalence (18) is a well

established one in the literature of logic, and it is listed

at the bottom of Table 2.

In the same way, the equivalence (17) between $(\exists x)$ $(\underline{a} \Rightarrow \underline{b})$

and $(\bar{\exists} x)$ $(\underline{b} \Rightarrow \underline{a})$ (also listed below Table 2) follows from

Fig. 2(a), where both of them are consequences of the fact that

$(\bar{\exists} x)$ $(\underline{a} \;\&\; \underline{b})$ is true. This equivalence for $(\exists x)$ is not

commonly used and is not found in the literature, but it is

extremely important for deriving the validity of categorical

syllogisms (see Section 4). It will be noticed that the reversed

SNS statement inside the bracket quantified by $(\bar{\exists} x)$ is different

from that quantified by $(\forall x)$. Because of this, when a pair of

QPL statements, one with $(\forall x)$ and the other with $(\bar{\exists} x)$, is reversed,

only one or the other leads to a non-D result. We shall discuss

this further in subsection 3(d) and section 4.

(d) Conclusions from two categorical statements

In this subsection, we shall combine two quantified statements

$(\mathcal{Q}_1 x)$ $(\underline{a} \Rightarrow \underline{b}')$, $(\mathcal{Q}_2 x)$ $(\underline{b} \Rightarrow \underline{c})$ and work out the resultant,

for $\mathcal{Q}_1$, $\mathcal{Q}_2 = \forall$ or $\exists$ and $\underline{b}' = \underline{b}$ or $\neg \underline{b}$. The results are

summarized in Table 3. The first entry appears to be a simple

---

Table 3. Sequence of two implications in succession connecting
the terms $\underline{a}$ and $\underline{c}$ via $\underline{b}$

---

extension of the PC equation $(\underline{a} \Rightarrow \underline{b})$, $(\underline{b} \Rightarrow \underline{c}) = (\underline{a} \Rightarrow \underline{c})$

to the universal quantifier state $(\forall x)$. However, this is not

such a trivial result, as its proof will show. Thus,

$$\text{For } (\forall x)\,(\underline{a} \Rightarrow \underline{b}),\ (\forall x)\,(a_T) \longmapsto (\exists x)\,(b_T)$$

$$\text{and } (\forall x)\,(a_F) \longmapsto (\exists x)\,(b_D) \tag{22a}$$

$$\text{For } (\forall x)\,(\underline{b} \Rightarrow \underline{c}),\ (\exists x)\,(b_T) \longmapsto (\exists x)\,(c_T)$$

$$\text{and } (\exists x)\,(b_D) \longmapsto (\exists x)\,(c_D) \tag{22b}$$

Combining the two, we get, for the relation between $\underline{a}$ and $\underline{c}$,

the correspondences

$$(\forall x)\,(a_T) \longmapsto (\exists x)\,(c_T),\quad (\forall x)\,(a_F) \longmapsto (\exists x)(c_D) \tag{23a}$$

which is identical in nature with the r.h.s of (22a) of

$(\forall x)\,(\underline{a} \Rightarrow \underline{b})$. Hence the correspondences in (23a) yield

$(\forall x)\,(\underline{a} \Rightarrow \underline{c})$, and we can write the analog of (8a) as below.

$$(\forall x)\,(\underline{a} \Rightarrow \underline{b}),\ (\forall x)\,(\underline{b} \Rightarrow \underline{c}) = (\forall x)\,(\underline{a} \Rightarrow \underline{c}) \tag{23b}$$

Table 3. Sequence of two implications in succession
connecting the terms a and c via b.

Sl. No.	First statement	Second Statement	Conclusion
1	$(\forall x)\ (a \Rightarrow b)$	$(\forall x)\ (b \Rightarrow c)$	$(\forall x)\ (a \Rightarrow c)$
2	$(\forall x)\ (a \Rightarrow \neg b)$	$(\forall x)\ (b \Rightarrow c)$	$\underset{\sim}{D}$
3	$(\exists x)\ (a \Rightarrow b)$	$(\forall x)\ (b \Rightarrow c)$	$(\exists x)\ (a \Rightarrow c)$
4	$(\exists x)\ (a \Rightarrow \neg b)$	$(\forall x)\ (b \Rightarrow c)$	$\underset{\sim}{D}$
5	$(\forall x)\ (a \Rightarrow b)$	$(\exists x)\ (b \Rightarrow c)$	$\triangle$
6	$(\forall x)\ (a \Rightarrow \neg b)$	$(\exists x)\ (b \Rightarrow c)$	$\underset{\sim}{D}$ and $\triangle$
7	$(\exists x)\ (a \Rightarrow b)$	$(\exists x)\ (b \Rightarrow c)$	$\triangle$
8	$(\exists x)\ (a \Rightarrow \neg b)$	$(\exists x)\ (b \Rightarrow c)$	$\underset{\sim}{D}$ and $\triangle$

This corresponds to the first mood in the first figure

(named _Barbara_) of the syllogisms in traditional logic (see

next section 4).

Considering the entry for Sl.No. 2, the first implication

gives the output $(\exists x)$ $(\neg \underline{b})$ and this, going into the second

implication, gives $(\exists x)$ (c_D) as the final output, thus leading

to the conclusion $\underset{\sim}{D}$.

Sl.No. 3 of Table 3 is very interesting. $(\exists x)$ $(\underline{a})$ as input

to the first statement gives $(\exists x)$ $(\underline{b})$ as output, and this, going

as input into $(\forall x)$ $(\underline{b} \Longrightarrow \underline{c})$ gives the output $(\exists x)$ $(\underline{c})$,

according to Sl.No. 3 of Table 2. Hence, the two statements

together yield the conclusion $(\exists x)$ $(a \Longrightarrow c)$, and we obtain

Eqn.(24).

$$(\exists x)\ (\underline{a} \Longrightarrow \underline{b}),\ (\forall x)\ (\underline{b} \Longrightarrow \underline{c})\ =\ (\exists x)\ (\underline{a} \Longrightarrow \underline{c}) \qquad (24)$$

This combination of statements is also in the first figure of the

syllogisms in traditional logic, and bears the name _Darii_ (see

Section 4).

All the rest of the combinations of two implications in

Table 3 lead to either $\underset{\sim}{D}$ or $\triangle$. (We use the heavy letter

symbol $\underset{\sim}{D}$ (and not D), for it refers to the $\underline{relation}$ $\underline{a}$ $\underset{\sim}{D}$ = $\underline{c}$

which yields c_D for both inputs a_T and a_F (see Ramachandran,

198**1**).

Of these, Sl.No. 5 is particularly interesting, for it has

the quantifier sequence ($\forall$, $\exists$) and yet leads to the QPL state $\triangle$.

This is because ($\forall$x) ($\underline{a}$ $\Longrightarrow$ $\underline{b}$) has the output ($\exists$x) ($\tilde{\underline{b}}$), and

this, going into ($\exists$x) ($\underline{b}$ $\Longrightarrow$ $\underline{c}$) will lead only to a $\triangle$-state

output as we have particularly pointed out in the last subsection

(see Sl.No. 7 of Table 2). The same reason makes Sl.No. 7 of Table

3 also have a $\triangle$-state conclusion. The occurrence of the

quantifier state $\triangle$ for the resultant of two QPL statements

under suitable conditions also plays an important part in the

general theory of categorical syllogisms, considered in the next

section.

Thus, we have obtained the remarkable conclusion that there is, in essence, only two forms of a pair of categorical statements, in the nature of $\underline{A}$, $\underline{E}$, $\underline{I}$ or $\underline{O}$, leading to valid conclusions, which are independent, namely Eqns (23b) and (24), listed as Sl.Nos. 1 and 3 in Table 3. We shall examine in the next section the 64 different ways (or moods) in which these statements can combine, as discussed in traditional logic, and indicate their validity, by bringing them to a form isomorphous with one of these two, which also will enable us to obtain the nature of the conclusion. Since the reverse of such an ordered pair of statements is, in general, not equivalent to the forward one, the test must be made with both the forward and reverse forms, and if either gives a valid conclusion, that should be taken as the conclusion of the pair. We shall prove in the next section that no ambiguity arises in this procedure.

But, it can happen that <u>both</u> forward and reverse senses may lead

to $\underset{\sim}{D}$ or Λ as conclusions, in which case the pair is invalid

(see Section 4 (C)).

One more general result for QPL statements of Type-1 remains

to be stated, since it will be applied in the treatment of

Section 4. This is that

$$(\forall x)\ (\underline{a} \Longrightarrow \underline{b}) \supset (\exists x)\ (\underline{a} \Longrightarrow \underline{b}) \tag{25}$$

Thus, it can happen, as with $(\forall x)\ (\underline{s} \Longrightarrow \neg\ \underline{m})$, $(\forall x)\ (\underline{m} \Rightarrow \underline{p})$,

that the pair leads only to the conclusion $\underset{\sim}{D}$ both in the

forward and in the reverse directions. In such a case, we can

take the reverse of the first statement to obtain $(\forall x)\ (\underline{m} \Longrightarrow \neg\ \underline{s})$,

and then apply (25) to this to obtain $(\exists x)\ (\underline{m} \Longrightarrow \neg\ \underline{s})$. If we

reverse it again to get $(\exists x)\ (\neg\ \underline{s} \Longrightarrow \underline{m})$, we are led to the

pair of statements in the l.h.s of (26), which have a conclusion,

by isomorphism with Eqn (24). We put $\neg\ \underline{s} = \underline{a}$, $\underline{m} = \underline{b}$ and $\underline{p} = \underline{c}$

in l.h.s. of (24) and obtain Eqn. (26)

$$(\exists x)\ (\neg\ \underline{s} \Longrightarrow \underline{m}),\ (\forall x)\ (\underline{m} \Longrightarrow \underline{p}) = (\exists x)\ (\neg\ \underline{s} \Longrightarrow \underline{m}) \tag{26}$$

The resultant syllogism, namely $(\forall x)$ $(\underline{m} \implies \underline{p})$,

$(\forall x)$ $(\underline{s} \implies \neg \underline{m}) = (\exists x)$ $(\neg \underline{s} \implies \underline{m})$ is one that is not

found in the list of eighteen in traditional logic and the

pair in the l.h.s. is usually listed as invalid, for instance,

in Delong (1971). Our technique has revealed a number of such

new valid syllogisms, and the next section gives the details

of the applications of the results presented in this subsection

to the 64 possible moods.

4. Theory of categorical syllogisms and the newly added examples.

(a) General principles

We shall not give an introduction to categorical syllogisms,

as they are well-understood in traditional logic (see e.g.

Church, 1966; Delong,1971). It connects a minor term $\underline{s}$ to a

major term $\underline{p}$ via the middle term $\underline{m}$, using two categorical

statements — namely the minor premise connecting $\underline{p}$ and $\underline{m}$ and

the major premise connecting $\underline{s}$ and $\underline{m}$. The two categorical

statements can be connected together in four different ways,

and the name "figure" is used for these, following Aristotle.

We have given the direction of the implication in each of the

four figures that are possible ~~at the bottom of~~ *in* Table 4. Following

tradition, we use only the four forms A, E, I, O for each of

the major and minor premises, so that there are 16 possible

"moods" in each figure, making a total of 64 in all the four

figures.

All the moods can be examined for validity, using the

following criteria:

(i) If both the major and the minor premises in the syllogism

are I or O, then the two statements have the pattern $\exists$, $\exists$ of

Sl.Nos. 7 or 8 of Table 3 and the resultant is D or $\triangle$, leading

to an "invalid" conclusion.

(ii) If one is A or E, and the other I or O, so that we

have the sequence $\forall$, $\exists$ or $\exists$, $\forall$, *and* [if it is not in isomorphism

with $(\exists x)$ $(\underline{a} \Rightarrow \underline{b})$, $(\forall x)$ $(\underline{b} \Rightarrow \underline{c})$ we attempt to obtain it

by reversing one, or both, the statements. We shall show below

that at most one of these possibilities leads to a valid conclusion,

and that conclusion is then assigned to the syllogism. If none

are valid, the syllogism is taken to have no valid conclusion,

and to lead always to indefiniteness.

(iii) If both are $\underline{A} \not{/} \underline{E}$, and we have the pattern $\forall_1$, $\forall_2$, we

test if it is in isomorphism with $(\forall x)$ $(\underline{a} \Rightarrow \underline{b})$, $(\forall x)$ $(\underline{b} \Rightarrow \underline{c})$.

If not, we try reversing one, or both, the statements, so as to

obtain four possibilities in all. We shall show that either

all four are invalid, or two are valid and two invalid. If two

are valid, then the two valid conclusions will be equivalent

under reversal, so that there is no ambiguity in the conclusion.

(iv) If, in (iii), all four are invalid, we obtain, using

the analog of Eqn. (25), one of the two possibilities — $\exists_1$, $\forall_2$

or $\forall_1$, $\exists_2$. We then follow the procedure in (ii) for the four

rearrangements of each. Then, it will be found that one, and

only one, possibility (of the eight) is valid and all the rest

are invalid. It will not happen that all eight are invalid in

this case.

We shall first discuss the results obtained by using (i)

to. (iv) in subsections (b) and (c) and prove these in subsection (d).

(b) Table of categorical syllogisms

If the procedures (i), (ii), (iii), (iv) of the previous

subsection are applied, as appropriate, to the 64 possible pairs

of statements of the forms $\underline{A}$, $\underline{E}$, $\underline{I}$, $\underline{O}$, 18 of the 19 traditional

moods (as listed e.g. by Church (1966) or Delong (1971)) are

obtained, provided we use, for the SNS statement of the conclusion,

only either $\underline{s} \Longrightarrow \underline{p}$ or $\underline{s} \Longrightarrow \neg \underline{p}$, i.e. the conclusion is

also one of the four forms $\underline{A}$ $\underline{E}$, $\underline{I}$, $\underline{O}$. These are given in

Table 4 on the left hand side. Following standard practice, the

symbols $\underline{A}$, $\underline{E}$, $\underline{I}$ or $\underline{O}$ of the major premise, minor premise, and conclusion are given in serial order, followed by the number (1, 2, 3 or 4) of the figure, for each valid mood. The mnemonic latin words for each mood is also given, which has its three vowels in the same order as the symbol. We shall give, in subsection 4(c) the demonstration of typical examples.

Table 4. Symbols and Names of Valid Moods of the Old and New Types

---

When this was done for all the 64 combinations, quite a number were found to have conclusions with implication $\neg \underline{s} \Rightarrow \underline{p}$ and $\neg \underline{s} \Rightarrow \neg \underline{p}$, with, of course, either $(\forall x)$ or $(\exists x)$ as quantifier. Thus, they have one of the forms $\underline{A}'$, $\underline{E}'$, $\underline{I}'$, $\underline{O}'$ (shown in (2a-d)) as conclusion. Therefore, these were systematically searched for, and then 18 more valid syllogisms were discovefed in the moods with symbols as listed in the right hand side of Table 4. The mnemonic words for these were coined and they are also listed in Tab.e 4. They all have four vowels, the first three corresponding to the forms of major premise, minor premise and conclusion, while the occurrence of the fourth vowel indicates that the syllogism is of the new type and has $\underline{A}'$, $\underline{E}'$, $\underline{I}'$ or $\underline{O}'$ as conclusion.

Table 4: Symbols and Names of Valid Moods of the
Old and New Types

Old Type		New Type	
Figure 1 : m ⟼ p, s ⟼ m, s ⟼ p			
AAA-1 ,	Barbara	AEI'-1 ,	Kaveriar
AII-1 ,	Darii	EEO'-1 ,	Electronic
EAE-1 ,	Celarent	IEI'-1 ,	Dimension
EIO-1 ,	Ferio	OEO'-1 ,	Ponderous
Figure 2 : p ⟼ m, s ⟼ m, s ⟼ p			
AEE-2 ,	Canestres	AAO'-2 ,	Bangalore
AOO-2 ,	Baroco	EEO'-2 ,	Neelotpal
EAE-2 ,	Cesare	IEI'-2 ,	Nivedita
EIO-2 ,	Festino	OAI'-2 ,	Moravia
Figure 3 : m ⟼ p, m ⟼ s, s ⟼ p			
AAI-3 ,	Darapti	AEI'-3 ,	Magnesia
AII-3 ,	Datisti	AOI'-3 ,	Sarojini
EAO-3 ,	Felapton	EEO'-3 ,	Development
EIO-3 ,	Feriso	EOO'-3 ,	Mesozoic
IAI-3 ,	Disamis	IEI'-3 ,	Indefinite
OAO-3 ,	Bocardo	OEO'-3 ,	Lodestone
Figure 4: p ⟼ m, m ⟼ s, s ⟼ p			
AEE-4 ,	Calemes	AAE'-4 ,	Management
EAO-4 ,	Fesapo	EEO'-4 ,	Generous
EIO-4 ,	Fresison	EOO'-4 ,	Leonora
IAI-4 ,	Dimatis	IEI'-4 ,	Silesia

(c) Illustrations of the procedures (i) to (iv) of Section 4(a)

We shall first give a few examples for the standard syllogisms in the four figures corresponding to the left half of Table 4, and then deal with one or two in the new syllogisms in the right half.

Considering the first figure, <u>Barbara</u> and <u>Darii</u> are the standard ones worked out in Section 3, with one of which all others are sought to be brought into isomorphism. We shall work out <u>Celarent</u> (<u>EAE</u>-1) which is straightforward. In this, $\underline{\underline{EA}}$ corresponds to the sequence of statements on the l.h.s. of (27a), which gives again an $\underline{E}$-statement from $\underline{\underline{s}}$ to $\neg\underline{\underline{p}}$, leading to the listing <u>EAE</u>-1.

$$(\forall x)\,(\underline{\underline{s}} \Longrightarrow \underline{\underline{m}}),\ (\forall x)\,(\underline{\underline{m}} \Longrightarrow \neg\underline{\underline{p}}) = (\forall x)\,(\underline{\underline{s}} \Longrightarrow \neg\underline{\underline{p}}) \qquad (27a)$$

On reversing both statements in (27a), we obtain the l.h.s. of (27b), which gives $(\forall x)\,(\underline{\underline{p}} \Longrightarrow \neg\underline{\underline{s}})$ as conclusion. This is

equivalent to the r.h.s. of (27a).

$$(\forall x)\ (p \Rightarrow \neg\, m),\ (\forall x)\ (\neg\, m \Rightarrow \neg\, s)$$
$$= (\forall x)\ (p \Rightarrow \neg\, s) \tag{27b}$$

On the other hand, reversing only one of the statements on the

l.h.s of (27a) leads to $(\forall x)\ (s \Rightarrow m),\ (\forall x)\ (p \Rightarrow \neg\, m)$

and $(\forall x)\ (\neg\, m \Rightarrow \neg\, s),\ (\forall x)\ (m \Rightarrow \neg\, p)$ which are

disconnected, and cannot lead to a valid conclusion. This

illustrates (iii), with the sequence $\forall_1,\ \forall_2$ which has a valid

conclusion.

From the examples in the second figure, we may take _Baroco_

(AOO-2). Its two statements are $(\forall x)\ (p \Rightarrow m),\ (\exists x)\ (s \Rightarrow \neg\, m)$

which are disconnected. However, it can be made to be a connected

pair by reversing the first statement alone, when we obtain (28a)

with a valid resultant.

$$(\exists x)\ (s \Rightarrow \neg\, m),\ (\forall x)\ (\neg\, m \Rightarrow \neg\, p) = (\exists x)\ (s \Rightarrow \neg\, p) \tag{28a}$$

The other two possibilities obtained by reversal of one or the

other of the $\underline{A},\underline{O}$ sequence yield only the disconnected pair

$(\forall x)$ $(\underline{p} \Rightarrow \underline{m})$, $(\exists x)$ $(\underline{s} \Rightarrow \neg \underline{m})$, ⌐or one with⌐ the resultant D and $\triangle$,

as in (28b).

$$(\forall x) \ (\underline{p} \Rightarrow \underline{m}), \ (\exists x) \ (\neg \underline{m} \Rightarrow \underline{s}) \ = \ \underline{D} \text{ and } \triangle \qquad (28b)$$

This illustrates (ii) with the sequence $\forall$, $\exists$, which has a valid

conclusion.

As an illustration of an invalid pair of statements (but

which is not $\exists$, $\exists$ as in (i)), we shall take the example $\underline{A} \ \underline{I}$ in

the fourth figure, which is

$$(\forall x) \ (\underline{p} \Rightarrow \underline{m}), \ (\exists x) \ (\underline{m} \Rightarrow \underline{s}) \ = \ \triangle \qquad (29a)$$

Although the SNS statements are connected, the quantifier sequence

is an $\forall$ followed by a $\exists$, which leads to the indefinite state

$\triangle$. We give the three other combinations obtained by reversing

one, or the other, or both the statements, below in (29 b,c,d),

all of which also lead to no valid conclusion.

$$(\forall x) \, (\neg \underline{m} \Rightarrow \neg \underline{p}), \, (\exists x) \, (\underline{m} \Rightarrow \underline{s}) \qquad \text{(Disconnected)} \qquad (29b)$$

$$(\forall x) \, (\underline{p} \Rightarrow \underline{m}), \, (\exists x) \, (\underline{s} \Rightarrow \underline{m}) \qquad \text{(Disconnected)} \qquad (29c)$$

$$(\exists x) \, (\underline{s} \Rightarrow \underline{m}), \, (\forall x) \, (\neg \underline{m} \Rightarrow \neg \underline{p}) \, = \, \underset{\sim}{D} \qquad (29d)$$

For an illustration of (iv), we may point out the example

of $\hat{\underline{A}\,\underline{E}}$ in the first figure, which was discussed at the end of

Section 3(c). It is invalid as is seen from (30a).

$$(\forall x) \, (\underline{s} \Rightarrow \neg \underline{m}), \, (\forall x) \, (\underline{m} \Rightarrow \underline{p}) \, = \, \underset{\sim}{D} \qquad (30a)$$

The reversal of its terms also gives only invalid conclusions,

as is readily verified. But, on applying Eqn. (25) and obtaining

an $(\exists x)$ for the first statement, we deduce the valid conclusion

$(\exists x) \, (\neg \underline{s} \Rightarrow \underline{p}) \, (\underline{I}')$ as shown earlier in (26). It can be

readily verified that the other connected combination that is

possible, leads only to D or $\triangle$, as in (30b) below:

$$(\forall x) \, (\neg \underline{p} \Rightarrow \neg \underline{m}), \, (\exists x) \, (\underline{m} \Rightarrow \neg \underline{s}) \, = \, \underset{\sim}{D} \text{ and } \triangle \qquad (30b)$$

On obtaining $(\exists x)$ $(\underline{m} \Rightarrow \underline{p})$ from the second statement

in (30a), we get the connected pair in the l.h.s. of (30c),

which again is $\underset{\sim}{\underline{D}}$ or $\triangle$. On reversing both the statements on

the l.h.s. of (30c), we obtain (30d), which has, however, the

conclusion $\triangle$, since the sequence of quantifiers in $\exists$, $\forall$.

The remaining four possibilities are disconnected, and therefore

do not lead to a valid conclusion.

$$(\forall x) \ (\underline{s} \Rightarrow \neg \underline{m}), \ (\exists x) \ (\underline{m} \Rightarrow \underline{p}) \ = \ \underset{\sim}{\underline{D}} \ \text{and} \ \triangle \qquad (30c)$$

$$(\exists x) \ (\underline{p} \Rightarrow \underline{m}), \ (\forall x) \ (\underline{m} \Rightarrow \neg \underline{s}) \ = \ \triangle \qquad (30d)$$

Thus, we find that $\underline{A}\,\underline{E}$ in the first figure has only the

conclusion $\underline{I}$'. In fact, this syllogism ($\underline{AEI}$'-1) was one of the

first examples of the new type of syllogism that was discovefed

during our studies, and it has been given the name Kaveriar

('Kaveri' being an important river close to Bangalore and 'ar'

being the termination for 'river' in Tamil).

It may be mentioned that, among the four arrangements

obtained by reversals of two categorical statements $(\frac{Q}{\sqrt{}_1}x)$ $(\underline{a} \Longrightarrow \underline{b}')$

and $(\frac{Q}{\sqrt{}_2}x)$ $(\underline{b} \Longrightarrow \underline{c})$, two will not form a connected chain from

$\underline{a}$ to $\underline{c}$ via the middle term $\underline{b}$, so that only two need be examined.

This is seen, for instance, in (27a, b) and (28a, b). Also, in

(30a, b) and (30c, d), this feature has been taken advantage of,

and only two possible arrangements, which are reverses of each

other, are examined. Although the manipulations in (27) to (30)

appear to be complicated, the proof of the uniqueness for the

conclusion given by a pair of statements connecting $\underline{s}$ to $\underline{p}$,

via the middle term $\underline{m}$, is quite simple. We give the proof

briefly in the next subsection.

(d) Proof of uniqueness of the procedures (i) to (iv) for obtaining
the resultant.

The proof is not demonstrative, but existential. We assume,

in each case, that one type of rearrangement (after reversal)

leads to a valid conclusion. We then show that no other

non-equivalent conclusion follows by any other possible rearrangement

In doing this, we consider only the other <u>connected</u> rearrangements

i.e which have either $a \longmapsto m, m \longmapsto b$ or $b \longmapsto m, m \longmapsto a$.

(i) $\exists , \exists$: In this case, no valid conclusion is possible,

for the resultant quantifier is always $\triangle$. Hence, there can be

no duplication.

(ii) $\forall , \exists$: Here again, if there is no valid resultant,

there is no question of duplication. Therefore, we assume that

there is one valid arrangement, in which $\exists$ must precede $\forall$, as

in

$$(\exists x) (\underline{a} \Longrightarrow \underline{k}), (\forall x) (\underline{\ell} \Longrightarrow \underline{b}) \tag{31a}$$

Note that, for validity, $\underline{k}$ must be equal to $\underline{\ell}$, i.e.

$$\underline{k} = \underline{\ell} = \text{either } \underline{m} \text{ for both, or } \neg \underline{m} \text{ for both} \tag{31b}$$

Now, on reversing both statements (which is a necessary condition

for correctedness), we obtain,

$$(\forall x) \ (\neg \underline{\underline{b}} \implies \neg \underline{\underline{\ell}}), \ (\exists x) \ (\underline{\underline{k}} \implies \underline{\underline{a}}) \ = \ \underline{D} \text{ and } \Delta \tag{31c}$$

The Δ arises from the sequence $\forall$, $\exists$ (see Table 3, Sl.Nos.5, 6)

and $\underline{D}$ arises because $\underline{\underline{k}} = \neg \underline{\underline{\ell}}$ if (31b) is satisfied.

The other two rearrangements will be disconnected, and hence

the valid conclusion is unique.

(iii, $\forall$, $\forall$ and conclusion exists: In this case, assuming

connectedness to exist, there is only one other rearrangement

possible for reversal. Hence, if

$$(\forall x) \ (\underline{\underline{a}} \implies \underline{\underline{m}}), \ (\forall x) \ (\underline{\underline{m}} \implies \underline{\underline{b}}) \ = \ (\forall x) \ (\underline{\underline{a}} \implies \underline{\underline{b}}) \tag{32a}$$

then

$$(\forall x) \ (\neg \underline{\underline{b}} \implies \neg \underline{\underline{m}}), \ (\forall x) \ (\neg \underline{\underline{m}} \implies \neg \underline{\underline{a}}) \ = \tag{32b}$$
$$= \ (\forall x)(\neg \underline{\underline{b}} \implies \neg \underline{\underline{a}})$$

and the two conclusions in (32a) and (32b) are equivalent. The

other two rearrangements are disconnected and hence invalid.

Therefore, in this case, although there is duplication, there is

no ambiguity.

(iv) $\forall$, $\forall$ and no valid conclusion : Since there is no

conclusion, the two statements have the pattern

$$(\forall_1 x)\ (\underline{a} \Rightarrow \underline{k}),\ (\forall_2 x)\ (\underline{\ell} \Rightarrow b)\ \text{with}\ \underline{k} = \neg\underline{\ell} \qquad (33)$$

Straightaway, obtaining an $(\exists x)$ statement from either $\forall_1$ or $\forall_2$,

and rearranging, after reversal, for connectedness, we obtain the

four possibilities — the other four are disconnected:

$$(\exists_1 x)\ (\underline{a} \Rightarrow \underline{k}),\ (\forall_2 x)\ (\underline{\ell} \Rightarrow \underline{b}) = \underset{\sim}{D} \qquad (34a)$$

$$(\forall_2 x)\ (\neg\underline{b} \Rightarrow \neg\underline{\ell}),\ (\exists_1 x)\ (\underline{k} \Rightarrow \underline{a}) = \Delta \qquad (34b)$$

$$(\forall_1 x)\ (\underline{a} \Rightarrow \underline{k}),\ (\exists_2 x)\ (\underline{\ell} \Rightarrow \underline{b}) = \underset{\sim}{D}\ \text{and}\ \Delta \qquad (34c)$$

$$(\exists_2 x)\ (\underline{b} \Rightarrow \underline{\ell}),\ (\forall_1 x)\ (\neg\underline{k} \Rightarrow \underline{a}) = (\exists x)\ (\underline{b} \Rightarrow \underline{a}) \qquad (34d)$$

$$= (\exists x)\ (\underline{a} \Rightarrow \underline{b})$$

It is seen that, with the condition $k = \neg\underline{\ell}$ in (33), only one

of the four, viz (34d), is valid, and the others are invalid.

By putting (iii) and (iv) together, we can say that the

sequence $\forall$, $\forall$ for a pair of categorical statements always

has a conclusion, either with the universal quantifier $\forall$, or

(if this is not possible) with the existential quantifier $\exists$.

Because of the proof of uniqueness of the valid conclusion

of a pair of categorical statements, in searching for the

conclusion of a given pair, if a valid conclusion is arrived at,

we need not check any further rearrangements.

(e) Summary of Section 4

It will be noticed that although the categorical syllogisms

covered by Table 4 in section 4(b) have only statements of the

form A, E, I, O for the two premises, the theory in section 4(a)

and its proof in section 4(d) make no such restrictions, and the

two statements considered in these can have all the eight forms

A, E, I, O , A', E', I', O' for them. Thus, quite generally,

there are 8X8 = 64 moods in each figure, leading in all to 256

possibilities. However, we are not considering all these pairs

of statements, since the procedures (i) to (iv) can always be

used to check for their validity, and to find the resultant

when any pair is valid. In effect, the procedures (i) to (iv)

have the following consequences:

(α) If the pair has only A, E, A', E' (i.e. of the type

$\forall, \forall$), then it always has a conclusion, either of the type $\forall$,

or of the type $\exists$.

(β) If it has only I, O, I', O' (i.e. of the type

$\exists, \exists$), then it is invalid and has no definite conclusion.

(γ) For the rest (of the type $\exists, \forall$ or $\forall, \exists$), some are

valid, and some invalid. When valid, the conclusion is always

of the type $\exists$, i.e. one of I, O, I', O'.

The above theory can be extended to a connected sequence of

n categorical statements, the QPL analog of the SNS sequence

considered in Eqn (11) of Section 2(e). As the study of these

has not yet been finalized, this is only presented in

outline in the next section.

5. Resultant of n connected categorical statements

(a) Three possible patterns leading to conclusions

Suppose we have a sequence of n connected statements

$$(\mathcal{Q}_1 x) (\underline{s} \Rightarrow \underline{b}_1), (\mathcal{Q}_2 x) (\underline{a}_2 \Rightarrow \underline{b}_2), \ldots (\mathcal{Q}_j x) (\underline{a}_j \Rightarrow \underline{b}_j)$$
$$\ldots (\mathcal{Q}_n x) (\underline{a}_n \Rightarrow \underline{p}) \tag{35a}$$

the condition for connectedness being that

$$\underline{b}_j = \underline{a}_{j+1}, \text{ or } \neg \underline{b}_j = \underline{a}_{j+1}, \ j = 1 \text{ to } (n-1) \tag{35b}$$

and

$$\mathcal{Q}_j = \forall \text{ or } \exists, \ \bar{j} = 1 \text{ to } n \tag{35c}$$

since we deal only with categorical statements of the forms

$\underline{A}, \underline{E}, \underline{I}, \underline{O}, \underline{A}', \underline{E}', \underline{I}', \underline{O}'$. We divide our discussion into two

categories — (i) All quantifiers are $(\forall x)$ and (ii) Some are

$(\exists x)$ and the remaining $(\forall x)$. The third case when all are $(\exists x)$

need not be considered, since even two implications in succession,

both $(\exists x)$, will lead to $(\triangle x)$ as consequence, so that there is

no valid conclusion when all are $(\exists x)$.

(b) Analogue of Barbara

(i) All $\mathcal{O}_j$ are $\forall$: In this case, by repeated application of

Barbara (Sl. No. 1 of Table 3) we obtain, using (35b),

$$(\forall x) (\underline{s} \Rightarrow \underline{b}_1), (\forall x) (\underline{b}_1 \Rightarrow \underline{b}_2), \ldots, (\forall x) (\underline{b}_j \Rightarrow \underline{b}_{j+1}) \qquad (36a)$$

$$\ldots, (\forall x) (\underline{b}_{n-1} \Rightarrow \underline{p}) = (\forall x) (\underline{s} \Rightarrow \underline{p})$$

and the necessary conditions in (35a) are that $\underline{b}_j = \underline{a}_{j+1}$ for

all j. It is readily verified that, on reversing the whole of

the l.h.s. of (36), we obtain again a connected sequence

$$(\forall x) (\neg \underline{p} \Rightarrow \neg \underline{b}_{n-1}), (\forall x) (\neg \underline{b}_{n-1} \Rightarrow \neg \underline{b}_{n-2}), \ldots,$$

$$(\forall x) (\neg \underline{b}_{j+1} \Rightarrow \neg \underline{b}_j), \ldots, (\forall x) (\neg \underline{b}_1 \Rightarrow \neg \underline{s}) \qquad (36b)$$

$$(\forall x) (\neg \underline{p} \Rightarrow \neg \underline{s})$$

and the conclusion in (36b) is the equivalent, under reversal,

of that in (36a). Thus, the resultant in r.h.s. of (36a) is unique.

(c) Two analogues of Darii in the forward and reverse directions.

(ii) Some $\mathcal{O}_j$ are $\exists$, others $\forall$: In this case, there cannot be

two $\exists$'s in succession, since it leads to a $(\triangle)$ and the

resultant conclusion of all the statements also will be in the

$\hat{\triangle}$-state and hence invalid. Also, a sequence $\forall$, $\exists$ cannot occur

anywhere, since this also leads to a $\triangle$-state (as per Sl.Nos

5 and 6 of Table 3). Hence, only $\mathcal{Q}_1$ or $\mathcal{Q}_n$ can be $\exists$.

(α) In the former case, the forward sequence (35a) is

implementable, with the additional condition $b_j = a_{j+1}$

for all j = 1 to (n-1), by repeated application of <u>Darii</u> (Sl.No.3

of Table 3), and the conclusion is $(\exists x)$ $(\underline{\underline{s}} \implies \underline{\underline{p}})$. For ready

reference, we give this, in (37a):

$$(\exists x) \ (\underline{\underline{s}} \implies \underline{\underline{b}}_1), \ (\forall x) \ (\underline{\underline{b}}_1 \implies \underline{\underline{b}}_2), \ldots, \ (\forall x) \ (\underline{\underline{b}}_{n-1} \implies \underline{\underline{p}})$$

$$\equiv (\exists x) \ (\underline{\underline{s}} \implies \underline{\underline{p}}) \tag{37a}$$

In this case, reversing the l.h.s. of (37a) leads to $\triangle$-state

for the conclusion because of the sequence $\forall$, $\exists$ at the end; but

this does not matter because by our theory, only one of the

forward and the reversed sequences need have a definite conclusion

for the set of categorical statements to have a valid resultant.

(β) However, in the reverse direction, $\exists$ can be the

quantifier of the first statement. So, we can look for a

resultant for a sequence of n statements, as in (36a) and (37a),

but with ($\exists$ x) quantifying the last one, under suitable conditions

connecting $\underline{b}_j$ and $\underline{a}_{j+1}$. Let us write it in the forward direction

as follows:

$$(\forall x) \, (\underline{s} \Rightarrow \underline{b}_1), \; (\forall x) \, (\underline{b}_1 \Rightarrow \underline{b}_2), \ldots, \; (\forall x) \, (\underline{b}_{n-2} \Rightarrow \underline{b}_{n-1})$$
$$(\exists x) \, (\underline{a}_{n-1} \Rightarrow \underline{p}) \tag{38a}$$

The conclusion of this is a $\triangle$-state; but when written in the

reverse direction, it takes the form

$$(\exists x) \, (\underline{p} \Rightarrow \underline{a}_{n-1}), \; (\forall x) \, (\neg \underline{b}_{n-1} \Rightarrow \neg \underline{b}_{n-2}), \ldots,$$
$$(\forall x) \, (\neg \underline{b}_2 \Rightarrow \neg \underline{b}_1), \; (\forall x) \, (\neg \underline{b}_1 \Rightarrow \neg \underline{s}) \tag{38b}$$

This is seen to have a form isomorphous with (37a) if

$\underline{a}_{n-1} = \neg \underline{b}_{n-1}$, and the resultant is $(\exists x) \, (\underline{p} \Rightarrow \neg \underline{s})$ / which

by homology,

is equivalent to $(\exists x) \, (\neg \underline{s} \Rightarrow \underline{p})$. Hence, we obtain the

third category of a sequence of n statements with a conclusion —

namely

$$(\forall x) (\underline{s} \Rightarrow \underline{b}_1), \; (\forall x) (\underline{b}_1 \Rightarrow \underline{b}_2), \ldots, \; (\forall x) (\underline{b}_{n-2} \Rightarrow \underline{b}_{n-1}),$$

$$(\exists x) (\neg \underline{b}_{n-1} \Rightarrow \underline{p}) = (\exists x) (\neg \underline{s} \Rightarrow \underline{p}) \tag{39a}$$

(For completeness sake, we can write (39a) in terms of

reverse implications, as

$$(\forall x) (\underline{s} \Leftarrow \underline{a}_1), \; (\forall x) (\underline{a}_1 \Leftarrow \underline{a}_2), \ldots, \; (\forall x) (\underline{a}_{n-2} \Leftarrow \underline{a}_{n-1})$$

$$(\exists x) (\underline{a}_{n-1} \Leftarrow \underline{p}) = (\exists x) (\underline{s} \Leftarrow \underline{p}) \tag{39b}$$

which is seen to be identical in structure with (37a), but with

reverse implications ($\Leftarrow$), instead of implications ($\Rightarrow$). Just

as (37a) has an $\exists$ only for the _first_ implication, (39b) also has

an $\exists$ only for the _first_ reverse implication, going from right

to left, and $\forall$'s for all the rest.)

The two cases (α) and (β) (given in (37a) and (39)

exhaust the possibilities under (ii) having some $\exists$'s present

in the sequence. Under (i), there is only one possibility,

which takes the form (36a). Thus, totally, there are only three

patterns (that are independent) which have valid conclusions,

for a sequence of n QPL implications in statements of Type-1.

———

References

1. Church, A. (1966) Article on "Logic" in Encyclopaedia Brittannica, Vol 14, p,216, Chicago.

2. Delong, H. (1971) "A Profile of Mathematical Logic", Addison-Wesley, Reading.

3. Ramachandran, G.N. (1979) "Syad-nyaya-system logic — Essential ideas connected with Propositional Calculus", Matphil Reports No. 1, pp 1-58.

4. Ramachandran, G.N. (1980) "Computerization of Logic" Golden Jubilee Commemoration Volume, Natl. Acad. Sci. India, pp 1-42.

5. Ramachandran, G.N. (1982) "Syad Nyaya System (SNS) — A new formulation of sentential logic and its isomorphism with Boolean algebra of genus 2" Current Science, 51, 625-636.

6. Ramachandran, G.N. (1983) (a,b) "Vector-matrix representation of Boolean algebras and application to extended predicate logic (EPL). Part I, 292-302; Part II, 335-341, Current Science, 52.

7. Ramachandran, G.N. (1983c) "Boolean algebra applied to statements of Type-1 in quantified predicate logic (QPL) and its consequences for categorical syllogisms of traditional Logic", Matphil Reports No. 33, pp 1 to 79.

8. Ramachandran, G.N., Johnson, R.E.C., and Thanaraj, T.A. (1980) "Electronic Syād-Nyāya-Yantra (ESNY-2) — Analog computer for sentential logic", Matphil Reports No. 13, pp 1-90.

9. Ramachandran, G.N., and Thanaraj, T.A.(1980a) "Outline of the Fortran Program NYAYA1", Matphil Reports No. 6, pp 1-32.

10. Ramachandran, G.N., and Thanaraj, T.A. (1980b) "Fortran Program for Sentential Logic", Matphil Reports No. 12, pp 1-61.

11. Ramachandran, G.N., and Thanaraj, T.A. (1981) "Matrix algebra for sentential logic and the Fortran program MATLOG", Matphil Reports No. 18, pp 1-64.

12. Ramachandran, G.N., Ramachandran, H., and Thanaraj, T.A. (1981) "Pocket Calculator program TINY1 for sentential logic", Matphil Reports No. 16, pp 1-43.

———

Vector-Matrix Representation of Boolean Algebra and Application to Extended Predicate Logic (EPL)

Part IV - General Theory for Statements of Type-1 in QPL

G.N. Ramachandran
Gita, 5, 10A Main Road
Malleswaram West
Bangalore 560 055

Vector-Matrix Representation of Boolean Algebras and application

to Extended Predicate Logic (EPL)

Part IV — General Theory for Statements of Type-1 in QPL[*]

G.N. Ramachandran

Gita, 5, 10A Main Road
Malleswaram West
Bangalore 560055

This report deals with the generalisation of the theory

in Part III dealing with categorical statements in quantified

predicate logic (QPL), extended to all statements of Type-1,

having the same quantifier $Q/(x)$. The method adopted is to

extend the logical graph of an SNS argument containing relations

of the type $\underline{a} \underset{\sim}{Z} = \underline{b}$ and $\underline{a} \underset{\sim}{Z} \underline{b} = \underline{c}$ to the case of an argument

containing Type-1 relations in QPL of the type $Q/(x) (\underline{a}(x) \underset{\sim}{Z} = \underline{b}(x))$

and $Q/(x) (\underline{a}(x) \underset{\sim}{Z} \underline{b}(x) = \underline{c}(x))$. The theory of SNS arguments

is briefly dealt with in Section 2, after which the elementary

unary and binary statements containing matrix operators $\underset{\sim}{Z}$ of

the above type in QPL Type-1 arguments are discussed in Section 3,

along with binary Boolean relations of the type $\underline{\underline{a1}} \cup \underline{\underline{a2}} = \underline{\underline{\underline{a}}}$

and $\underline{\underline{a1}} \vee \underline{\underline{a2}} = \underline{\underline{\underline{a}}}$.

This is then followed by detailed discussion of the structure

of the logical graph of Type-1 argument and how various preliminary

alterations of this structure make it possible to convert the

given graph into a "reduced" graph in which every step can be

written in an algorithmic manner with the following structure:

input QFL state(s), QFL Type-1 relation, leading to QFL state

of output, of the particular elementary relation, forming part

of the reduced graph. After discussing the various manipulations

and steps involved in this. a summary of the implementation

procedure is discussed in Section 5 along with a proof that a

reduced graph of the above type is always implementable for

any finite QFL Type-1 logical graph, (*see addendum).

The paper in effect proves the result that QFL logic of

Type-1 has no incompleteness, or indefiniteness, associated with

it, except that the QFL state of the output may be also one of

the four non-definite states of extended predicate logic: namely

$\triangle$ (1 1 1) = indefinite, $\emptyset$ = (0 0 0) = contradiction,

$\lessgtr$ = (0 1 0) = some, but not all and not none, $\ominus$ = (1 0 1) =

all or none. Incidentally, in Section 2 it is shown that a

circular SIS argument, which leads to paradoxes, can always be

converted into a graph without circularity, leading to some one

of the SIS states being a possible input for the argument. The

generalization of this to QPL Type-1 circular arguments is employed

in the theory of Section 4.

This particular report is Draft-II of a paper being prepared

with this title for publication. For the present, it is being

xeroxed for private circulation and for opinion of qualified

persons. It is not claimed that there are no errors or omissions

in this particular draft.

———

Addendum

(The proof for the implementability of any finite QPL Type-1

logical graph is extendable to the general case of any finite

EPL logical graph, containing both Type-1 and Type-2 statements.

This follows from the fact that the graph structure of SIS, QPL

Type-1, and general EPL graphs, are isomorphic and can be treated

for proof of implementability in exactly the same way. This is

proposed to be shown in Part V.)

CONTENTS

1. Introduction

In Part III [1] we considered the algebraic theory for combining

two categorical statements in QPL to obtain the resultant, which

is also a categorical statement. This was then extended to the

resultant of n categorical statements of type $(\mathcal{Q}'_j x)(\underline{a}_j(x) \Rightarrow$

$\underline{a}_{j+1}(x))$; (By having a_j and a_{j+1} to be affirmative or negative, with $j = 1$ to n.)

the four possible SNS connective operators $\underline{I}$, $\underline{IN}$, $\underline{NI}$ and $\underline{NIN}(\equiv \underline{J})$

are all taken into account. For the quantifier $\mathcal{Q}_j$, only two

possibilities in QPL need be considered, namely $\forall$ and $\exists$.

More generally, a statement of Type-1 has the form

$$(\mathcal{Q}x)\,(\underline{s}(x)) \text{ , where } \mathcal{Q} = \forall \text{ or } \exists \text{ and } \underline{s}(x) \text{ is}$$
$$\text{an SNS statement} \tag{1}$$

If $\underline{s}(x) \equiv (\underline{a}(x) \Rightarrow \underline{b}(x))$, then we get the restricted example

of categorical statements. More generally, however, $\underline{s}(x)$ can have

various other forms and the only condition we put is that it

must be a logical statement composed of SNS terms and connectives,

which can take up the four possible truth values T, F, D, X of

SNS logic. The theory of SNS statements and sentences has been

discussed in Ref [1] and it can be shown that any general SNS

argument can be written in a form of a logical graph in which the

elementary statements are either unary or binary. These two

examples of elementary statements in SNS can be represented

symbolically as follows:

Unary $\underline{s}(x) \equiv (\underline{a}(x) \underset{\sim}{Z} = \underline{b}(x))$,

      where $\underset{\sim}{Z}$ is one of the ten possible SNS

      operators, $\underset{\sim}{E}, \underset{\sim}{N}, \underset{\sim}{A}, \underset{\sim}{O}, \underset{\sim}{I}, \underset{\sim}{J}, \underset{\sim}{A}^{c}, \underset{\sim}{O}^{c}, \underset{\sim}{I}^{c}, \underset{\sim}{J}^{c}$

      (~~see below for limitations on these~~). (See ref 4 (2)

    and Section 3(c)(ii))

Binary Forward

      $\underline{s}(x) \equiv (\underline{a}(x) \underset{\sim}{Z} \underline{b}(x) = \underline{c}(x))$,

      where $\underset{\sim}{Z}$ is one of the above ten matrix

      connectives, or one of the ~~three~~ two Boolean

      connectives $\underset{\sim}{U}, \underset{\sim}{V}, \not\!\!W$ (see Ref [1] and Section   (3)

    3(a), below)

Binary Reverse

$$\underline{s}(x) \equiv (\underline{c}(x) \overset{\leftarrow}{\underset{\sim}{Z}} \underline{a}(x) = \underline{b}(x)),$$

in which $\underset{\sim}{Z}$ can be only one of the

above ten matrix connectives, (See P.((,h) and   (4a)

Section 3(c)(iii)).

According to the nature of $\underline{c}(x)$, the binary reverse relation

(4a) becomes [one of] the following, using [unary] forward relations:

$$\underline{c} = T \longmapsto \underline{a} \underset{\sim}{Z} = \underline{b} \quad ; \quad \underline{c} = F \longmapsto \underline{a} \underset{\sim}{Z}^{c} = \underline{b} \qquad (4b, 4c)$$

$$\underline{c} = D \longmapsto \underline{a} \underset{\sim}{D} = \underline{b} \quad ; \quad \underline{c} = X \longmapsto \underline{a} \underset{\sim}{X} = \underline{b} \qquad (4d, 4e)$$

More generally, $\underline{s}(x)$ in Eq.(1) can be any general SNS statement

connecting any number of terms <u>via</u> any number of unary and/or

binary logical SNS relations. In all these cases, the final

output of the SNS argument has one of the four SNS states when

each input term contained in the argument is specified to have one

of these four truth values. This general theorem is the result

of the theory of SNS logic (see Ref [4]); see also Section 2.2

below for an outline of the proof using our Boolean algebraic

representation.)

In this article, we shall generalize the algebra for any

general SNS argument to the case when the quantifier $(\forall/x)$ is

appended before the bracket containing the SNS sentence, so that

each term in the argument is a QPL statement for the variable x .

$$(\alpha \ (x) \cong (\forall / x) \ (\underline{a} \ (x)),)$$

The inputs are QPL terms /containing both /quantifier state as well
  (a)

as an SNS truth value and we ask for both the SNS truth value,

(when we are)
as well as the quantifier state, for the final output(s), /given

the corresponding quantifies for each of the input terms contained

in the full set of statements in the QPL argument. As a preliminary

to this, we shall give a fairly full, but condensed, account of

the way in which any general SNS argument is worked out in the

forward direction. The terms and connectives of any such argument

can be represented by a logical graph in which both the terms

as well as the connective operators are nodes, which are connected

by directed edges indicating the direction in which the logic of

the argument flows. This is discussed in the next section for an

SNS argument and the general way in which the outputs of such

a graph are worked out is indicated therein. Certain recondite

points such as the reversal of the direction of flow of the

argument when the "impossible" state X arises are only briefly

commented upon. Similarly, the case of circular arguments, in

which the logical graph has a closed circuit of edges are also

only briefly discussed. (The difficulties with circular arguments

arises only when the "doubtful" state D or the "impossible" state

X arises as a consequence.) We shall comment on these at the

appropriate place and reserve a full discussion of such pathological

situations for a separate report.

2. Logical graph of an SNS argument

(a) Elementary statements

The logical graphs corresponding to the SNS parts of the

elementary QPL statements in Eqs. (2), (3) and (4) are shown in

Fig. 1, and the corresponding equations are given below in

Fig. 1   Elementary statements of SNS logic.
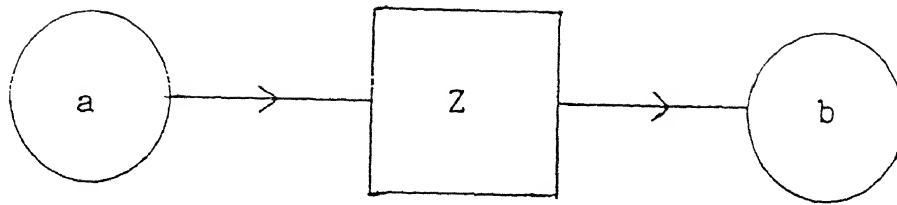
(a). Unary relation as in Eqs (2) and (5)
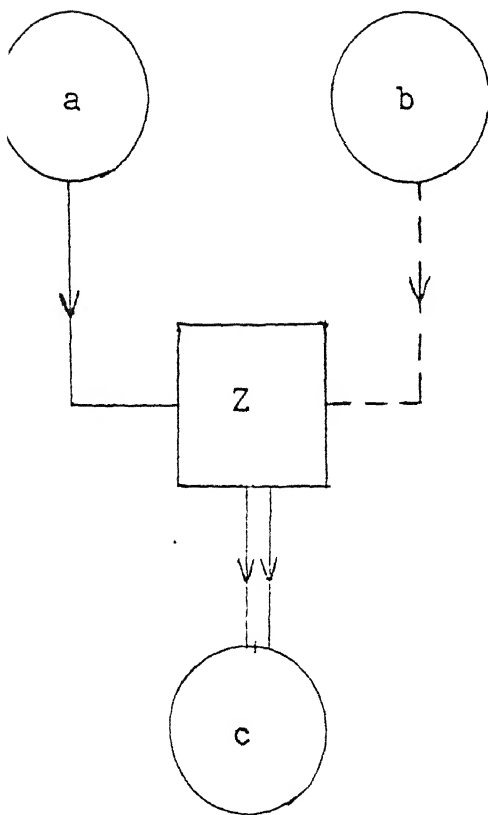
(b). Binary forward relation
as in Eqs (3) and (6)
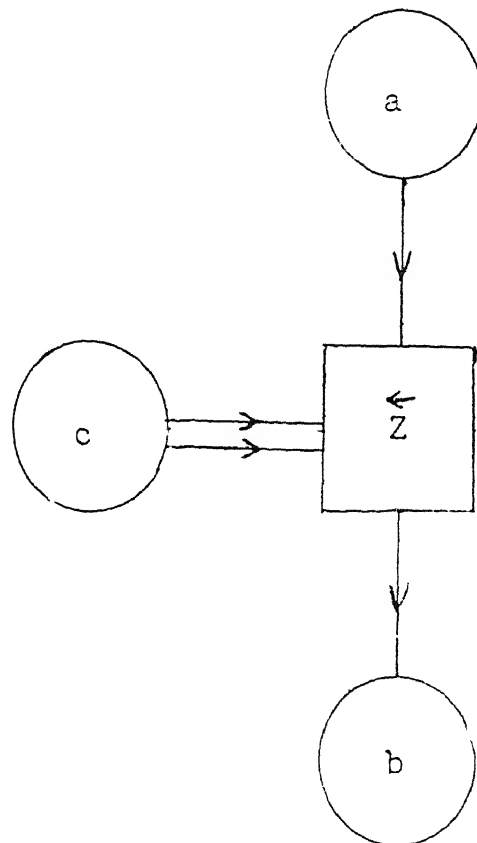
(c). Binary reverse relation
as in Eqs. (4) and (7)

FIG. 1. Elementary statements of SNS logic

Eqs. (5), (6) and (7):

Unary (SNS) relation

$\underline{\underline{a}}\, \underset{\sim}{Z} = \underline{\underline{b}}$; Given the input term $\underline{\underline{a}}'$, and the nature of the

connective $\underset{\sim}{Z}$, to find the output term $\underline{\underline{b}}'$ (5a, b)

The matrix equation for this is

$$\langle a' \mid Z \mid = \langle b' \mid \qquad (5c)$$

whose expanded form is

$$\sum_{\lambda = \alpha, \beta} a'_\lambda \, Z_{\alpha\lambda} = b'_\alpha \quad ; \quad \sum_{\lambda = \alpha, \beta} a'_\lambda \, Z_{\beta\lambda} = b'_\beta \qquad (5d)$$

Binary forward (SNS) relation:

$\underline{\underline{a}}\, \underset{\sim}{Z}\, \underline{\underline{b}} = \underline{\underline{c}}$; Given the input terms $\underline{\underline{a}}'$ and $\underline{\underline{b}}'$, and the

connective $\underset{\sim}{Z}$, to find $\underline{\underline{c}}'$ (6a, b)

For matrix operators, the equation is

$$\langle a' \mid Z \mid b' \rangle = c'_\alpha \, , \quad \langle a' \mid Z^c \mid b' \rangle = c'_\beta \qquad (6c, d)$$

where

$$\langle a' \mid Z \mid b' \rangle = \sum_\lambda \sum_\mu a'_\lambda \, Z_{\lambda\mu} \, b'_\mu \qquad (6e)$$

For non-matrix binary operators (see Ref. [1]), the equations

are:

$$Z = U \ : \quad a'_\alpha \oplus b'_\alpha = c'_\alpha \ ; \quad a'_\beta \oplus b'_\beta = c'_\beta \tag{6f}$$

$$Z = V \ : \quad a'_\alpha \otimes b'_\alpha = c'_\alpha \ ; \quad a'_\beta \otimes b'_\beta = c'_\beta \tag{6g}$$

Binary reverse(SNS) relation:

$$\underset{\sim}{c} \overleftarrow{Z} \underline{a} = \underline{b} \ ; \quad \text{Given } \underline{c}', \underline{a}', \text{ to find } \underline{b}' \tag{7a, b}$$

For the four truth values T, F, D, X, respectively,

of $\underline{c}'$, we get the unary relations, as in (4a-d), and

the matrix equations for these are of the same form

as in (5c, d). (7c)

It is to be noted that any unary forward relation is expressible

as a binary relation, with $\underline{c}' = $ T or F. Hence, the introduction

of the unary relation in our Boolean-algebraic representation of

logic is consistent with the conventional treatments of propositional

calculus; but the consequences of including also the SUS states

D and X are covered by our matrix algebraic representation as

given above. In particular, $\underline{a}\ \underline{Z} = \underline{b}$ is equivalent to $(c_T\ \overleftarrow{\underline{Z}}\ \underline{a}) =$

As will be discussed in particular in connection with QFL

statements of Type-1 (see Section 3(c)(ii)), the ten common matrix

operators $\underline{Z}$ can be classified into three types — namely the

A-type $(\underline{A},\ \underline{Q}^C,\ \underline{I}^C,\ \underline{J}^C)$, the I-type $(\underline{Q},\ \underline{A}^C,\ \underline{I},\ \underline{J})$, and the

E-type $(\underline{E},\ \underline{E}^C \equiv \underline{U})$. Of these, the I-type and E-type operators

never give a unary output which is X for $\underline{a}\ \underline{Z} = \underline{b}$, for $\underline{a} = T$ or F,

while the A-type operators can do so. For example, $\underline{a}' = \overline{F} = (0\ \ 1)$

input to $\underline{a}\ \underline{A} = \underline{b}$, leads to

$$\underline{b}' = (0\quad 1) \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = (0\quad 0) = X \qquad (8)$$

If the structure of the unary relation , as in Fig. 1(a) is adopted

for the A-type operators also, then the contradictory state X

will be propagated through the graph. To avoid this, in the

analogous unary relation for Type-1 statements, as shown in

Fig. 6a, a new construction called "modified $\underset{\sim}{A}$-construction"

is adopted by which the /input $(0 \quad 1) = a_{1T}^{\prime}$, and the necessary

[above "input": actual]

input $(1 \quad 0) = a_{2T}^{\prime}$ for $\underset{\sim}{A}$, are compared by the consistency

"vidya" operator $\underset{\sim}{V}$ and this leads to an output which may be

X or otherwise. On the other hand, the output from $\underset{=}{a} \underset{\sim}{A} = \underset{=}{b}_{\wedge}$

goes as $\underset{=}{b}^{\prime} = T = (1 \quad 0)$ into the graph for further processing.

(As this great convenience of the modified $\underset{\sim}{A}$-construction

became very clear only for Type-1 statements, reference is here

made to Fig. 6b and its explanation in Section 3(c)(ii) and

it is not explained here. However, it may be mentioned that the

net result is that for all operators of the type shown in Fig.1,

by making suitable $\underset{\sim}{A}$-constructions and $\underset{\sim}{V}$-constructions (see

later in this section), it is possible to relegate all contradictory

states X to outputs and the graph can be analysed and implemented

throughout. The proof of this is given later in this section.)

(b) Structure of a general logical graph in SLS logic.

We shall not make the treatment in this section completely

general. Instead, we shall consider two or three typical logical

graphs of this type and indicate how the steps from the inputs

to the final output are worked out in each case. Thereafter, we

shall try to indicate how the whole process can be generalized

and worked out for any logical graph.

To start with, we shall consider a graph which has no circuits

and has therefore the structure of a "tree" (Fig.2). The nature

of the six connective operators $\underset{\sim}{Z1}$ to $\underset{\sim}{Z6}$ is also indicated

within Fig. 2. It is to be noted that the edges leading to $\underline{c}$ in

---

Fig.2. Logical graph having a tree structure containing unary
       connectives and binary forward and binary reverse
       connectives.

$\underline{\underline{a}} \underset{\sim}{Z} \underline{\underline{b}} = \underline{\underline{c}}$, and starting from $\underline{\underline{c}}$ in $\underline{\underline{c}} \overset{\leftarrow}{\underset{\sim}{Z}} \underline{\underline{a}} = \underline{\underline{b}}$, are marked by

double lines (see Fig. 1), and the input from $\underline{\underline{b}}$ to $\underset{\sim}{Z}$ is distinguished

from that from $\underline{\underline{a}}$, in $\underline{\underline{a}} \underset{\sim}{Z} \underline{\underline{b}}$, by a dotted line for the former
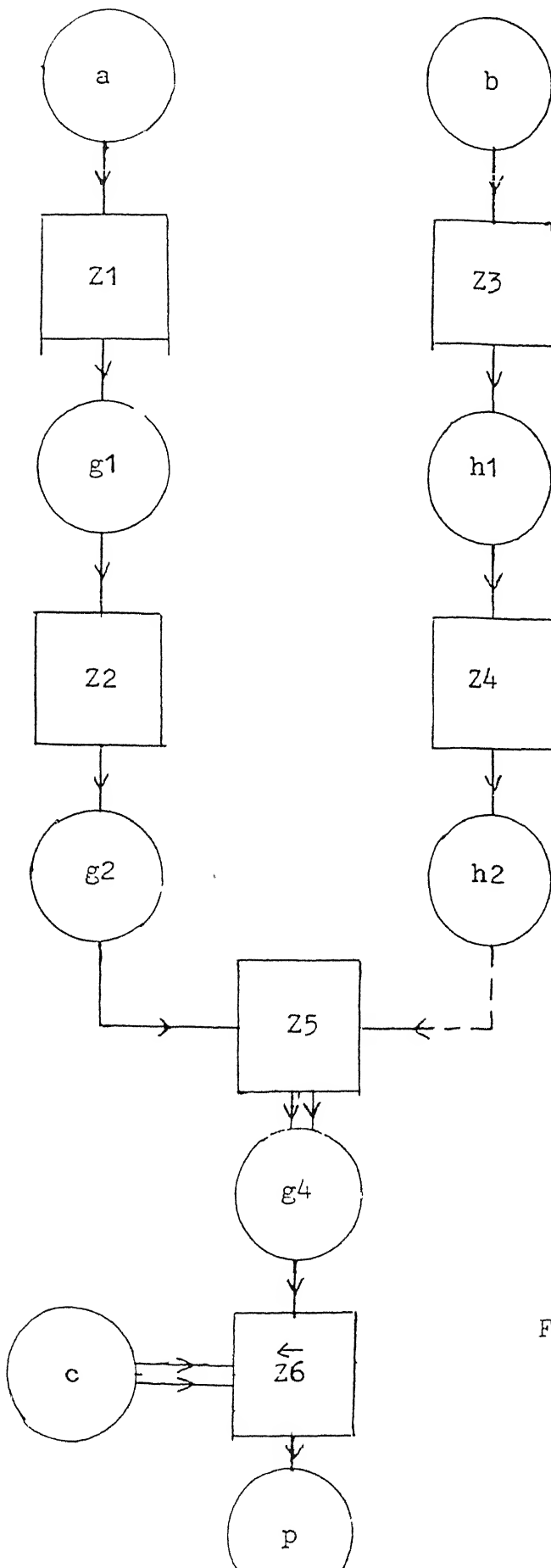
$$\underset{\sim}{Z1} = \underset{\sim}{I}$$
$$\underset{\sim}{Z2} = \underset{\sim}{E}^c (N)$$
$$\underset{\sim}{Z3} = \underset{\sim\sim}{IN} (A^c)$$
$$\underset{\sim}{Z4} = \underset{\sim\sim}{NI} (O)$$
$$\underset{\sim}{Z5} = \underset{\sim}{A}$$
$$\underset{\sim}{Z6} = \underset{\sim}{I}$$

Inputs: $\underline{a}$, $\underline{b}$, $\underline{c}$

Output: $\underline{p}$

Fig. 2. Logical graph having a
tree structure containing
unary connectives and
binary forward and reverse
connectives.

The equations connecting the various terms that are contained in this graph are given below.

$$\underline{a}\; \underset{\sim}{Z1} = \underline{g1} \;\; ; \;\; \underline{g1} \; \underset{\sim}{Z2} = \underline{g2} \qquad\qquad (8a,\; 8b)$$

$$\underline{b}\; \underset{\sim}{Z3} = \underline{h1} \;\; ; \;\; \underline{h1} \; \underset{\sim}{Z4} = \underline{h2} \qquad\qquad (8c,\; 8d)$$

$$\underline{g2} \; \underset{\sim}{Z5} \; \underline{h2} = \underline{g4} \;\; ; \;\; \underline{c}\; \overset{\leftarrow}{\underset{\sim}{Z6}}\; \underline{g4} = \underline{p} \qquad\qquad (8e,\; 8f)$$

It is to be noted that only matrix-type connectives are used in this graph. Further, Eq. (8f) effectively yields the equation $\underline{c}\; \underset{\sim}{Z7} = \underline{p}$, in which $\underset{\sim}{Z7} = \underset{\sim}{I}$ or $\underset{\sim\sim}{AN}$, according as $\underline{c}$ = T or F.

Using Eqs (5), (6) and (7) and the 2X2 Boolean matrix representations of the various connectives (as given in Table 2 of Ref.[2]), we can work out the results shown in Table 1 below for the SNS truth values of the output $\underline{p}$, and of the intermediate terms $\underline{g1}$, $\underline{g2}$, $\underline{h1}$, $\underline{h2}$, $\underline{g4}$, corresponding to the eight possibilities, obtained by putting each of $\underline{a}$, $\underline{b}$, $\underline{c}$ equal to T and F.

Table 1. SNS truth values of the terms in Fig. 1 for the different T and F inputs for $\underline{a}$, $\underline{b}$, $\underline{c}$.

Table 1. SNS truth values of the terms in Fig.1 for
the different T and F inputs for a, b, c.

Sl. No.	Inputs			Intermediate output-inputs					Z7	Output p
	a	b	c	g1	g2	h1	h2	g4		
1	T	T	T	T	F	F	T	F	I	T
2	T	T	F	T	F	F	T	F	AN	X
3	T	F	T	T	F	D	D·	F	I	T
4	T	F	F	T	F	D	D	F	AN	X
5	F	T	T	D	D	F	T	D	I	D
6	F	T	F	D	D	F	T	D	AN	F
7	F	F	T	D	D	D	D	D	I	D
8	F	F	F	D	D	D	D	D	AN	F

It will be noticed that, in four cases, p has the definite

state T or F; and in two cases it has the doubtful state D.   What

is interesting is that, in two cases, namely Sl. No. 2; a = T,

b = T, c = F, and Sl. No. 4: a = T, b = F, c = F, the output p

has the impossible state X, and this (However,) impossible state (indicating a

contradiction) occurs only at the last step involving z_6 and c.

We can therefore conclude that, in these two cases, if all the

earlier steps, as well as the inputs a and b, are perfectly valid,

then c can only be T and not F (leading to Sl. Nos 1 and 3).

Vice versa, if c is known to be definitely F, then a can only be

F and not T (as in Sl. Nos 6 and 7).   It is readily verified that,

in all these four cases thus corrected, the output p is either T or

F, and not X.   (We shall not examine further the theory of

reversing the argument from a contradictory state (X) for the

output, leading to "conditional" consequences containing the

exclusive or, like the above — "Either c is to be changed to T or

a must be changed to F" — which occurs in this argument. This is

reserved for a separate communication.)

Table 2. List of intermediate terms and output p and q
          for the logical graph shown in Fig. 3

Sl. No.	Inputs a	b	Intermediate output-inputs g1	p1	h1	p2	Outputs p	q
1	T	T	T	D	F	F	F	T
2	T	F	T	X	F	F	X	T
3	F	T	D	D	D	D	D	D
4	F	F	D	F	D	D	F	D

(c) SNS logical graphs with circuits

We shall first consider an example of a graph with a closed circuit of edges (Fig. 3), in which the circuit does not have throughout the same direction for the flow of the argument.

The nature of the connectives is indicated in the graph itself.

The results for the various intermediate terms and for the outputs $\underline{p}$ and $\underline{q}$
/corresponding to possible truth values T and F for the inputs $\underline{a}$ and $\underline{b}$ are given in Table 2.

Table 2. List of intermediate terms and outputs $\underline{p}$ and $\underline{q}$ for the logical graph shown in Fig. 3

Fig. 3. A simple logical graph having a closed circuit of edges, in which the outputs from two different paths are combined by the Boolean connective $\underset{\sim}{V}$

It is to be noted that $\underline{p1}$ and $\underline{p2}$ are the truth values giving information regarding the state of $\underline{p}$ coming <u>via</u> two different paths, which are combined by the vidya operator $\underset{\sim}{V}$ to give the values of p_α and p_β that is common to them, so as to give $\underline{p} = (p_\alpha \, p_\beta)$

(∂c Eq $(6\,\mathfrak{g}1)$).
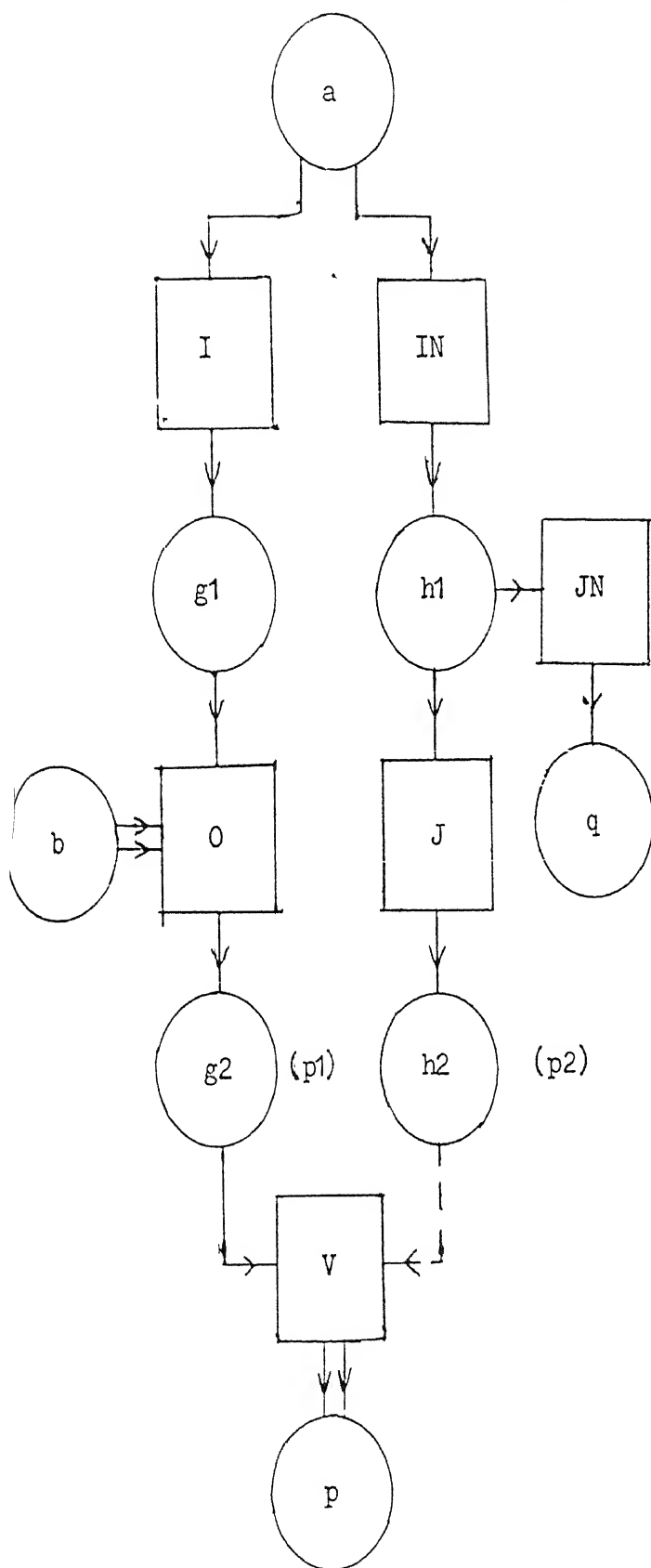
$$\underline{a} \; \underset{\sim}{\underline{I}} \; = \; \underline{g1} \qquad \text{(Step 1)}$$

$$\underline{\underline{b}} \; \overset{\leftarrow}{\underset{\wedge}{0}} \; \underline{g1} \; = \; \underline{\underline{g2}} \; = \; \underline{\underline{p1}} \quad (\text{ "} \quad 2)$$

$$\underline{a} \; \underset{\sim}{\underline{I}} \; \underset{\sim}{\underline{N}} \; = \; \underline{\underline{h1}} \qquad (\text{ "} \quad 3)$$

$$\underline{\underline{h1}} \; \underset{\sim}{\underline{J}} \; = \; \underline{\underline{h2}} \; = \; \underline{\underline{p2}} \quad (\text{ "} \quad 4)$$

$$\underline{\underline{p1}} \; \underset{\sim}{\underline{V}} \; \underline{\underline{p2}} \; = \; \underline{\underline{p}} \qquad (\text{ "} \quad 5)$$

$$\underline{\underline{h1}} \; \underset{\sim\sim}{\underline{JN}} \; = \; \underline{\underline{q}} \qquad (\text{ "} \quad 6)$$

Fig.3. A simple logical graph having a closed circuit of edges, in which the outputs from two different paths are combined by the Boolean connective $\underset{\sim}{\underline{V}}$.

It will be noticed that no contradiction arises for the

output $\underset{=}{q}$. On the other hand, $\underset{=}{p}$ = X for the inputs $\underset{=}{a}$ = T, $\underset{=}{b}$ = F;

but the impossible state X arises for the first time for $\underline{p1}$,

produced by Step 2 in the graph. Hence, one of the two inputs

to $\overset{\leftarrow}{\underset{\sim}{0}}$, namely $\underset{=}{b}$ and $\underline{g1}$, must be changed. If $\underset{=}{b}$ is made T, then

$\underline{g2}$ (= $\underline{p1}$) is D, for $\underset{=}{a}$ = T, and we get the case of Sl.No. 1 for the

inputs, with the X-state being eliminated. On the other hand, if

we know for certain that $\underset{=}{b}$ = F, the equation for step 2 becomes

$\overset{\leftarrow}{F \underset{\sim}{0}}$ $\underline{g1}$ = $\underline{g2}$ (= $\underline{p1}$), or $\underline{g1} \underset{\sim}{0}^{c}$ = $\underline{g2}$, which requires that $\underline{g1}$ = F.

On reversing step 1 in Fig. 3, we have $\underline{g1} \underset{\sim\sim\sim}{NIN}$ = $\underset{=}{a}$ and this yields

$\underset{=}{a}$ = F for the input $\underline{g1}$ = F. Thus, we get a conditional consequence

of the contradiction X for $\underline{p1}$ (= $\underline{g2}$), namely, "Either $\underset{=}{b}$ is to be

changed from F to T, or $\underset{=}{a}$ to be changed from T to F", a consequence

very similar to that of an X arising in Table 1 for Fig. 2.

The other interesting feature in Fig. 3 is the occurrence of

the SNS Boolean operator $\underset{\sim}{V}$, as mentioned above. It removes the

doubt in one of $\underline{p1}$ and $\underline{p2}$ when the other input has a definite

state T or F, in Step 5 (as in S1. No. 1   of Table 2). If,

however, one of $\underline{p1}$ and $\underline{p2}$ were T and the other F (as does not

happen in Table 2) it would lead to

$$\underline{p} = T \underset{\sim}{V} F = (1 \quad 0) \oslash (0 \quad 1) = (0 \quad 0) \equiv X \tag{9}$$

If so, such a contradiction (X) for the state (truth value) of $\underline{p}$

would indicate that "Either $\underline{p1}$ has to be changed from T to F, or

p2 has to be changed from F to T" and the consequence in either

case worked back in the reverse sense of the argument, to the

inputs. An example of this will be shown in the next subsection.

(d)   SNS logical graph having a directed circuit

(i) Condensing of n unary equations in sequence

Suppose we have (n-1) statements, all unary, as in Eq (10)

below:

$$\underline{a} \; \underset{\sim\sim}{Z1} = \underline{g1}, \quad \underline{g1} \underset{\sim\sim}{Z2} = \underline{g2}, \; \ldots \ldots, \; \underline{g(n-2)} \; \underset{\sim}{Z(n-1)} = \underline{b} \tag{10}$$

Then, they can be readily combined to give a single equation

$$\underline{a} \; \underline{Z} = \underline{b} \; , \quad |Z| = |Z1| \; Z2| \; . \; . \; . \; . \; |Z(n-2)| \qquad (11a, \; b)$$

If we are not interested in the intermediate terms $\underline{\underline{g1}}$ to $\underline{\underline{g(n-2)}}$,

then (11a) can be employed for all calculations pertaining to

a logical graph, of which (10) is a subgraph. The equations

(11a, b) can also be written in the reverse sense, from $\underline{b}$ to $\underline{a}$,

for which the relevant equation, obtained by reversing the sense

of all the (n-1) equations in (10) is

$$\underline{b} \; \underline{Z}^{t} = \cdot \underline{a} \; , \quad |Z^{t}| = |Z(n-2)^{t}| \; . \; . \; . \; . |Z2^{t}| \; Z1^{t}| \qquad (12a, \; b)$$

The analogues of the condensed equations (11a) and (11b) are

very relevant to QPL statements of type-1, and they will be

discussed in Section   below.

(ii) Cyclic equations

We shall consider now a cyclic set of equations in SNS logic,

obtained by augmenting (10) by adding one more equation $\underline{b} \; \underline{Zn} = \underline{a}$.

This yields a two-equation circular argument shown in Fig. 4(a):

$$\underline{a}\ \underline{Z}' = \underline{b}\ ,\quad \underline{b}\ \underline{Z}'' = \underline{a} \tag{12a, b}$$

with

$$\underline{Z}' = \underline{Z1}\ \underline{Z2}\ \cdots\ \underline{Z(n\text{-}1)}\ ,\quad \underline{Z}'' = \underline{Zn} \tag{13a, b}$$

We shall consider a few combinations of $\underline{Z}'$, $\underline{Z}''$ in (12) to

see how the circularity of the argument affects them. If $\underline{Z}' = \underline{Z}''$

$= \underline{I}$, $a_T \longmapsto b_T$, $b_T \longmapsto a_T$; but $a_F \longmapsto b_D$, $b_D \longmapsto a_D$. Hence,

there is only one solution possible for the two equations (12a, b)

viz. $\underline{a}$ = T, $\underline{b}$ = T. All other possibilities lead to the doubtful

state.

If $\underline{Z}' = \underline{I}$, $\underline{Z}'' = \underline{IN}$, and we start with $\underline{a}$ = T, we obtain

$a_T \longmapsto b_T$, but $b_T \longmapsto a_D$ and on continuing the cycle $a_D \longmapsto b_D$

$b_D \longmapsto a_D$. Hence, starting with a_T does not lead to a solution

with either T or F for $\underline{a}$ and $\underline{b}$. On the other hand, if $\underline{a}$ = F,

$a_F \longmapsto b_D$, $b_D \longmapsto a_D$ and once again $\underline{a}$ = D, $\underline{b}$ = I the only

solution obtained. Hence, the graph of Fig. 4(a) has a solution

Fig.4. (a) Directed circuit with two unary connections.

(b) Inclusion of the operator $\underline{V}$ and removal of the
directed circuit. Note the feed-back of the total

(a)   (b)

Fig.4. (a) Directed circuit with two unary connectives.

(b) Inclusion of the operator $\underline{V}$ and removal of the
directed circuit. Note the feed-back line from a to a1

with the definite state, but only with the indefinite state D,

when $\underset{\sim}{Z}' = \underset{\sim}{I}$ and $\underset{\sim}{Z}'' = \underset{\sim\sim}{IN}$. We will now consider another such

pathological case, with $\underset{\sim}{Z}' = \underset{\sim}{E}$, $\underset{\sim}{Z}'' = \underset{\sim}{N}$. Then if $\underset{=}{a} = T$, we get

the unending sequence $a_T \longmapsto b_T \longmapsto a_F \longmapsto b_F \longmapsto a_T \longmapsto$, ...

We get, for both $\underset{=}{a}$ and $\underset{=}{b}$, the repeating sequence . . . T,F,T,F,. . . .

To avoid this, we can use the vidya operator, as in Fig. 4(b).

Since we start from $\underset{=}{a}$ and end with $\underset{=}{a}$ in Fig. 4(a), we call the two

terms at the beginning and the end of the circuit as $\underline{\underline{a1}}$ and $\underline{\underline{a2}}$

respectively and test for their consistency by introducing the

binary $\underset{\sim}{V}$ between them. When this is done, the graph does not have

a closed circuit with the direction of flow of logic being the

same throughout. In Fig. 4(b), it is clockwise from $\underline{\underline{a1}}$, via $\underset{\sim}{Z1}$, $\underset{\sim}{b}$,

$\underset{\sim}{Z2}$ and $\underline{\underline{a2}}$, to $\underset{\sim}{V}$, but it is counter-clockwise from $\underline{\underline{a1}}$ to $\underset{\sim}{V}$. We

shall show that no infinite repetition occurs for this.

(iii) Feedback line

Thus, taking $\underset{\sim}{Z}' = \underset{\sim}{I}$, $\underset{\sim}{Z}'' = \underset{\sim}{I}$ as in the example above, we

obtain $\underline{\underline{a2}} = T$ starting from $\underline{\underline{a1}} = T$; and $\underline{\underline{a1}} \underset{\sim}{V} \underline{\underline{a2}} = T$. The curved

This yields a two-equation circular argument shown in Fig. 4(a):

$$\underline{a} \, \underline{Z}' = \underline{b} \, , \quad \underline{b} \, \underline{Z}'' = \underline{a} \qquad \qquad (12a, \ b)$$

with

$$\underline{Z}' = \underline{Z1} \, \underline{Z2} \, \ldots \ldots \underline{Z(n-1)} \, , \quad \underline{Z}'' = \underline{Zn} \qquad \qquad (13a, \ b)$$

We shall consider a few combinations of $\underline{Z}'$, $\underline{Z}''$ in (12) to

see how the circularity of the argument affects them. If $\underline{Z}' = \underline{Z}''$

$= \underline{I}$, $a_T \longmapsto b_T$, $b_T \longmapsto a_T$; but $a_F \longmapsto b_D$, $b_D \longmapsto a_D$. Hence,

there is only one solution possible for the two equations (12a, b)

viz. $\underline{a} = T$, $\underline{b} = T$. All other possibilities lead to the doubtful

state.

If $\underline{Z}' = \underline{I}$, $\underline{Z}'' = \underline{IN}$, and we start with $\underline{a} = T$, we obtain

$a_T \longmapsto b_T$, but $b_T \longmapsto a_D$ and on continuing the cycle, $a_D \longmapsto b_D$

$b_D \longmapsto a_D$. Hence, starting with a_T does not lead to a solution

with either T or F for $\underline{a}$ and $\underline{b}$. On the other hand, if $\underline{a} = F$,

$a_F \longmapsto b_D$, $b_D \longmapsto a_D$ and once again $\underline{a} = D$, $\underline{b} = D$ is the only

solution obtained. Hence, the graph of Fig. 4(a) has no solution

---

Fig.4. (a) Directed circuit with two unary connectives.
   (b) Inclusion of the operator $\underline{V}$ and removal of the
       directed circuit. Note the feed-back line from a to a1

dotted line indicates that this final output T for a is to be

fed back to $a1$, as in a computer cycle. Such a feedback leads

to everything being satisfactory and consistent in this example.

If now we take $Z' = I$, $Z'' = IN$, $a1 = T$ leads to $a2 = D$

and $a = a1 \lor a2 = T$, which is what is to be fed back to $a1$.

The fed-back value of $a1$ is identical with the starting one.

Note that, in the main graph, without the dotted feedback line,

$a1$ is the input, while $a2$ is only an intermediate input-output,

leading to the output a.

It is therefore to be noted that the feedback line is another

elementary component of a logical graph that we must include in

addition to the set of three connectives shown in Fig. 1.

Consider now the third case of subsection (ii), namely

$Z' = E$, $Z'' = N$. Then, we obtain the following for Fig. 4(b)

$$(a1 = T) \longmapsto (a2 = F) \longmapsto a = X \qquad (13a)$$

$$(a1 = F) \longmapsto (a2 = T) \longmapsto a = X \qquad (13b)$$

Hence, both the inputs T and F for $\underline{\underline{a1}}$ lead to contradictions,

so that the set of equations,

$$\underline{\underline{a1}} \; \underline{E} = \underline{b} \; , \quad \underline{b} \; \underline{N} = \underline{\underline{a2}} \; , \quad \underline{\underline{a1}} \; \underline{V} \; \underline{\underline{a2}} = \underline{a}$$

$(14 a, b, c)$
$c - d - e$
$(\cancel{19d, b, e})$

have no solution, with a definite T or F, for the input $\underline{a1}$.

This means that the first two equations in $(\cancel{14a,b})$ $(18 a, b)$ are

mutually contradictory and the consequence of the state X for $\underline{a}$

irrespective of the state T or F for $\underline{\underline{a1}}$ indicates the following:

"Either E must be changed to $\underline{E}^c (=\underline{N})$ in $(\cancel{14c})$ $(14a)$, or $\underline{N}$ must be changed to

$\underline{N}^c (=\underline{E})$ in $(\cancel{14b})$ $(14b)$", — a conditional conclusion very similar to what

we obtained for   the logical graphs in Fig.2 and Fig. 3,

when a contradiction X arises.

(iv) Single-term cyclic equations

Eq.(12), containing two unary relations can be condensed

still further to give an one-term circular argument, as in (15a),

whose graph is shown in Fig.5(a).

$$\underline{a} \; \underline{Z} = \underline{a} \; , \quad \underline{Z} = \underline{Z}' \; \underline{Z}'' \qquad (15a, b)$$

Fig.5 (a) Closed directed circuit with a single term and only
one connective
(b) Same as (a), but avoiding the directed circuit, but
with a feed-back line.

(a)   (b)
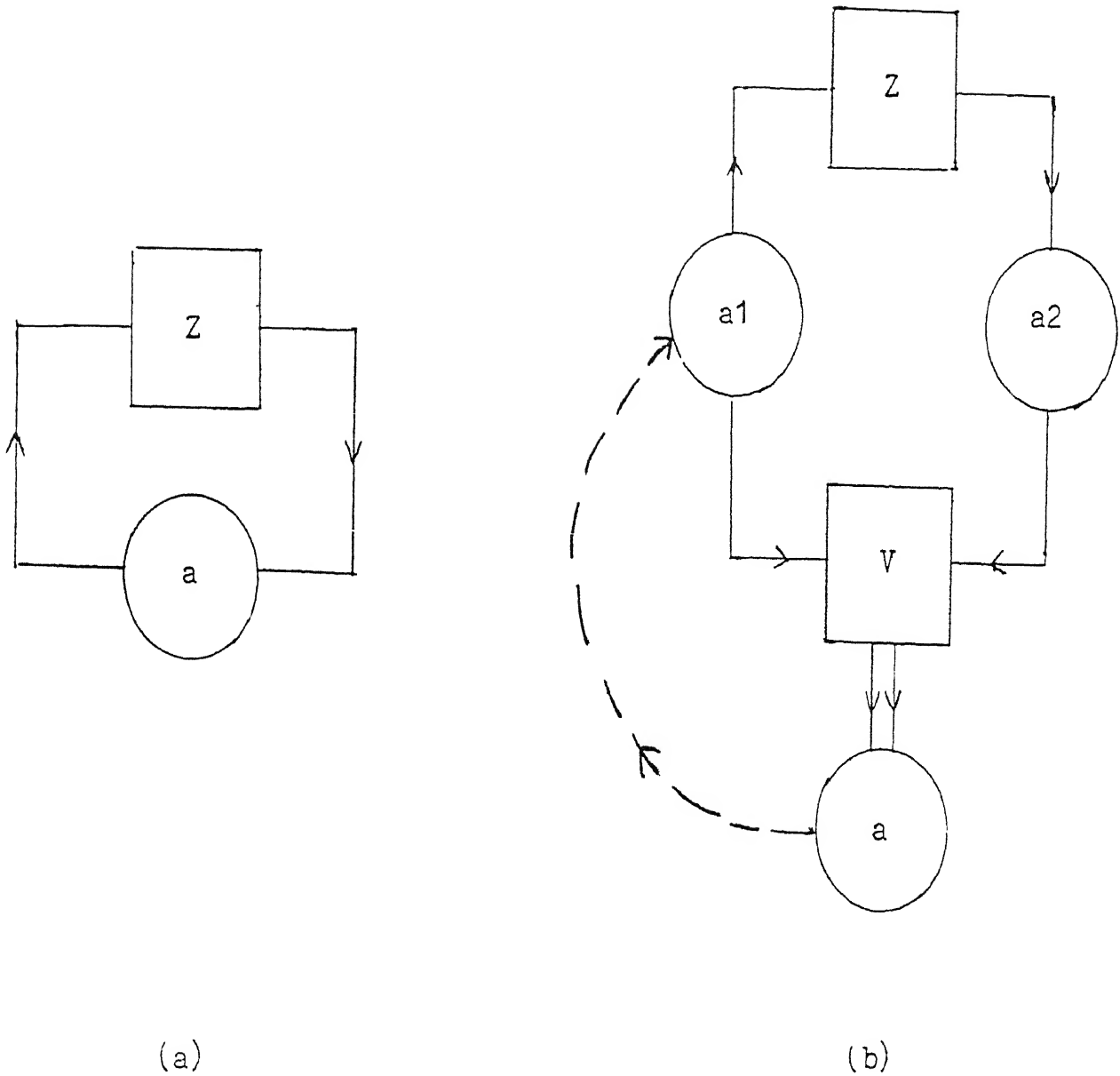
Fig. 5. (a) Closed directed circuit with a single term and
only one connective

(b) Same as (a), but avoiding the directed circuit,
but with a feed-back line.

We shall consider the behaviour of this for typical cases of

Z being I, IN, E and N. We denote, for convenience, the input

a in (15a) by $a1$ and the output by $a2$, and the output is fed as

input straightaway for the next cycle. Then, we obtain, for

$Z = I$, the following:

$$a_T \longmapsto a_T \longmapsto a_T \longmapsto \ldots \qquad (15c)$$

$$a_F \longmapsto a_D \longmapsto a_D \longmapsto \ldots \qquad (15d)$$

Hence, $a = T$ is a solution for this case, having a definite

truth value.

If now $Z = I\,N$,

$$a_T \longmapsto a_F \longmapsto a_D \longmapsto a_D \ldots \qquad (16a)$$

$$a_F \longmapsto a_D \longmapsto a_D \longmapsto \ldots \qquad (16b)$$

so that, the only stable solution of (15a) for $Z = IN$ is $a = D$, which

has the doubtful state.

For the case when $Z = E$,

$$a_T \longmapsto a_T \longmapsto \ldots \; ; \; a_F \longmapsto a_F \longmapsto \ldots \qquad (17a, b)$$

and both $\underline{a}$ = T and $\underline{a}$ = F are possible solutions.

Only for $\underset{\sim}{Z}$ = $\underset{\sim}{N}$ do we get a pathological result, as in

(18a, b):

$$a_T \longmapsto a_F \longmapsto a_T \longmapsto \dots ; \quad a_F \longmapsto a_T \longmapsto a_F \longmapsto \dots \qquad (18a,b)$$

Thus, we get an endless alternation in the truth value of $\underline{a}$,

irrespective of whether we start from T or F.

However, this is not a paradox, but a contradiction, as is

shown by converting the graph in Fig. (5a) into that in Fig. 5(b),

with the vidya operator checking for contradiction, and including

also the feedback line (as in Fig. 4(b)). For a general $\underset{\sim}{Z}$, the

logical graph in Fig. 5(b) corresponds to the equations:

$$\underline{\underline{a1}} \underset{\sim}{Z} = \underline{\underline{a2}} \ , \quad \underline{\underline{a1}} \underset{\sim}{V} \underline{\underline{a2}} = \underline{a}, \quad \underline{a} \longmapsto \underline{\underline{a1}} \ (if \ \underline{a} \neq X) \qquad (19a,b,c)$$

The following results for $\underset{\sim}{Z}$ = $\underset{\sim}{N}$ are readily worked out:

$$(\underline{\underline{a1}} = T) \longmapsto (\underline{\underline{a2}} = F) \longmapsto (\underline{a} = X) \qquad (20a)$$

$$(\underline{\underline{a1}} = F) \longmapsto (\underline{\underline{a2}} = T) \longmapsto (\underline{a} = X) \qquad (20b)$$

Hence the argument contained in the graph shown in Fig. 5(b)

with $Z = N$ is illogical and self-contradictory, exactly as is

the case with Fig. 4(b). If we wish to remove this contradiction,

⌐Eq (13) for⌐

it can only be done by changing the argument itself, i.e. by

changing Z to Z^c — in this case by changing N to $N^c (\equiv \Sigma)$. When

we do this, (20a, b) go over to (17a, b) which are perfectly normal.

Another remedy is to admit the doubtful state D as input, when we

obtain (for $Z = N$ in Fig. 5(b)):

$$(\underline{\underline{a1}} = D) \longmapsto (\underline{\underline{a2}} = D) \longmapsto (\underline{a} = D) \longmapsto (\underline{\underline{a1}} = D) \quad \text{(consistent)} \quad (21)$$

The equation $\underline{a}\,\underline{N} = \underline{a}$ is the algebraic representation,

in our notation, of the famous Cretan liar paradox: "What I say is

false", which leads to an endless repetition of the truth values

⌐which stands for⌐

$T,F,\ T,F,\ \ldots$ for the statement $\underline{a}$ $\Big/$ "What I say". But, we have

and

got two solutions (a) $\underline{a} = X$, or (b) $\underline{a} = D$, which satisfy this

equation. The truth value X for $\underline{a}$ means that the statement is

"impossible", or "in alid, and there can be no such statement in

logic (propositional calculus). The second possible solution

$\underline{a} = D$ means that "we do not know whether the statement $\underline{a}$ is true

or false", or "the statement $\underline{a}$ may be true, or may be false",

a possible way of expressing its doubtful logical state which is

permissible in SNS logic. Since the state D occurs very frequently

in the analysis of arguments in PC, as in those represented by

the graphs in Figs. 2 and 3, there should be no objection to having

it as the state of the only term in the circular argument $\underline{a} \underset{\sim}{N} = \underline{a}$.

It will be shown below that, if circular arguments in directed

circuits are removed by introducing the vidya operator, as in

Figs 4(b) and 5(b), then any general logical graph in PC can be

converted into a set of elementary logical equations, which can

then be implemented in sequence (see Section $2(t)$). In such a case,

the consequences of the inputs having given truth values can

always be worked out and the logical states of the outputs can be

determined by straightforward algorithms. Such algorithms can even

be converted into a computer program. This is discussed in

the next subsection.

We shall not describe here the procedure whereby contradictions,

when they arise, are used to work backwards into the argument

and find out the precise location of the place where the contradictio

arises, and the consequent change that should be introduced into

truth values, or nature of connectives, in earlier steps in the

argument. The general treatment of this is a big project and

has not yet been completed. It will be discussed in a later part

of this series. Here, we shall only point out, in the next subsection,

the way in which SNS graphs are implemented and worked out,

because this is vital for giving a general theory of the implementati

of a QPL argument completely consisting of Type-1 statements.

(e) Algebraic implementation of a general SIS graph

Any general SIS argument can be built up employing only

the three types of elementary statements as shown in Fig. 1.

We assume further that there are no circular arguments in it,

consisting of a full directed circuit of edges. If one such

occurs, as in Fig. 4(a), we avoid circularity by employing two

terms such as $\underline{a1}$ and $\underline{a2}$ in Fig. 4(b), connected by the binary

vidya operator, to give $\underline{a1} \underset{\sim}{V} \underline{a2} = \underline{a}$. We avoid also multiple

inputs into a term (say $\underline{a}$) by using the vidya operator $\overset{\text{(as in)}}{\underset{\wedge}{\underline{a1}}} \underset{\sim}{V} \underline{a2} = \underline{a1}'$,

$\underline{a1}' \underset{\sim}{V} \underline{a3} = \underline{a2}'$ etc. (See also Section 4 below). On the other

hand, there can be multiple outputs from a term to different

connectives $\underline{Z1}$, $\underline{Z2}$ etc. Thus from the same term $\underline{g}$ we can have

Eqs of the type $\underline{g} \underset{\sim}{I} = \underline{p1}$, $\underline{g} \underset{\sim}{A} \underline{h} = \underline{p2}$, etc. On the other hand,

an operator, if it is unary can only have one input and one output,

and if it is binary can only have two inputs and one output.

For such an argument, the flow of logic in it can be

expressed in the form of a set of equations, as shown, for instance,

for Figs. 2, 3 and 4, and these equations are executable in

sequence, using the given truth values of all inputs, so as to

yield the truth values (in SIS) of all the outputs. The proof of

this is remarkably simple, as given below, using mathematical induction.

Firstly, it can be readily verified that all graphs of this nature, having upto 10 nodes are implementable. We use the term "implementable" to indicate that a set of SNS logical equations representing the graph can be written down, and executed in the sequential order in which they are listed. The input(s) for the algebraic equation representing any step contained in it would have been obtained in one of the preceding steps, or it (they) will be fresh inputs for the argument.

For larger graphs having n nodes, we prove the result by induction on the number n. We assume that all SNS logical graphs $\mathcal{L}_m$ without circular arguments as subgraphs, with $m \leqslant n$ nodes, are implementable, and prove from this that all such graphs $\mathcal{L}_{n+1}$ with (n+1) nodes are implementable. The proof is as follows.

The graph $\mathcal{L}_{n+1}$ has at least one output, say $\underline{p}$. Then, this has arisen either from one term (say $\underline{tj}$) via a unary connection $\underset{\sim}{Z}$ (as in Fig. 1(a)), or from two terms $\underline{tk}$ and $\underline{t\ell}$ via a binary connective $\underset{\sim}{Z}$ or $\overset{\leftarrow}{\underset{\sim}{Z}}$ (as in Figs. 1(b) or 1(c)). In all cases, remove

the connective ($\underline{Z}$ or $\underleftarrow{\underline{Z}}$) and the output term $\underline{\underline{p}}$, and obtain a graph $\mathscr{G}_{n-1}$, or $\mathscr{G}_{n-2}$, as the case may be. By assumption, this graph $\mathscr{G}_{n-1}$ or $\mathscr{G}_{n-2}$ is implementable.

Then, add the following equations at the end of the list of equations for the reduced graph (namely $\mathscr{G}_{n-1}$ or $\mathscr{G}_{n-2}$):

For $\mathscr{G}_{n-1}$: $\underline{tj}$ $\underline{Z}$ = $\underline{\underline{p}}$ (22a)

For $\mathscr{G}_{n-2}$: $\underline{tk}$ $\underline{Z}$ $\underline{\underline{t\ell}}$ = $\underline{\underline{p}}$, or $\underline{tk}$ $\underleftarrow{\underline{Z}}$ $\underline{\underline{t\ell}}$ = $\underline{\underline{p}}$ (22b,c)

It is quite obvious that the augmented list of equations for $\mathscr{G}_{n+1}$ is implementable.

The above is an existential proof of the theorem that "Given the graph of any argument in PC, we can always write down for it a set of elementary SIS equations (of the types of (22a,b,c)), which can be executed in serial order". A Fortran program to perform the execution has been outlined, but not yet written. However, the above theorem indicates that the sequence of equations is theoretically capable of being written out. The discussion of the algorithm is reserved for a later publication, as it is common to SIS and QFL, and for arguments which use combinations of CL, SIS and QFL.

3. Logical graph of a QPL argument composed of Type-1 statements.

(a) Elementary statements of this type

A general statement of Type-1 (in QFL) has the form of

Eq (1), and the three forms of elementary statements of this type

are given in Eqs. (2), (3), (4) of Section 1. Since the variable

x is the same throughout in an argument composed solely of Type-1

statements, we shall employ the following abbreviated notation

for these:

$$\text{Unary} \qquad : \quad \mathcal{Q}/(\underline{a} \; \underset{\sim}{Z} = \underline{b}) \qquad\qquad (23a)$$

$$\text{Binary forward} \quad : \quad \mathcal{Q}/(\underline{a} \; \underset{\sim}{Z} \; \underline{b} = \underline{c}) \quad (\text{Matrix}) \qquad (23b)$$

$$\underline{a} \; \mathbb{Z} \; \underline{b} = \underline{c} \quad (\text{Boolean}) \qquad (23c)$$

$$\text{Binary reverse} \quad : \quad \mathcal{Q}/(\underline{c} \; \underset{\sim}{Z} \; \underline{a} = \underline{b}) \qquad\qquad (23d)$$

In these, $\mathcal{Q}/ = \forall$ or $\exists$ only; but $\underset{\sim}{Z}$ can be any one of the ten

matrix operators in all three of (23a, b, d), while it can also

be one of the two Boolean connectives, "upon" and "vidya", for

the binary forward relation in (23c). When this occurs, the

corresponding connectives $\underset{\cup}{|}$ and $\bigvee$ of BA-3 become operative,

and the equations become

$$\underline{a1} \ \text{\Large W} \ \underline{a2} = \underline{a} \ ; \ \underline{a1} \ \text{\Large W} \ \underline{a2} = \underline{a} \qquad (23e,f)$$

where $\underline{a1}$, $\underline{a2}$ stand for the canonical form in BA-3 of the two

QPL inputs in the standard form. (See Part II, Section 6, for a

description of these forms, and of the canonizer and standardizer

which interconvert these representations of a QPL term. As an

illustration, $\forall(a1_T) \ \text{\Large W} \ \exists(a2_T) \equiv (1 \ 0 \ 0) \otimes (1 \ 1 \ 0) =$

$(1 \ 0 \ 0) \equiv \forall(a_T)$ and $\forall(a1_T) \ \text{\Large W} \ \exists(a2_F) = (1 \ 0 \ 0) \otimes$

$(0 \ 1 \ 1) = (0 \ 0 \ 0) = \emptyset \ (\underline{a})$, the contradictory state for $\underline{\underline{a}} \ \alpha$.

It may be mentioned that the EFL contradictory state arises

only as the output of the operator $\text{\Large W}$, as is the case with $\underset{\approx}{V}$ in

SNS. It is also obvious that the variable x is the same for $\underline{a1}$,

$\underline{a2}$ and $\underline{a}$ in (23e,f). Upon inputting $\mathcal{O}\!\!\!/_{a1}(\underline{a1}') \equiv \underline{a1}'$ and

$\mathcal{O}\!\!\!/_{a2}(\underline{a2}') \equiv \underline{a2}'$ in Eqs (23) or (23), the output $\underline{a}'$ is obtained

via the Boolean sum or Boolean product of the two 3-vectors, as

the case may be.

We shall indicate how the quantifier $\mathcal{O}\!\!\!/$, and the connective

operator Z , are treated in parallel channels, with a small

interaction of Z with $\mathcal{Q}_{V}$, when the QPL input(s) are put in (23a,b,c)

and the QPL output is worked out. The relevant formulae are given

in outline, along with the graphical flow chart representation

of the three forms of elementary statements, in Fig. 6.

Fig. 6. Elementary connective relations in QPL Type-1 sentences.

We shall illustrate the derivation of the formulae by taking the

example of a categorical statement, discussed in detail in Part III

[Ref. 1] , which has the form $\mathcal{Q}_{V}(\underline{a}\ I = \underline{b})$. Suppose the input is

$\mathcal{Q}_{V_a}(\underline{a}')$. Then, the flow of the logic on implementing

Eq. (23a) with $Z = I$ to yield the output $\mathcal{Q}_{V_b}(\underline{b}')$ is best

understood <u>via</u> the formulae in (24) below.

$$\mathcal{Q}_{V_a}(\underline{a}')\ ,\quad \mathcal{Q}_{V}(\underline{a}\ I)\ \longmapsto\ \mathcal{Q}_{V}'_a(\underline{a}'\ I)\ \longmapsto\ \mathcal{Q}_{V_b}(\underline{b}')\qquad\qquad (24)$$

where

$$\mathcal{Q}_{V}\otimes\mathcal{Q}_{V_a} = \mathcal{Q}_{V}'_a\ ,\quad \mathcal{Q}_{V}'_a\ \oplus\mathcal{Q}_{V_I} = \mathcal{Q}_{V_b}\qquad\qquad (25a,b)$$

and

$$\underline{a}'\ I = \underline{b}'\ ,\ \mathcal{Q}_{V_I} = \exists\qquad\qquad (25c,d)$$

$$\underline{a}' \ \underset{\sim}{Z} = \underline{b}' \ , \ (\mathcal{V} \otimes \mathcal{V}_a) \oplus \mathcal{V}_Z = \mathcal{V}_b$$
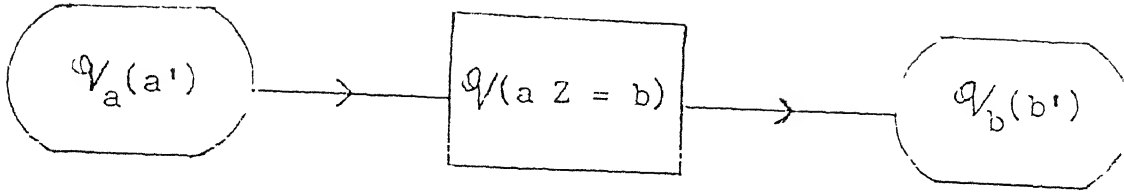
$$\mathcal{V}_Z = \forall \quad \text{for} \quad \underset{\sim}{Z} = \underline{E}\text{-type}$$

$$\mathcal{V}_Z = \exists \quad \text{for} \quad \underset{\sim}{Z} = \underline{I}\text{-type}$$

$$\exists (\underline{a} \underset{\sim}{A} = \underline{b}) \equiv \exists (\underline{b} \underset{\sim}{A} = \underline{a}) \equiv \exists (\underline{a} \underset{\sim}{I} = \underline{b}) \equiv \exists (\underline{b} \underset{\sim}{I} = \underline{a})$$

For $\forall (a \underset{\sim}{A} = b)$, see Fig. 6(b)

Fig.6(a) Unary QPL relation of Type-1
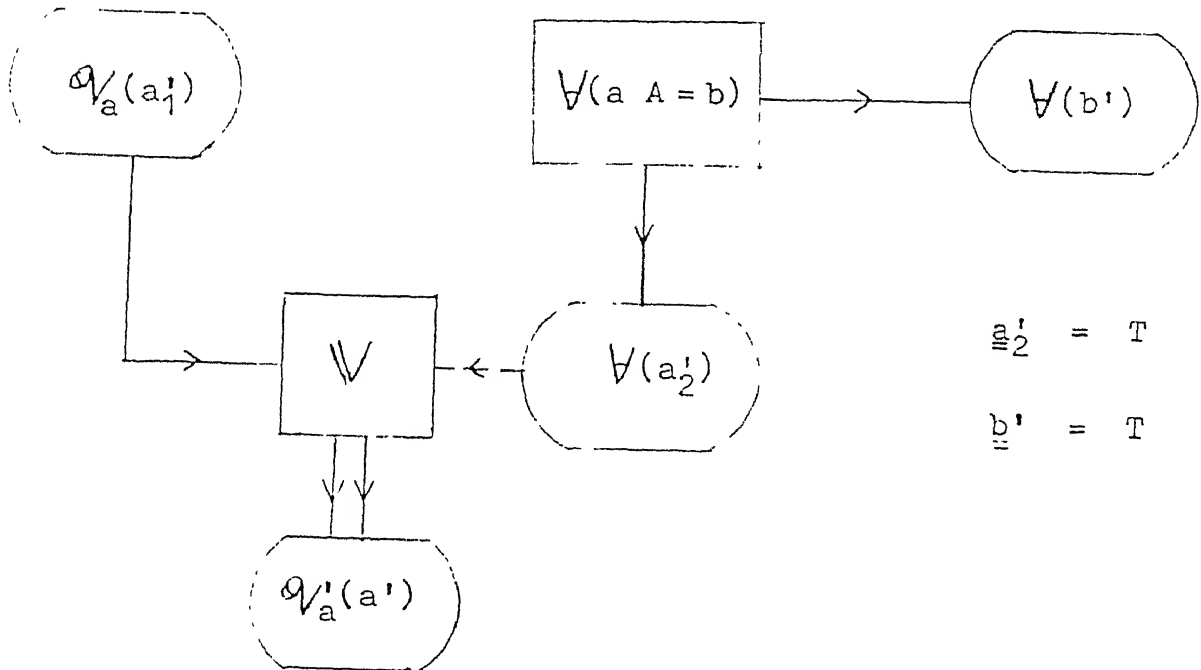
Fig. 6(b) Unary Type-1 relation with $(\forall, \underline{A})$ and input $\mathcal{V}_a(\underline{a}'_1)$. The modified $\underset{\sim}{A}$-construction is shown, with outputs $\forall (b')$ and $\mathcal{V}'_a(\underline{a}')$.

$\mathcal{V}_a(a')$ $\mathcal{V}_b(b')$

$\mathcal{V}(a\ Z\ b = c)$

$\mathcal{V}_c(c')$
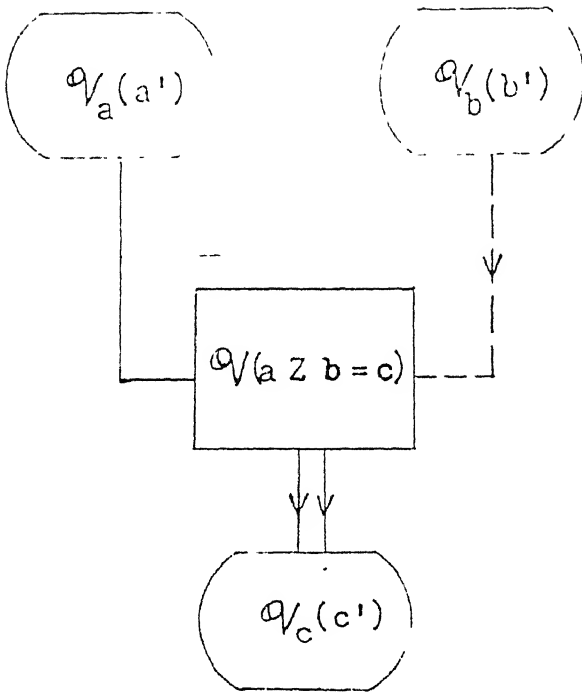
a' Z b' = c'

See text for formulae
regarding $\mathcal{V}_c$ for the
three types of Z ,
namely A-type, Q-type
and E-type (Eqns. 30,
32, 33, 34).

Fig. 6(c)　Binary forward QPL relation of Type-1.

$\mathcal{V}_a(a')$

$\mathcal{V}_c(c')$ $\mathcal{V}(c\ \overleftarrow{Z}\ a = b)$

$\mathcal{V}_b(b')$

a' Z = b', if c' = T

a' Z^c = b', if c' = F

$(\mathcal{V} \oplus \mathcal{V}_c \oplus \mathcal{V}_a)$

$\oplus (\mathcal{V}_Z$ or $\mathcal{V}_Z c) = \mathcal{V}_b$
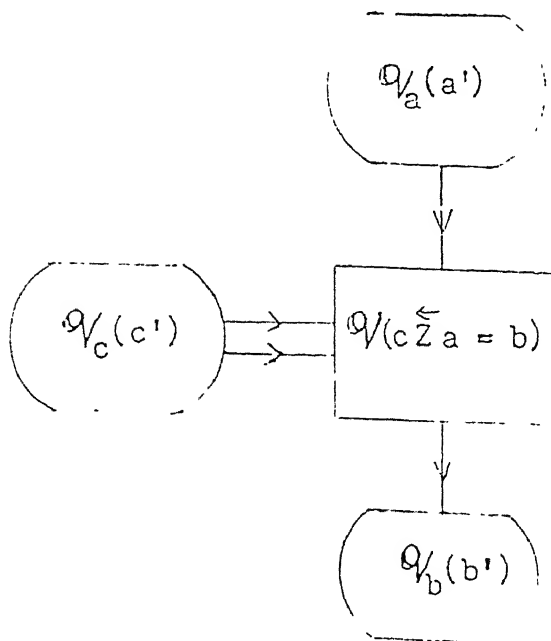
where $\mathcal{V}_Z$(or $\mathcal{V}_Z c$) is the
same as in Fig.6(a),
including the modified
A-construction.

Fig. 6(d)　Binary reverse QFL relation of Type-1.

An intuitive justification of the four equations (25a,b,c,d)

is as follows. The SNS equation (25c) is the simplest to see,

and it gives the output $\underline{b}'$ for $\underline{b}$, given the input $\underline{a}'$ for $\underline{a}$

and the operator equation $\underline{a} \underline{I} = \underline{b}$. As regards (25a), this employs

a new connective, which has been given the name "into" and having

the symbol $\bigotimes$, with all the quantifiers ($\mathcal{Q}$, $\mathcal{Q}_a$, $\mathcal{Q}'_a$; ....)

having only the two states $\forall \equiv (1 \ 0 \ 0)$ and $\exists \equiv (1 \ 1 \ 0)$.

The property of this operator $\bigotimes$ is best described by Table 3

which gives the multiplication table for the equation $\mathcal{Q}'_a = \mathcal{Q} \bigotimes \mathcal{Q}$

Table 3. Table for the "into" product $\mathcal{Q} \bigotimes \mathcal{Q}_a$

§(b) <u>Explanation of the new connective operator "into"</u>

// The contents in Table 3 are simply explained as follows.

If the equation contains $\mathcal{Q} = \forall$, and the input quantifier $\mathcal{Q}_a$

is also $\forall$, then the effective quantifier $\mathcal{Q}'_a$ in Eq (24) is seen

to be also necessarily $\forall$. If, on the other hand, either one of

$\mathcal{Q}$ or $\mathcal{Q}_a$ is $\forall$ and the other is $\exists$, then only the one that is

Table 3. Table for the "into" product[+] $\mathcal{Q} \otimes \mathcal{Q}_a$

$\mathcal{Q}$ \ $\mathcal{Q}_a$	$\forall$	$\exists$	$\triangle$
$\forall$	$\forall$	$\exists$	$\triangle$
$\exists$	$\exists$	$\triangle$	$\triangle$
$\triangle$	$\triangle$	$\triangle$	$\triangle$

[+]The table lists the EPL states of $\mathcal{Q}' = \mathcal{Q} \otimes \mathcal{Q}_a$, for the permissible input states of $\mathcal{Q}$ and $\mathcal{Q}_a$ in Sentences of Type-1.

logically less restrictive of the two (namely $\exists$) will be the

quantifier of the resultant. This is because, if we take that

the equation $\mathcal{Q}/ (\underline{a} \, \underline{I} = \underline{b})$ is valid for $\mathcal{Q}/ = \forall$, while the

input state $\mathcal{Q}/_a$ is only $\exists$, then the resultant equation giving

the output $\mathcal{Q}/'_a (\underline{a}'I)$ will only be valid for $\mathcal{Q}/'_a = \exists$. In the

same way, if the equation is valid for $\mathcal{Q}/ = \exists$, while the

input is with $\mathcal{Q}/_a = \forall$, then also the resultant $\mathcal{Q}/'_a$ will only

be $\exists$.

The fourth possibility, in which the equation has $\mathcal{Q}/_a = \exists$,

and the input also has $\mathcal{Q}/_a = \exists$, requires special comment. The

situation is best understood from Fig. 2 of Ref [1], in which

the two inputs of $\exists$ may, or may not, have any common elements.

When they do not have any elements in common, the input $\mathcal{Q}/_a = \exists$,

when fed into $\mathcal{Q}/ = \exists$ for the equation, can lead to the resultant $\overline{\varPhi}$

("for none") for $\mathcal{Q}/'_a = \mathcal{Q}/ \otimes \mathcal{Q}/_a$. On the other hand, it is also

possible for the resultant to be $\mathcal{Q}/'_a = \exists$, if the intersection

is non-empty. Thus, the combination " $\mathcal{Q}/$ into $\mathcal{Q}/_a$" in Eq. (25a)

can lead to either $\exists$ or $\overline{\Phi}$. Consequently, the effective

quantifier for $\mathcal{Q}'_a$ is

$$\mathcal{Q}'_a = \exists \oplus \overline{\Phi} = (1 \quad 1 \quad 0) \oplus (0 \quad 0 \quad 1) = (1 \quad 1 \quad 1) = \Delta \qquad (26)$$

This is why the fourth entry, corresponding to $\mathcal{Q} = \exists$, $\mathcal{Q}_a = \exists$,

in Table 3 is Δ.

The remaining entries in Table 3 are readily seen to arise

from the following considerations. If one of either $\mathcal{Q}$ or $\mathcal{Q}_a$ is

Δ, the resultant $\mathcal{Q}'_a$ in $\mathcal{Q}_u(\underline{a}\ \underline{I})$ is completely indefinite (Δ)

and the fact that the other is $\forall$, $\exists$ or Δ is irrelevant. Hence,

we have the entries in the last row and column of Table 3.

It will be noticed that the entries in Table 3 are closely

similar to those for the Boolean sum "upon" in $\mathcal{Q} \oplus \mathcal{Q}_a$. All

of them (for $\mathcal{Q} \otimes \mathcal{Q}_a$) agree with the EPL state of $\mathcal{Q} \oplus \mathcal{Q}_a$,

except one — namely $\exists \otimes \exists$, which is Δ, while $\exists \oplus \exists = \exists$.

This new result, viz that $\exists \otimes \exists = \Delta$, derived here for $\underline{a}\ \underline{I} = \underline{b}$,

is equally true for all matrix operators $\underline{Z}$ in $\mathcal{Q}(\underline{a}\ \underline{Z} = \underline{b})$, with

the input $\mathcal{Q}_a(\underline{a}')$, leading to $\mathcal{Q}'_a(\underline{a}'\ \underline{Z} = \underline{b}')$, as will be shown

below. What is more, the operator $\bigotimes$ is equally required for

binary relations (both forward and reverse), as stated in the

formulae listed in Fig. 6. It turns out that the connective

$\bigotimes$ plays a dominant role in the theory of not only QPL sentences

of Type-1, but also in its extension to EPL sentences.

From the structure of Table 3, it is readily verified that

$\bigotimes$ is both associative and commutative. Further, it is also seen

that the following distributive formulae between $\bigotimes$ and $\bigoplus$ are valid:

$$(\mathcal{U}_1 \oplus \mathcal{U}_2) \otimes \mathcal{U}_3 = (\mathcal{U}_1 \otimes \mathcal{U}_3) \oplus (\mathcal{U}_2 \otimes \mathcal{U}_3) \qquad (27a)$$

$$\mathcal{U}_3 \otimes (\mathcal{U}_1 \oplus \mathcal{U}_2) = (\mathcal{U}_3 \otimes \mathcal{U}_1) \oplus (\mathcal{U}_3 \otimes \mathcal{U}_2) \qquad (27b)$$

Boolean addition ($\bigoplus$) is, however, not distributive with respect

to the "into" ($\bigotimes$) operation, so that, for example, sometimes

$$(\mathcal{U}_1 \otimes \mathcal{U}_2) \oplus \mathcal{U}_3 \neq (\mathcal{U}_1 \oplus \mathcal{U}_3) \otimes (\mathcal{U}_2 \oplus \mathcal{U}_3) \qquad (27c)$$

This can be verified from $\mathcal{U}_1 = \forall$, $\mathcal{U}_2 = \forall$, $\mathcal{U}_3 = \exists$, when

the l.h.s. gives $\exists$, while the r.h.s yields Δ .

The explanation of Eq. (25b) and the value $\exists$ for $\mathcal{Q}'_I$ in

(25d) are both given below for the particular case of the connective

operator $\underset{\sim}{Z} = \underset{\sim}{I}$. A general treatment for all connectives is given

in the next section. It will be noticed that, in the expression

$\mathcal{Q}'_a$ ($\underline{a}'$ I), $\mathcal{Q}'_a$ can be one of $\forall$, $\exists$ or Δ. We shall consider each

of these in turn. As discussed in Part III, the equation

$\forall$ ($\underline{a}'$ I = $\underline{b}'$) yields only the output $\exists$ ($\underline{b}'$) (and not $\forall$($\underline{b}'$), while

the expression $\exists$ ($\underline{a}'$ I = $\underline{b}'$) also yields $\exists$ ($\underline{b}'$) as output (see

Table 2, p 25 of Ref [1]). Thus, irrespective of $\mathcal{Q}'_a$ being $\forall$ or $\exists$,

$\mathcal{Q}'_b$ is only $\exists$, for $\mathcal{Q}'_a$ ($\underline{a}'$ I) = $\mathcal{Q}'_b$($\underline{b}'$). On the other hand,

if $\mathcal{Q}'_a$ is Δ, then it stands to reason that $\mathcal{Q}'_b$ also will only

have Δ. All these properties are readily satisfied by Eq (25b),

with the condition $\mathcal{Q}'_I = \exists$, as given in Eq (25d). These two

equations are empirical and have been designed to fit the properties

of the logical formulation of a categorical implicational statement.

However, it has been written in a form which is a special case of

what is true for a unary QPL relation in general. This section only

clarifies the method of approach and we shall give a full treatment

for the general case in the next section.

(c) Justification of Fig.6 for all elementary connective relations in QPL.

(i) Binary relations for general Z

We shall consider first Fig. 6(c) giving this type of elementary relation. Anologous to Eq (24) the flow of logic may be pictured as follows. We are given the binary relation $\mathcal{Q}(a\ Z\ b=c)$ in which the inputs are $\mathcal{V}_a(a')$ and $\mathcal{V}_b(b')$, and we are required to find out both the quantifier and SNS states of $\mathcal{V}_c(c')$.

$$\mathcal{V}_a(a'),\quad \mathcal{V}_b(b'),\quad \mathcal{V}(a\ Z\ b) \longmapsto \mathcal{V}_c(c') \tag{28}$$

For this we first put in the two inputs "into" the corresponding places in the relation, and we obtain:

$$\mathcal{V}_a(a') \otimes \mathcal{V} = \mathcal{V}'_a(a') \tag{29a}$$

$$\mathcal{V}_b(b') \otimes \mathcal{V} = \mathcal{V}'_b(b') \tag{29b}$$

Thereafter the way in which these quantifiers go to form $\mathcal{V}_c(c')$ is very complicated and requires separate treatment for the three types of matrix connectives, namely the E-type, A-type and O-type as in (30).

$$\text{E-type}\quad (E,\ N = E^c) \tag{30a}$$
$$\text{A-type}\quad (A,\ O^c,\ I^c,\ J^c) \tag{30b}$$
$$\text{O-type}\quad (O,\ A^c,\ I,\ J) \tag{30c}$$

After redefining $\underline{a}$ and $\underline{b}$, the SNS relation can be converted

to the form $\underline{a} \, \underline{\underset{\sim}{Z}} \, \underline{b} = \underline{c}$, with $\underline{\underset{\sim}{Z}} = \underline{\underset{\sim}{E}}$, $\underline{\underset{\sim}{A}}$, or $\underline{\underset{\sim}{O}}$, for each of the

three types of connectives in $(30a,b,c)$. Then the formula for

$\underline{c}'$ is the same for all three, namely

$$\underline{a}' \, \underline{\underset{\sim}{Z}} \, \underline{b}' = \underline{c}' \tag{31}$$

The formula for $\mathcal{V}_c$ on the other hand, is best given in the

form of an algorithm for each case, as in (32), (33), (34) below.

$\underline{\underset{\sim}{Z}} = \underline{\underset{\sim}{A}}$

If $\underline{a}' = F$, $\underline{b}' = F$, then $\underline{c}' = F$ and $\mathcal{V}_c = \mathcal{V}'_a \otimes \mathcal{V}'_b$ (32a)

If one of $\underline{a}'$, $\underline{b}'$ is F and the other is T, then $\underline{c}' = F$
and $\mathcal{V}_c = \mathcal{V}'_a$ or $\mathcal{V}'_b$, according as $\underline{a}'$ or $\underline{b}'$ is F (32b)

If both of $\underline{a}'$, $\underline{b}'$ are T, then $\mathcal{V}_c = \mathcal{V}'_a \oplus \mathcal{V}'_b$. If
however, both $\mathcal{V}_a$ and $\mathcal{V}_b$ are $\reflectbox{E}$, then $\mathcal{V}_c = \mathcal{V}'_a \otimes \mathcal{V}'_b$ (32c)

$\underline{\underset{\sim}{Z}} = \underline{\underset{\sim}{O}}$

If $\underline{a}' = T$, $\underline{b}' = T$, then $\underline{c}' = T$ and $\mathcal{V}_c = \mathcal{V}'_a \otimes \mathcal{V}'_b$ (33a)

If one of $\underline{a}'$, $\underline{b}'$ is T and the other is F, then $\underline{c}' = T$
and $\mathcal{V}_c = \mathcal{V}'_a$ or $\mathcal{V}'_b$, according as $\underline{a}'$ or $\underline{b}'$ is T (33b)

If both $\underline{a}'$ and $\underline{b}'$ are F, then $\mathcal{V}_c = \mathcal{V}'_a \oplus \mathcal{V}'_b$.
However, if $\mathcal{V}_a$ and $\mathcal{V}_b$ are both $\reflectbox{E}$, then $\mathcal{V}_c = \mathcal{V}'_a \otimes \mathcal{V}'_b$. (33c)

E-type:

The formula for c' is $a' \perp b' = c'$ as in Eq (31) and the

formula for the quantifier is as follows:

$$\mathcal{Q}'_a \oplus \mathcal{Q}'_b = \mathcal{Q}_c \tag{34a}$$

However, if both a' and b' have $\mathcal{Q} = \exists$, then

$$\mathcal{Q}'_a \otimes \mathcal{Q}'_b = \mathcal{Q}_c \tag{34b}$$

:Note that there is a perfect symmetry between the formulae

in Eq (32) for $\underset{\sim}{A}$ and Eq (33) for $\underset{\sim}{O}$ and that what is valid

for the input state T for $\underset{\sim}{O}$ is valid for F for $\underset{\sim}{A}$ and vice versa,

a phenomenon we had noticed in SNS theory of the relations involving

the matrix operators for $\underset{\sim}{O}$ and $\underset{\sim}{A}$. We are not giving the proofs

of the formulae in Eqs (32, 33 and 34) here, but shall give a short

summary of the proof in Appendix 1.

Having thus considered all the ten SNS matrix connectives $\underset{\sim}{Z}$

for Type-1 QPL binary forward statements, we shall now consider

the two Boolean binary connectives $\mathbb{U}$ and $\mathbb{V}$. As mentioned

in Section 3(a), these connectives straightaway connect the

two QFL inputs $\mathcal{O}/_{a1}(\underline{a1}')$ and $\mathcal{O}/_{a2}(\underline{a2}')$ to give the output $\underline{a}'$

in the form of a Boolean QFL relation in BA-3, namely

$\underline{a1}' \not{Z} \underline{a2}' = \underline{a}'$, where $\mathbb{Z}$ is $\mathbb{U}$ or $\mathbb{V}$ as the case may be. Hence

the procedure to be followed is to convert the standard form of

the inputs into the three-vector canonical form and then apply

the connective $\mathbb{U}$(or $\mathbb{V}$) between them. These are implementable

using (35a,b), involving the Boolean sum and product, respectively,

of the two 3-vectors:

$$\mathbb{U}: \quad \underline{a1}' \oplus \underline{a2}' = \underline{a}' \; ; \quad \mathbb{V}: \quad \underline{a1}' \otimes \underline{a2}' = \underline{a}' \qquad (35a,b)$$

of these, $\mathbb{V}$ is extensively used also in connection with binary

reverse operators and with the unary forward connective relation

$\mathbb{V}(\underline{a} \triangleq \underline{b})$ — see Section 3(c) and 3(b) below.

(ii) Unary relations for all matrix connectives

We shall first consider all the ten matrix connectives

occurring in a unary connective relation, although such a

relation invariably arises from a binary reverse relation, as

in Fig.6(d), having $\underline{c}$ = T, or F, as the case may be. This is

because we have exhaustively considered the unary connectives

of the $\underset{\sim}{I}$-type in Part III, and the considerations taken into

account therein, and in Eqs (24) and (25) above, are all valid,

and useful, for the discussion below in this section.

Firstly, we convert the four $\underset{\sim}{Q}$-type connectives into the

$\underset{\sim}{I}$-type by the well-known relation

$$\neg \underline{a} \vee \underline{b} \equiv \underline{a} \Rightarrow \underline{b}, \quad \text{or} \quad \underset{\sim}{Q} \underset{\sim}{N} = \underset{\sim}{I} \tag{36}$$

This is because the relation actually takes effect as an

implication, with all its properties as discussed in Part III.

However, the $\underset{\sim}{A}$-relation has peculiar properties in its unary

form. With the existential quantifier, $\exists(\underline{a} \,\&\, \underline{b})$ has the

significance of $\exists(\underline{a} \Rightarrow \underline{b})$ (as described in Part III), or

$$\exists(\underline{a} \underset{\sim}{A} = \underline{b}) \equiv \exists(\underline{a} \underset{\sim}{I} = \underline{b}) \tag{36a}$$

Since the matrix for $\underset{\sim}{A}$ is symmetric, $(\underline{a} \underset{\sim}{A} = \underline{b}) \equiv (\underline{b} \underset{\sim}{A} = \underline{a})$, which

with the equivalence in (36a) gives

$$\exists(\underline{a} \underset{\sim}{A} = \underline{b}) \equiv \exists(\underline{b} \underset{\sim}{A} = \underline{a}) \equiv \exists(\underline{b} \underset{\sim}{I} = \underline{a}) \tag{36b}$$

Thus, we obtain the strange result (used extensively in Part III)

that

$$\exists(\underline{a} \Rightarrow \underline{b}) \equiv \exists(\underline{b} \Rightarrow \underline{a}) \tag{37c}$$

although

$$\forall(\underline{a} \Rightarrow \underline{b}) \equiv \forall(\neg \underline{b} \Rightarrow \neg \underline{a}) \tag{37d}$$

In view of all these, we use Eqn (37a) to treat $\underset{\sim}{A}$-connectives

with the existential quantifier $\exists$ as implications, and no further

discussion of $\exists(\underline{a} \underset{\sim}{A} = \underline{b})$ is necessary. However, the elementary

QFL statement $\forall(\underline{a} \underset{\sim}{A} = \underline{b})$, which arises from (or is equivalent to)

the statements in (38) requires special discussion.

"We are given that $\forall(\underline{a} \,\&\, \underline{b})$ is true. Given ${}^{\mathcal{Q}}\!/\!_{a}(\underline{a}')$
what is ${}^{\mathcal{Q}}\!/\!_{b}(\underline{b}')$?" $\tag{38}$

It will be noticed that the connective $\underset{\sim}{A}$ in $\underline{\underline{a}} \underset{\sim}{A} \underline{\underline{b}}$ asserts

that $\underline{both}$ $\underline{\underline{a}}$ $\underline{and}$ $\underline{\underline{b}}$ are true. Hence, if $\underline{\underline{a}}'$ is F, there

can be a contradiction. Further, the state of $\underline{\underline{b}}$(namely $\underline{\underline{b}}'$ = T)

is independent of what state $\underline{\underline{a}}'$ has. Extending these

considerations to QPL, and including the $\mathcal{Q}\!\!\!/\,'$s in (38), we obtain

the flow chart of the logic for this particular case $\bigvee(\underline{\underline{a}} \underset{\sim}{A} = \underline{\underline{b}})$,

as shown in Fig. 6(b).

The explanation of this figure is as follows. As mentioned

above, the expression $\bigvee(\underline{\underline{a}} \underline{A} = \underline{\underline{b}})$ asserts that both $\bigvee(\underline{\underline{a}})$ and

$\bigvee(\underline{\underline{b}})$ are true; hence the output $\bigvee(\underline{\underline{b}}')$ with $\underline{\underline{b}}'$ = T. On the

other hand $\bigvee(\underline{\underline{a}})$ may or may not be consistent with $\mathcal{Q}\!\!\!/_a(\underline{\underline{a}}')$.

Hence, we designate these by $\mathcal{Q}\!\!\!/_a(\underline{\underline{a}}'_1)$ and $\bigvee(\underline{\underline{a}}'_2)$, and apply the

vidya operator $\bigvee_{\Lambda}$/for consistency between them, to obtain the
to check   *and*

output $\mathcal{Q}\!\!\!/'_a(a')$. (As in the case of SMS logical graphs, we

reserve the discussion of the $\underline{consequences}$ of the consistency

test using $\bigvee$ for a later study. We only show that the statement

$\forall (\underline{a} \; \underset{\sim}{A} = \underline{b})$ is implementable with whatever input that is provided.)

The discussion of the results for $\mathcal{Q}(\underline{a} \; \underset{\sim}{E} = \underline{b})$, with

$\mathcal{Q} = \forall$ or $\exists$, and input $\mathcal{Q}_a(\underline{a}')$ as in Fig. 6(a) is simple. The

net effective quantifier that goes in is obtained $\underline{via}$ the "into"

operator as $\mathcal{Q}'_a = \mathcal{Q}_a \oplus \mathcal{Q}$, exactly as for $\mathcal{Q}(\underline{a} \; \underset{\sim}{I} = \underline{b})$.

However, the output, with $\underset{\sim}{E}$, has the same quantifier as the

input (being equivalent to it). Hence, for $\underset{\sim}{Z} = \underset{\sim}{E}$ in Fig. 6(a),

$$\underline{a}' \; \underset{\sim}{E} = \underline{b}' \; , \qquad \mathcal{Q}_a \oplus \mathcal{Q} = \mathcal{Q}_b \qquad\qquad (39a, \; b)$$

However, for uniformity and convenience in programming, (39b) is

written in the same form as for $\underset{\sim}{Z} = \underset{\sim}{I}$, namely

$$(\mathcal{Q}_a \oplus \mathcal{Q}) \oplus \mathcal{Q}_E = \mathcal{Q}_b \; , \quad \mathcal{Q}_E = \forall \qquad\qquad (39c)$$

We consider in the next subsection the algebraic treatment

of the binary reverse relations which give rise to unary relations.

(iii) Binary reverse relations for matrix connectives

This kind of relation, for statements of Type-1 in QPL, is described by the algebraic expression $\overset{Q}{/}(\underset{\sim}{c}\ \overset{\leftarrow}{\underset{\sim}{Z}}\ \underset{=}{a} = \underset{=}{b})$. The SNS expression within the bracket has the significance of Eq (13) in Part I (Ref. [ ]), in that it asks for $\underset{=}{b}'$, given $\underset{=}{a}\ \underset{\sim}{Z}\ \underset{=}{b} = \underset{=}{c}$ and the inputs $\underset{=}{c}'$ and $\underset{=}{a}'$. Now that the quantifier $\overset{Q}{/}$ has been added, the inputs are $\overset{Q}{/}_c(\underset{=}{c}')$ and $\overset{Q}{/}_a(\underset{=}{a}')$, and we ask for the output $\overset{Q}{/}_b(\underset{=}{b}')$, as shown in Fig.6(d), for a binary reverse relation of Type-1.

In SNS, the information regarding the state of $\underset{=}{c}'$ leads to the equations

$$\underset{=}{a}'\ \underset{\sim}{Z} = \underset{=}{b}'\ , \quad \text{if}\ \underset{=}{c}' = T; \quad \underset{=}{a}'\ \underset{\sim}{Z}^c = \underset{=}{b}'\ , \quad \text{if}\ \underset{=}{c}' = F \qquad (40)$$

which are seen to be readily capable of being taken over into QFL for Type-1 sentences (as written in Fig. 6(d)), since $\mathcal{Q}$ is common for $\underset{=}{a}$, $\underset{=}{b}$, $\underset{=}{c}$ and $\underset{\sim}{Z}$. If, however, the quantifier $\overset{Q}{/}_c$ is different from $\overset{Q}{/}$, it is clear that $\overset{Q}{/}(\underset{=}{c}\ \overset{\leftarrow}{\underset{\sim}{Z}}\ \underset{=}{a} = \underset{=}{b})$ gets modified to

$$\overset{Q}{/}'(\underset{=}{c}'\ \overset{\leftarrow}{\underset{\sim}{Z}}\ \underset{=}{a} = \underset{=}{b})\ , \quad \text{with}\quad \overset{Q}{/}' = \overset{Q}{/} \otimes \overset{Q}{/}_c \qquad (41a,b)$$

This gives rise to two QPL Type-1 equations analogous to (40),

namely (42a) and (42b), with the inputs also mentioned in each case:

and
$$\mathcal{Q\!V}'(\underline{a}\ \underleftarrow{Z} = \underline{b})\ \text{if}\ \underline{c}' = T\ ,\quad \mathcal{Q\!V}_a(\underline{a}') \tag{42a}$$

$$\mathcal{Q\!V}'(\underline{a}\ \underleftarrow{Z}^c = \underline{b})\ \text{if}\ \underline{c}' = F\ ,\quad \mathcal{Q\!V}_a(\underline{a}') \tag{42b}$$

Equations (42a) and (42b) are identical in structure to the unary

QPL relation of Type-1 shown in Fig. 6(a). Hence, they lead to

the outputs $\mathcal{Q\!V}_b$ and $\underline{b}'$ as stated in Fig. 6(d). The formula for

$\underline{b}'$ is clear, and the formula for $\mathcal{Q\!V}_b$ arises from (41b) and the

formula for unary relations namely

$$(\mathcal{Q\!V}' \otimes \mathcal{Q\!V}_a) \oplus \mathcal{Q\!V}_Z = \mathcal{Q\!V}_c \tag{43}$$

We may restate these here for ready reference. Given the inputs

$$\mathcal{Q\!V}_c(\underline{c}')\ ,\quad \mathcal{Q\!V}_a(\underline{a}')\ \text{in}\ \mathcal{Q\!V}(\underline{c}\ \underleftarrow{Z}\ \underline{a} = \underline{b}) \tag{44a,b,c}$$

the output is

$$\mathcal{Q\!V}_b(\underline{b}'),\ \text{with}\ \underline{a}'\ \underleftarrow{Z} = \underline{b}'\ \text{for}\ \underline{c}' = T \tag{44d}$$

$$\text{and}\ \underline{a}'\ Z^c = \underline{b}'\ \text{for}\ \underline{c}' = F \tag{44e}$$

and

$$(\mathcal{Q\!V}_c \otimes \mathcal{Q\!V}' \otimes \mathcal{Q\!V}_a) \oplus \mathcal{Q\!V}_{Z\ \text{or}\ Z^c} = \mathcal{Q\!V}_b \tag{44f}$$

4. Structure and implementation of a general logical graph for Type-1 statements.

In this section, we shall consider the structure of a general graph representing the flow of logic in a QPL argument composed solely of Type-1 statements, and then show that such a graph is implementable with any set of QPL inputs, from the fact that each of the elementary statements of which it is composed is implementable. We shall therefore comment on how to work out combinations of the elementary statements and then prove the general theorem that, for any finite logical graph, employing QPL statements of Type-1, the EFL states of the outputs can all be worked out in an algorithmic way — i.e. by following a sequence of operations in a stepwise fasion — which can therefore be computerized. Before proceeding to such combinations, we shall indicate the characteristics of the structure of a general QFL graph, following the lines adopted earlier for an SMS logical graph. It is to be noted that, although all inputs are QPL states, the intermediate terms and the outputs

may have any one of the EPL states and this particularly arises

when they are outputs of the vidya operator.

(a) Preliminary steps for binary reverse connectives.

In a QPL graph, every unary operator has one input and one

output in the relation $\mathcal{Q}/(\underline{a} \ Z = \underline{b})$, which feature can also be

generated by a binary reverse relation $\mathcal{Q}/(\underline{c} \ \overleftarrow{Z} \ \underline{a} = \underline{b})$, with

$\underline{c} = T$. In the latter case, there are two inputs, $\underline{c}'$ and $\underline{a}'$,

and the nature (truth value) of $\underline{c}'$ generates the unary operator

$(Z \text{ or } Z^c)$ connecting $\underline{a}$ and $\underline{b}$ in the unary chain. In view of the

fact that the T and F states of $\underline{c}$, with $Z = \underline{A}$-type or $\underline{I}$-type,

can lead to either an $\underline{A}$-type, or an $\underline{I}$-type, relation, and since

the further processing of the section of the graph dealing with

this unary relation is quite different in the two cases, the first

step in the unravelling of the logic of a QPL graph will be to

work out the unary relations arising out of every binary reverse

relation. The steps involved in this are repeated in Eqs.(45a,b,c).

If $(\underline{c} \overleftarrow{\underset{\sim}{Z}} \underline{a}) = \underline{b}$, and if $\underline{c}' = \mathcal{O}\!\!\!/_c(T)$, then $\mathcal{O}\!\!\!/'(\underline{a} \underset{\sim}{Z} = \underline{b})$ (45a)

and   if $\underline{c}' = \mathcal{O}\!\!\!/_c(F)$, then $\mathcal{O}\!\!\!/'(\underline{a} \underset{\sim}{Z}^c = \underline{b})$ (45b)

where $\mathcal{O}\!\!\!/' = \mathcal{O}\!\!\!/ \otimes \mathcal{O}\!\!\!/_c$ (45c)

While doing this, if $\underline{c}'$ is an independent original input, the

rest of the manipulations mentioned in Eq (45) are carried out

as such. If, however, it is not so, and it *is* an output of same

step in the logical graph (argument) like $\mathcal{O}\!\!\!/(\underline{g} \underset{\sim}{Z}' = \underline{c})$ leading

to $\mathcal{O}\!\!\!/_c(\underline{c}')$, then a "$\underline{c}$-construction" must be made as follows.

The term $\mathcal{O}\!\!\!/_c(\underline{c}')$ is called $\mathcal{O}\!\!\!/_{c1}(\underline{c1}')$ and the input $\underline{c}'$ in (45)

is called $\mathcal{O}\!\!\!/_{c2}(\underline{c2}')$ and these two are connected by a vidya

operator $\bigvee$ as shown in Fig. 7 to yield an output $\mathcal{O}\!\!\!/_c(\underline{c})$.

_____

Fig.7   Schematic diagram of $\underline{c}$-reconstruction

_____

Thus effectively, the two equations

$\mathcal{O}\!\!\!/_g(\underline{g} \underset{\sim}{Z}' = \underline{c})$; $\mathcal{O}\!\!\!/(\underline{c} \overleftarrow{\underset{\sim}{Z}} \underline{a} = \underline{b})$ (46a,b)
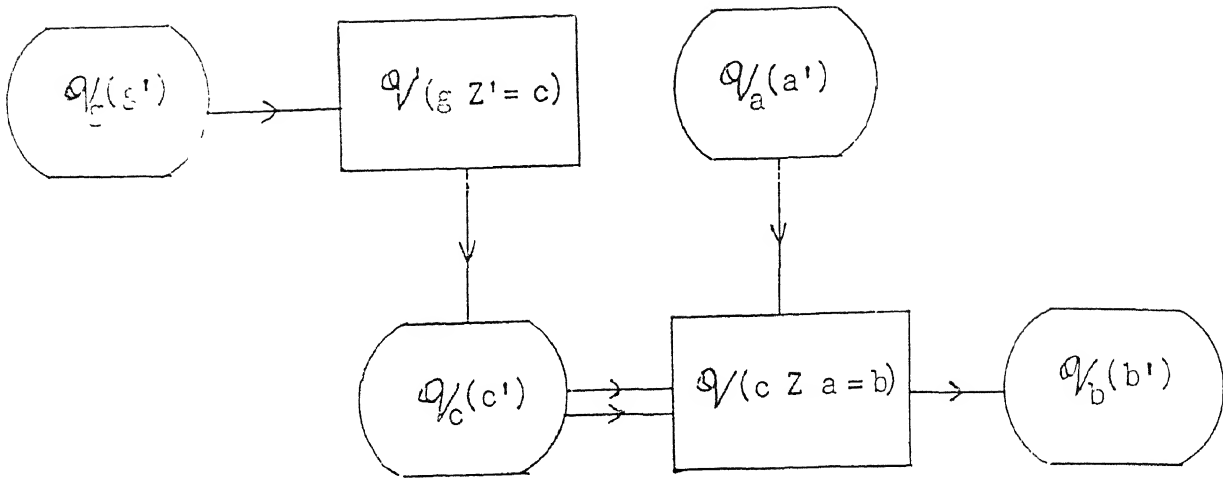
Fig. 7(a)   Portion of graph before $\underline{c}$-reconstruction
representing Eqs. (46a,b)
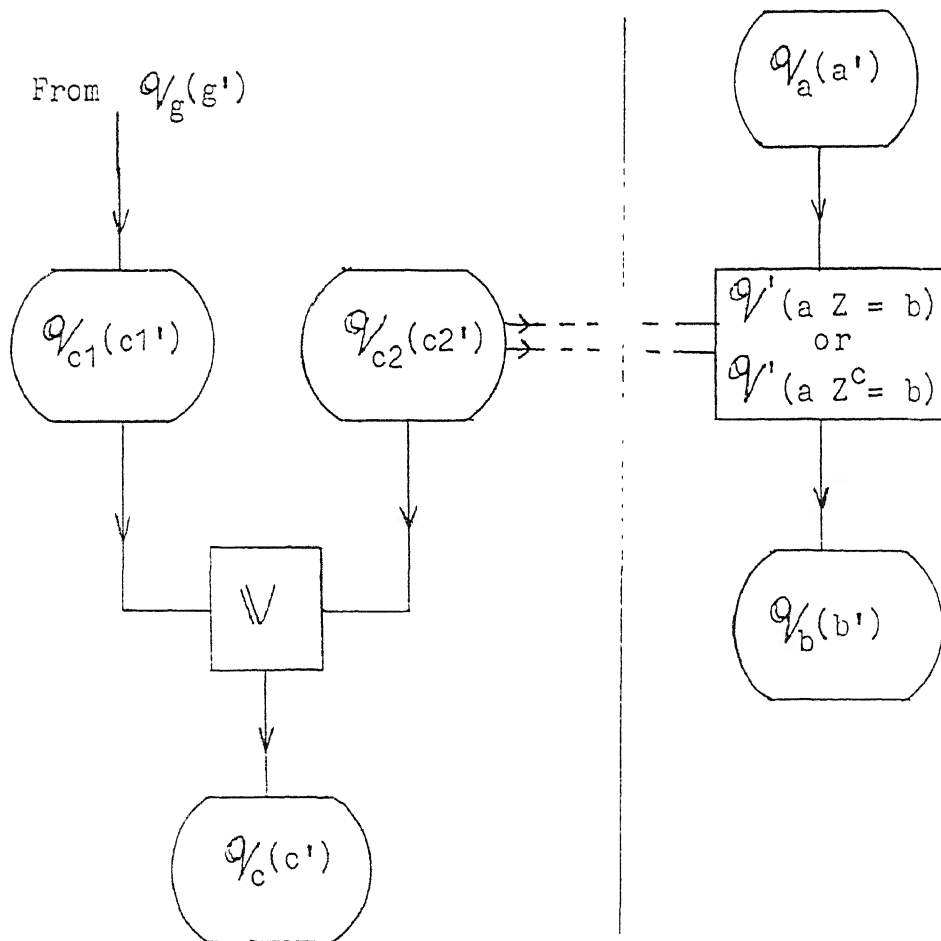
Fig.7(b)   Left: Part of the $\underline{c}$-reconstructed graph with the consistency
check operator $\mathbb{V}$. Right: Unary relation between $\underline{a}$ and $\underline{b}$
generated by $\mathcal{V}_{c2}(c2') \equiv \mathcal{V}_c(c')$.

are converted into the set of equations

$$\mathcal{Q}/_g(\underline{g}' \; \underset{\sim}{Z}' = \underline{c1}') \; ; \quad \mathcal{Q}/(\underline{c2}' \; \overset{\leftarrow}{\underset{\sim}{Z}} \; \underline{a}' = \underline{b}') \qquad (47a,b)$$

with the vidya relation

$$\underline{c1}' \; \bigvee \; \underline{c2}' = \underline{c}' \qquad (47c)$$

In this way all binary reverse relations are dispensed with

and each one attached to a consistency check operator $\bigvee$ and the

graph is reduced to one having only unary forward and binary

forward relations — the latter, however, containing both one

of the ten matrix operators and one of the two Boolean operators,

in particular, the vidya operator $\bigvee$.

(b) Preliminary steps related to A-type unary connectives

As mentioned above, the treatment of a unary forward relation

when it has the general form $\bigvee(\underline{a} \; \underset{\sim}{A} = \underline{b})$ requires the $\underset{\sim}{A}$-construction

as shown in Fig. 6(b). Here, the unary $\underset{\sim}{A}$-relation is taken back

into the binary reverse $\underset{\sim}{A}$-relation from which it arose and the

logic of that is interpreted — namely that "For all $\chi,$ both $\underline{a}$ and $\underline{b}$

are true". (As will be obvious, if the connective is any of the

other three possible ones of the $\underset{\sim}{A}$-type, a simple transformation

of either $\underline{a}$, or $\underline{b}$, or both, by a negation brings it to the

standard form $\forall(\underline{a} \; \underset{\sim}{A} = \underline{b}))$. Then, these two results $\underline{a}$ is T and

$\underline{b}$ is T are applied independently. The first one, namely that

"a is always true" is given by the expression $\forall(\underline{a}_2')$, with $\underline{a}_2' = T$,

in Fig. 6(b). This is compared with the actual state of the

QPL term $\mathcal{Q}_a(\underline{a}_1')$ by the consistency operator $\mathbb{V}$. On the other

hand, the second result that "$\underline{b}$ is true" is fed into the graph by

means of the term $\forall(\underline{b}')$ with $\underline{b}' = T$, as shown in Fig. 6(b).

In this way, the graph in Fig. 6(c), for the case $\mathcal{Q} = \forall$ and

$Z = \underset{\sim}{A}$, has been modified so that no contradictions are fed from

the $\underset{\sim}{A}$-type relation into the graph. If any such arises, as can

happen if $\underline{a}' = \underline{a}_1' = F$, it will be in one of the final outputs,

namely $\mathcal{Q}_a'(\underline{a}')$, as shown in Fig. 6(b).

(γ) Consistency check for multiple inputs to a term

As mentioned in connection with SMS logicl graphs, there

can only be one edge in the graph, leading as input from a QPL

term (say $\mathcal{Q}_g(\underline{g})$) to an operation (unary or binary) containing

$\underline{z}$ or $\mathbb{Z}$; but more than one outputs, from two or more different

operators, can lead to the same term. An example of three such

is as follows:

$$\mathcal{Q}_1(\underline{\underline{g}}\ \underline{z}_1 = \underline{p1}) \ , \quad \text{yielding} \quad \mathcal{Q}'_{p1}(\underline{p1}') \tag{48a}$$

$$\mathcal{Q}_2(\underline{h1}\ \underline{z}_2\ \underline{h2} = \underline{p2}) \ , \quad \text{yielding} \quad \mathcal{Q}'_{p2}(\underline{p2}') \tag{48b}$$

$$\mathcal{Q}_3(\underline{k}\ \underline{z}_3 = \underline{p3}) \ , \quad \text{yielding} \quad \mathcal{Q}'_{p3}(\underline{p3}') \tag{48c}$$

In (48a,b,c) $\underline{p1}$, $\underline{p2}$, $\underline{p3}$ all refer to the same QPL term $\underline{p}$, but

yield information coming from three different sources. Then,

it follows (as in SMS) that $\underline{p}$ is the 3-vector that is obtained

by applying the vidya operator $\bigvee$ between $\underline{p1}$ and $\underline{p2}$ to obtain,

say, $\underline{p2}'$ and then between $\underline{p2}'$ and $\underline{p3}$ to obtain $\underline{p}$ (see Eqns (49 a,b)).

$$\underline{p1}\ \bigvee\ \underline{p2} = \underline{p2}' \ , \quad \underline{p2}'\ \bigvee\ \underline{p3} = \underline{p} \tag{49a,b}$$

The generalization to n outputs $\underline{\underline{p1}}$, $\underline{\underline{p2}}$, ..., $\underline{\underline{pn}}$ all leading

to the same term $\underline{\underline{p}}$ is obvious. To reduce space, we shall write

the final output after applying (n-1) vidya operations to these

in the form

$$\underline{\underline{p1}} \ \text{\Large W} \ \underline{\underline{p2}} \ \text{\Large W} \ \underline{\underline{p3}} \ \ldots \ \text{\Large W} \ \underline{\underline{pn}} = \underline{\underline{p}} \tag{50}$$

We shall call this process as the "W-construction".

We shall examine the EPL state of $\underline{\underline{p}}$ in general, first when

the inputs $\underline{\underline{p1}}$ and $\underline{\underline{p2}}$ belong to QPL or $\triangle$ (which occur in Part III

and in the earlier sections of this paper so far), in $\underline{\underline{p1}} \ \text{\Large W} \ \underline{\underline{p2}} = \underline{\underline{p}}$.

It must be remembered that, although we have only for the quantifiers

$\mathcal{Q} = \forall$, $\exists$ and $\triangle$ in $\mathcal{Q}(\underline{\underline{a}})$, the occurrence of $\underline{\underline{a}} = T$, or F, leads

to the following possibilities for $\underline{\underline{a}}$, namely $\forall \equiv (1 \ 0 \ 0)$,

$\exists \equiv (1 \ 1 \ 0)$, $\wedge = (0 \ 1 \ 1)$, $\Phi = (0 \ 0 \ 1)$, $\triangle = (1 \ 1 \ 1)$.

A table of $\underline{\underline{a1}} \ \text{\Large W} \ \underline{\underline{a2}} = \underline{\underline{a}}$ with these EPL states as inputs is shown

in Table 4.

Table 4. List of the vidya product $\underline{\underline{a1}} \ \otimes \ \underline{\underline{a2}}$

Table 4. List of the vidya product $\underline{\underline{a1}} \otimes \underline{\underline{a2}}$

$\underline{\underline{a1}}$ \ $\underline{\underline{a2}}$	∀ (1 0 0)	∃ (1 1 0)	∧ (0 1 1)	Φ (0 0 1)	△ (1 1 1)
∀	∀	∀	∅	∅	∀
∃	∀	∃	Σ	∅	∃
∧	∅	Σ	∧	Φ	∧
Φ	∅	∅	Φ	Φ	Φ
△	∀	∃	∧	Φ	△

$$\Sigma \equiv (0\ 1\ 0), \qquad \emptyset \equiv (0\ 0\ 0)$$

It will be noticed that two new states (both belonging to

EPL) occur for the vidya products $\underline{a1} \ \bigvee \ \underline{a2}$, of QPL terms

$\underline{a1}$ and $\underline{a2}$, namely $\emptyset \equiv (0 \ \ 0 \ \ 0)$, standing for "contradictory"

and $\xi \equiv (0 \ \ 1 \ \ 0)$, for "some".

Of these, the former comes as a result of the consistency

check made by $\bigvee$, and if $\quad \underline{a} \equiv \underline{a1} \ \bigvee \ \underline{a2} = (0 \ \ 0 \ \ 0)$, then it

means that $\underline{a1}$ and $\underline{a2}$ are mutually inconsistent, as may be readily

verified from the two pairs (for all, for none), *(for none, there exists)*, and (for all,

not for all), for which $\underline{a} = \emptyset$. When this actually occurs in a

graph, then any term to which it leads, <u>via</u> a path of unary or

binary forward connectives, can be shown to have also the state $\emptyset$

This is readily verified for all $\underset{\sim}{Z}$ and $\not{Z}$ in the elementary

connective operations *(except $\bigvee$)*.

The occurrence of ξ is even more restricted — it happens

just for one combination, namely "There exists " vidya "not for all";

but it must be remembered that this state is not necessarily

In fact,

producable only by such a combination of QFL states. ~~Instead,~~

as discussed in Part II, $\xi \equiv (0\ 1\ 0)$ is one of the three __basic__

states of the EA-3 representation of the four QFL states

$\forall$, $\exists$, $\wedge$ and φ . That the state ξ can occur in very simple

arguments is illustrated in the next subsection.

(d) Occurrence of the state ξ in a simple two-step argument
     containing the vidya operator.

We shall illustrate this by an actual pair of statements

in mathematics. Suppose $\mathcal{V}$ is any positive integer and π a prime

number. Then the following QFL statements are obviously true

for the sentence $\underline{s}(x) \equiv \left(\mathcal{V}(x) \implies \pi(x) \right)$, where x ranges over

all positive integers.

$$(\exists x)\ (\mathcal{V}(x) \implies \pi(x)) \equiv \exists(s_T) \equiv (1\ 1\ 0) \qquad (51a)$$
$$(\exists x)\ (\mathcal{V}(x) \implies \neg\pi(x)) \equiv \exists(s_F) \equiv \wedge(s_T) \equiv (0\ 1\ 1) \quad (51b)$$

Both (51a) and (51b) are QPL statements about the same sentence

s̲ , and hence the common knowledge contained in both of them

is to be extracted by applying the vidya operator $\overline{\bigvee}$ between them.

This gives

$$\exists(s_T) \; \overline{\bigvee} \; \exists(s_F) \equiv (1 \quad 1 \quad 0) \otimes (0 \quad 1 \quad 1) = (0 \quad 1 \quad 0)$$

$$\equiv \Sigma(s_T) \equiv \Sigma(s_F) \tag{52}$$

The conclusion ~~statement~~ $\Sigma(s̲)$ thus obtained, stated in words, is that the

statement "If x is a positive integer, then it is a prime" is

true for some integers, but not for all integers (and obviously

not for no integers).

The form of the above statement illustrating the symbolized

logical term $\Sigma(s̲)$ is exactly the same as that asserted in the

famous Gödel incompleteness theorem — namely that there are

statements in predicate logic which are true, but which cannot be

proved to be true for all x, or)

proved to be false for all x . As is well-known, Gödel has given

a carefully constructed theorem which has this property. However,

as will be seen from above, the Godel's state is identical with

the state $\zeta = (0 \ 1 \ 0)$ in our extended predicate logic. As

mentioned above, the BA-3 representation of quantified predicate

logic uses as basic states the following three, namely

$(1 \ 0 \ 0) \cong$ For all, $(0 \ 1 \ 0) \cong$ For some, $(0 \ 0 \ 1) \cong$ for none.

The middle one "for some" excludes both "for all" and "for none";

and in this way <u>three</u> mutually exclusive states, or basic vectors,

of bA-3 are constructed. Since EPL is based on BA-3, there should

occur, under suitable circumstances, any number of examples of QPL

terms having the logical state $\zeta(0 \ 1 \ 0)$, and the occurrence

of this does not indicate any "incompleteness" in the structure

of predicate logic. In fact, just as we found that the doubtful

state $D(1 \ 1)$ is a necessary consequence of the complete formulation

of propositional calculus with $T(1 \ 0)$ and $F(0 \ 1)$ alone, so

also both types of indeterminate states, namely $\triangle(1 \ 1 \ 1)$ and

$\mathcal{Z}$(0 1 0) can occur, in addition to the well defined four

states, when QPL is extended. When arguments are set up of

various kinds, even if only the four QPL states are input, the

intermediate terms and the output terms can have any one of the

eight states of BA-3 of the extended formulation (EFL).

Thus, the unary and binary forward operators involving the

matrix connectives $(\underset{\sim}{Z})$ can lead only to five of the states, with

$\triangle$ in addition to the four standard QPL states. We have now

seen that two others — namely $\emptyset$ and $\mathcal{Z}$ — arise from the

application of the vidya operator. There only remains one out of

the eight, namely $\ominus$ (1 0 1), standing, for "For all or for none"

but which can arise as $\forall \oplus \bar{\Phi}$, by the application of the operator "upu" (\U))

which has not been introduced so far, {We shall ~~consider this~~

~~below, along with a~~ discuss~~ion of~~ how the four non-QPL states,

$\mathcal{Z}, \ominus, \triangle, \emptyset$, can be incorporated in the general treatment of

statements of Type-1. (For statements of Type-2 as discussed

in Part II, all the eight states of BA-3 are completely taken

into account in the matrix-algebraic treatment given in that

Part. As will be discussed in the next section, statements of

both Type-1 and Type-2 can occur together in an argument, and the

algorithm for treating such arguments will be discussed in the

next paper (Part V) of this series.)

(e) Incorporation of EPL states in Type-1 arguments

We have seen in the previous subsections of this Section 4 that the use of the four QPL canonical states $\forall$, $\exists$, $\bar{\Phi}$ and Λ in arguments containing Type-1 statements could lead to one of the remaining states of EPL, namely Δ , $\emptyset$, Σ . We shall therefore discuss the procedures for incorporating all the four additional states of EPL (including Θ) in such a Type-1 argument.

(i) Implementation of Δ as input

The state $\Delta \equiv (1 \quad 1 \quad 1) \equiv \exists \oplus \bar{\Phi}$ is the completely indefinite state in QPL, which arises in the implementation of categorical statements (QPL implications), as shown in Part III. It is readily verified that, on putting this Δ as one of the inputs for any binary forward matrix connective operation, the output is also Δ , for all ten Z's; so also for their unary relations. Also, it can be shown that, for $c = \Delta$, corresponding to $\forall(\supset)$ in a binary reverse relation $\frac{q}{\forall}(c \overset{\leftarrow}{Z} a = b)$, the interpretation of D as T $\oplus$ F leads to an output Δ , as in Eqns (53a,b):

$$\underline{c}' = D, \quad \underline{c} \overset{\leftarrow}{\underset{\sim}{Z}} \underline{a} = \underline{b} \longmapsto \underline{a}' \underset{\sim}{Z} \oplus \underline{a}' Z^c = \underline{b}' = \underline{a}' D = (1 \quad 1) = D \quad (53a)$$

and
$$\mathcal{Q}(\underline{b}') = \triangle \qquad (53b)$$

Thus, $\triangle$ as an input in any step of a Type-1 QPL argument leads

to $\triangle$ as output for all matrix connectives. For the Boolean

connective $\mathbb{U}$ also $\triangle \oplus \underline{a} = \triangle$, so that $\triangle$ goes through as

to the output in the binary "upon" relation. Only for $\mathbb{V}$ is

$\triangle \otimes \underline{a} = \underline{a}$, for all states of $\underline{a}$, because $(1 \quad 1 \quad 1) \otimes$

$(a_\gamma \quad a_\delta \quad a_\epsilon) = (a_\gamma \quad a_\delta \quad a_\epsilon)$.

Thus, the introduction of the non-QPL state $\triangle$ into Type-1

statements introduces no difficulty.

(ii) Implementation of Σ and $\ominus$ as inputs

In the last subsection (i), we interpreted $\mathcal{Q}(D)$, where

$\mathcal{Q}$ is $\mathbb{V}$ or $\exists$, as equivalent to the EA-3 state $(1 \quad 1 \quad 1)$, corresponding

to $\triangle$ in PL. In the case of Σ and $\ominus$, $\Sigma(a_T) \equiv \Sigma(a_F)$ and

$\ominus(a_T) \equiv \ominus(a_F)$. This is because if only some (but not all)

have the property $\underline{a}$, some others will have the property $\neg \underline{a}$

so that we may reasonably associate $\lesssim$, when input into a Type-1 sentence, as giving a_D and hence having the properties of Δ .

(On the other hand, $\lesssim$ as input for a Type-2 sentence can lead to quite definite and useful conclusions, as will be shown in Part V.) In the same way, the sentence, "For all, or for none, $\underline{a}$ is T", is the same as "For none, or for all, $\underline{a}$ = T", which is equivalent to "For all, or for none, $\underline{a}$ is F", so that $\ominus(\underline{a})$, when input into a Type-1 sentence, also has the properties of D for the SMS term $\underline{a}$, and hence, we may take it as having the properties of Δ when input into Type-1 sentences.

Hence, we assume that all types of uncertainties $\mathcal{Q}(a_D)$, in which $\mathcal{Q}$ is any of the 7 quantifiers, other than $\emptyset$, of EFL, go in as $\mathcal{Q} \equiv \Delta$ (indefinite) into the next step of the argument, if it is a Type-1 sentence, except for the operator $\bigvee$. As already mentioned, $\underline{a1} \bigvee \underline{a2} = \underline{a}$ is a sentence which can occur in both Type-1 and Type-2 arguments.

(iii) Implementation of $\emptyset$ as input

Just as with $\triangle$ which has three 1's, $\emptyset$ has three 0's.

Hence, it also goes through as an impossible state $\emptyset$ for the

output of all elementary connective operations for which it is

an input, except in $\underline{a1} \; \text{\textbackslash U} \; \underline{a2} = \underline{a}$. Hence, in a program $\emptyset$ as

input can be straightaway incorporated as an output state $\emptyset$.

(We do not explain the exception for $\text{\textbackslash U}$, since we have not used

this logical connective "upon" anywhere. It can be shown that

it (i.e. the binary $\text{\textbackslash U}$ relation) is relevant to the superposition

of information from "rumours"; but this discussion is deferred

to Part V.) Further, as stated before, $\emptyset$ can occur only as the

output of the binary $\text{\textbackslash V}$-relation in our method of approach of

treating Type-1 arguments.

We shall now proceed to the treatment of a chain of unary

operations.

(f) Preliminary steps for unary chains

We consider a chain of unary relations connecting, say,

$\underline{a}$ with $\underline{b}$ (or $\neg\underline{b}$), $\underline{b}$(or $\neg\underline{b}$) with $\underline{c}$(or $\neg\underline{c}$),..., $\underline{g}$(or $\neg\underline{g}$)

with matrix connectives $\underset{\sim}{Z}_1$, $\underset{\sim}{Z}_2$,..., $\underset{\sim}{Z}_n$, and quantifiers

$\mathcal{Q}_1$, $\mathcal{Q}_2$,..., $\mathcal{Q}_n$. We have already discussed the procedure

by which $\underset{\sim}{A}$-type unary connectives can be made not form part of a

unary chain, so that we are left with only $\underset{\sim}{I}$-type and $\underset{\sim}{E}$-type

connectives. Of these, $\underset{\sim}{I}$-type relations have been exhaustively

considered in Part III, and it has been shown that the above

chain of n implications can be reduced to be equivalent to a single

Type-1 implicational statement connecting $\underline{a}$ with $\underline{g}$. In saying

this, we consider the "invalid", or $\triangle$-state, conclusion also to be

a single implication, for this purpose, and write it as $\mathcal{Q}(\underline{a} \underset{\sim}{D} = \underline{b})$.

For working out n implications like the above in sequence,

we cannot input $\mathcal{Q}'(\underline{a}')$ and work out $\mathcal{Q}'(\underline{b}')$ etc sequentially

in an algorithmic manner, one after the other in succession.

Instead, the n Type-1 implications have to be "<u>theoretically</u>"

combined to get $\mathcal{Q}/$ and Z in $\mathcal{Q}/(\underline{a}\ Z = \underline{g})$, from $\mathcal{Q}/_1$, Z_1;

$\mathcal{Q}/_2 Z_2$;..., $\mathcal{Q}/_n$, Z_n; and <u>thereafter</u> $\mathcal{Q}/_a$ and $\underline{a}'$ can be input to

$\mathcal{Q}/(\underline{a}\ Z = \underline{c})$ for obtaining the output $\mathcal{Q}/_g$ and $\underline{g}'$ of the unary

chain. Because of this, we shall give below, in outline, the

formulae for combining two unary Type-1 implications to obtain

the resultant of the same type, and then show how the formulae

can be iterated.

(i) Combination of two ~~or more~~ Type-1 implications

We first consider two QPL implications of Type-1 in

succession namely $\mathcal{Q}/_1(\underline{a} \implies \underline{b})$ or $\mathcal{Q}/_1(\underline{a} \implies \neg\ \underline{b})$ and $\mathcal{Q}/_2(\underline{b} \implies \underline{c})$

in succession, where $\mathcal{Q}/ = \forall$ or $\exists$. The resultant

$\mathcal{Q}/_3(\underline{a}(\text{or}\ \neg\ \underline{a}) \implies \underline{b}(\text{or}\ \neg\ \underline{b}))$ for all these cases is summarized

in Table 5. By putting $\underline{a} = \neg\ \underline{a}$ and $\underline{c} = \neg\ \underline{c}$, all possible

combinations which occur in practice can be obtained from the

above eight possibilities.

Table 5. Resultant of two Type-1 implications in succession.

Table 5. Resultant of two Type-1 implications in succession

Sl. No.	First implication	Second implication	Resultant	Name of categoric: syllogism
1	$\forall(a\ I = b)$	$\forall(b\ I = c)$	$\forall(a\ I = c)$	<u>Barbara</u>
2	$\forall(a\ I = \neg b)$	$\forall(b\ I = c)$	$\forall(a\ D = c) \equiv \Delta$	Invalid
3	$\forall(a\ I = b)$	$\exists(b\ I = c)$	$\Delta(a\ I = c) \equiv \Delta$	Invalid
4	$\forall(a\ I = \neg b)$	$\exists(b\ I = c)$	$\exists(\neg a\ I = c)$	<u>Dimensior</u>
5	$\exists(a\ I = b)$	$\forall(b\ I = c)$	$\exists(a\ I = c)$	<u>Darii</u>
6	$\exists(a\ I = \neg b)$	$\forall(b\ I = c)$	$\exists(a\ D = c) \equiv \Delta$	Invalid
7	$\exists(a\ I = b)$	$\exists(b\ I = c)$	$\Delta(a\ I = c) \equiv \Delta$	Invalid
8	$\exists(a\ I = \neg b)$	$\exists(b\ I = c)$	$\Delta(a\ D = c) \equiv \Delta$	Invalid

[+]In Table 4 of Part III. Note the occurrence of the "New Type" of syllogism, <u>Dimension</u> ($IEI' - 1$)

[*]For all "invalid" resultants, the output c is Δ, irrespective of the QPL input state α.

It will be noticed that only three combinations in Table 5

lead to valid conclusions for the resultant. Of these, <u>Barbara</u>

and <u>Darii</u> are classical, but we have, in addition, one of the

new types of syllogisms derived in Part III, namely <u>Dimension</u>

(IEI'-1). This can only be worked out by reversing the sequence

of the first two implications and then reversing, once again,

the valid resultant thus obtained. However, in the form in

which it is stated in Table 5, it is algorithmically applicable

for working out the valid resultants of sequences of n implications,

and this leads to the same results as those shown in Section 5

of Part III. Thus, the forward sequence $\forall(\neg a\, I \Rightarrow \neg b)$,

$\exists(b\, I \Rightarrow c)$, $\forall(c \Rightarrow d)$ can be worked out as follows to

yield $\exists(a \Rightarrow d)$:

$$\forall(\neg a\, I \Rightarrow \neg b),\quad \exists(b\, I \Rightarrow c) = \exists(a \Rightarrow c)$$

<div align="center">(Analogue of <u>Dimension</u>) (54a)</div>

$$\exists(a \Rightarrow c),\quad \forall(c \Rightarrow d) = \exists(a \Rightarrow d)\quad \text{(Analogue of <u>Darii</u>) (54b)}$$

(ii) Inclusion of E-type unary connectives

Since A-type Type-1 statements, with both $\forall$ and $\exists$ as quantifier, do not occur in unary chains, we need consider only the E-type connective relations that may occur in between implications in a unary chain. Considering first combinations of two E-type relations, the following equations are obvious:

$$\mathcal{Q}_1(a\ Z_1 = b), \quad \mathcal{Q}_2(b\ Z_2 = c) \ = \ \mathcal{Q}_3(a\ Z_3 = c) \tag{55a}$$

where

$$\mathcal{Q}_1 \otimes \mathcal{Q}_2 = \mathcal{Q}_3 \ ; \quad Z_1\ Z_2 = Z_3 \tag{55b}$$

with

$$\mathcal{Q}_1,\ \mathcal{Q}_2 \ = \ \forall, \exists \quad \text{and} \quad Z_1,\ Z_2 = E \text{ or } E^c (\equiv N) \tag{55c}$$

Combinations of an I-type and an E-type relations also follow simple rules, as in (56) and (57) below.

$$\mathcal{Q}_1(a\ I = b), \quad \mathcal{Q}_2(b\ Z_2 = c) \ = \ \mathcal{Q}_3(a\ I = x) \tag{56a}$$

where

$$(\mathcal{Q}_1 \oplus \exists) \otimes \mathcal{Q}_2 = \mathcal{Q}_3 \tag{56b}$$

and

$$x = c \text{ if } Z_2 = E, \quad \text{and} \quad x = \neg c \text{ if } Z_2 = N(E^c) \tag{56c}$$

$$\mathcal{Q}_1(\underline{a} \ \underline{Z}_1 = \underline{c}), \quad \mathcal{Q}_2(\underline{b} \ \underline{I} = \underline{c}) = \mathcal{Q}_3(\underline{y} \ \underline{I} = \underline{c}) \qquad (57a)$$

where

$$(\mathcal{Q}_1 \otimes \mathcal{Q}_2) \oplus \rceil = \mathcal{Q}_3 \qquad (57b)$$

and

$$\underline{y} = \underline{a}, \ \text{if} \ \underline{Z}_1 = E, \ \text{and} \ \underline{y} = \rceil \underline{a}, \ \text{if} \ \underline{Z}_1 = \underline{N}(E^c) \qquad (57c)$$

To work out a chain of $\underline{I}$-type or $\underline{E}$-type relations, we consider their associativity.

(iii) Associativity of $\hat{n}$ unary connective relations in a unary path.

This can be proved quite generally. For this, we first take three unary relations $\mathcal{Q}_1(\underline{a} \ \underline{Z}_1 = \underline{b})$, $\mathcal{Q}_2(\underline{b} \ \underline{Z}_2 = \underline{c})$ and $\mathcal{Q}_3(\underline{c} \ \underline{Z}_3 = \underline{d})$ and denote the equivalent single unary relation by $\mathcal{Q}(\underline{a} \ \underline{Z} = \underline{d})$. For convenience, we write it as a product of three operators

$$(\mathcal{Q}_1, \underline{Z}_1) = Z_1(\text{say}), \ (\mathcal{Q}_2, \underline{Z}_2) = Z_2(\text{say}) \ \text{and} \ (\mathcal{Q}_3, \underline{Z}_3) = Z_3(\text{say})$$

in Eq (58a and b), with brackets in the two different ways of association:

$$((\mathcal{Q}_1, \underline{Z}_1) \ (\mathcal{Q}_2, \underline{Z}_2)) \ (\mathcal{Q}_3, \underline{Z}_3) \equiv (Z_1 \ Z_2) \ Z_3 = Z \ (\text{say}) \qquad (58a)$$

$$(\mathcal{Q}_1, \underline{Z}_1) \ ((\mathcal{Q}_2, \underline{Z}_2) \ (\mathcal{Q}_3, \underline{Z}_3)) \equiv Z_1(Z_2 \ Z_3) = Z' \ (\text{say}) \qquad (58b)$$

By taking the data in Table 7, and in Eqns (55), (56) and

(57), it is readily verified that for all $\mathcal{Q}'$'s being $\forall$ and $\exists$,

and all $\underset{\sim}{Z}$'s being $\underset{\sim}{I}$, $\underset{\sim}{E}$ or $\underset{\sim}{N}$, if

$$ Z \equiv (\mathcal{Q}, \underset{\sim}{Z}) \text{ and } Z' \equiv (\mathcal{Q}' \underset{\sim}{Z}') \; ; \quad Z \equiv Z' \text{ in (58a,b)} \quad (59) $$

In other words, unary connective relations in succession are

associative. (The detailed proof ~~of associativity~~ is long, but

quite simple, and it is omitted).

(iv) Implementation of a unary bouquet

We have already seen that there cannot be more than one

incoming edge to a term from an operator (for shortness, we use

the symbol t for a term $\mathcal{Q}(\underline{t})$ and Z for an operator $(\mathcal{Q}, \underset{\sim}{Z})$

producing a unary or binary relation). However, more than one

edge can connect a term t_1 to different operators $(Z_1, Z_2, Z_3,$ say).

Hence, the unary paths emerging from a term t_1 can have a "bouquet"

structure, as shown in Fig. 8(a).

Fig.8. Reduction of a general bouquet of unary paths to a bouquet
         of reduced single unary relations.
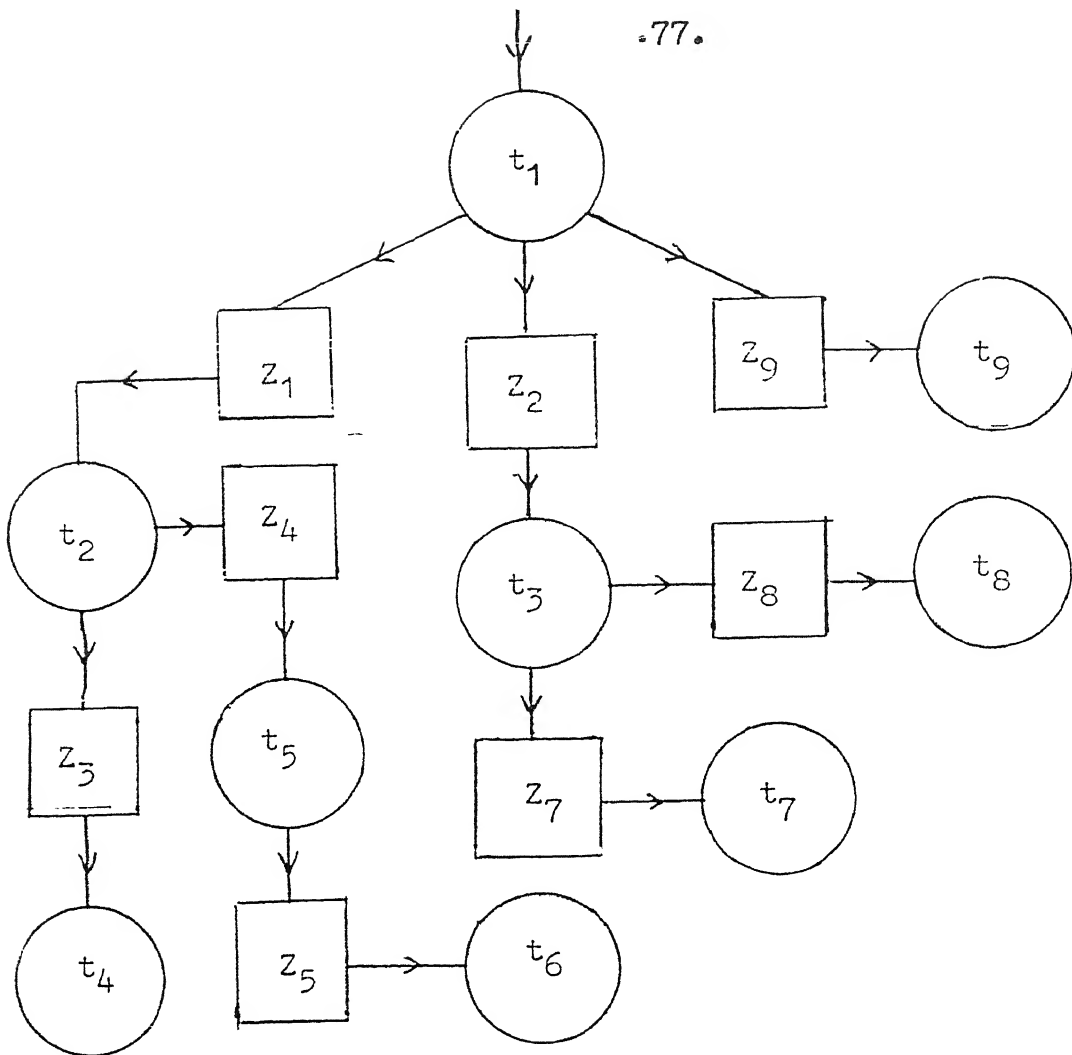
Fig. 8(a) Bouquet of edges starting from t_1 along unary paths and ending in t_4, t_6, t_7, t_8 and t_9.
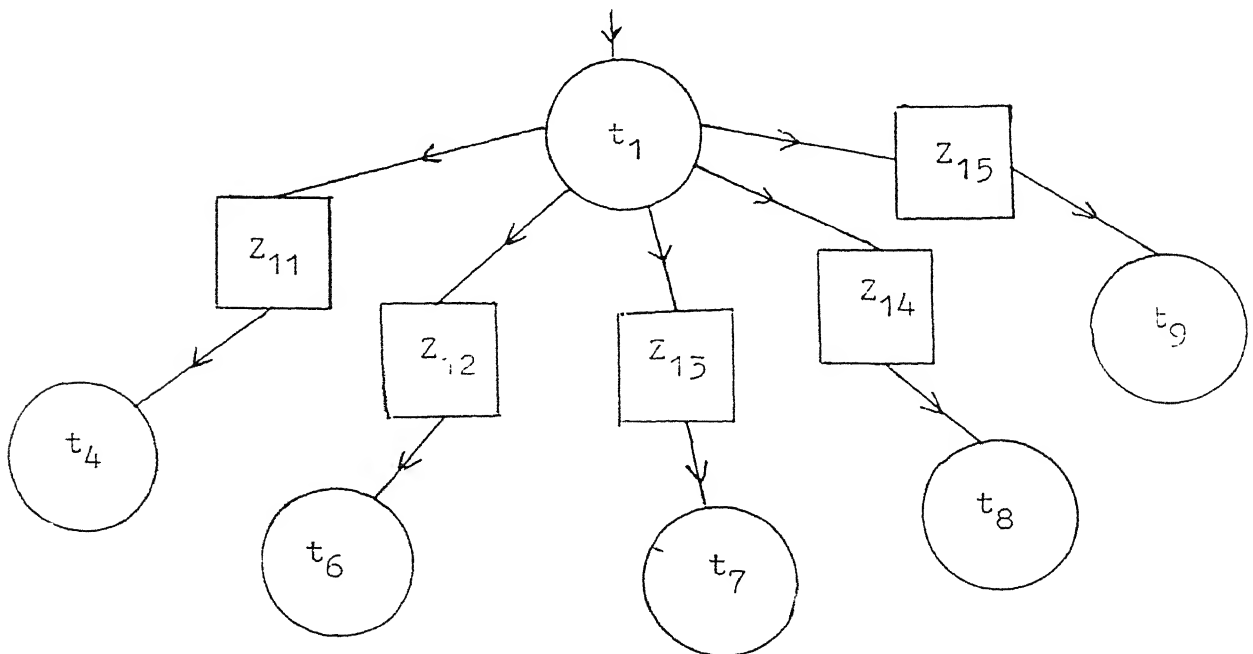
Fig. 8(b) Equivalent single connective unary paths, all starting from t_1 and ending in t_4, t_6, t_7, t_8 and t_9.

The word "bouquet" is suggestive of t_1 forming the stalk, and

the different directed paths from it forming branching stems,

leading to "flowers" (t_4, t_6, t_7, t_8,t_9) as termini of the

various unary paths from t_1. It is $\overset{a}{\wedge}$simple theorem in the theory

of graphs that form trees (of which bouquets are a special class)

that two vertices are connected by a unique path in a tree. Hence,

the term forming the stalk t_1 is connected by a unique unary

path to each flower, forming its terminus.

We now show, using the theory developed earlier in this

section 4(f), that a general bouquet of unary paths of Type-1 relations

emerging from t_1, as in Fig. (8a), is reducible to a set of single

Type-1 unary relations from t_1 straight to the terminus of each

of the unary paths. This follows from the fact that we can

collapse n unary QFL relations (of Type-1, containing only I-type

and E-type connectives) into a single relation of the same type.

The proof follows from two successive relations being reducible

to a single one, which can be applied iteratively to any number.

The associativity of this process, stated above, leads to the

uniqueness of the single QPL Type-1 relation.

Thus, using the contracted notation mentioned in Section 4(f)

(iii), if we follow the unary path from t_1 to t_2, t_5 and t_6, then

$$t_1 \; Z_1 = t_2 \; , \quad t_2 \; Z_4 = t_5, \quad t_3 \; Z_5 = t_6 \tag{60a}$$

so that

$$t_1 \; Z_{12} = t_6 \; , \quad \text{where} \quad Z_{12} = Z_1 \; Z_4 \; Z_5 \tag{60b}$$

The relation (60b) is shown in Fig.8(b). Similarly, each one of

the paths from t_1 to t_2, t_7, t_8, t_9 can be reduced to single

unary relations by following the paths from its origin to its

terminus. Thus, Fig. 8(a) is reduced to Fig. 8(b), using the

unary operators Z_{11}, Z_{12}, Z_{13}, Z_{14}, Z_{15}, one at a time for

obtaining the output term at each terminus, with $Z_{11} = Z_1 Z_3$,
$Z_{12} = Z_1 Z_4 Z_5$, $Z_{13} = Z_2 Z_7$, $Z_{14} = Z_2 Z_8$, $Z_{15} = Z_9$.

Thereafter, the input (say $\mathcal{V}_1'(t_1')$) can be fed to each relation

and the outputs $\mathcal{Q}/_4'(\underline{t}_4')$, etc can be worked out. These latter

will be either final outputs, or will be fed to binary relations,

which are algorithmically implementable from input(s) to output.

Thus, by the "bouquet reduction" procedure, the general graph

of an argument is converted to a form in which it consists only

of single unary relations, connecting input and/or output

$\overline{\text{terms}}$ of binary relations. This reduced graph has only relations

of the types in (61a), written in the abbreviated version

$$t_1 \ Z = t_2 \quad \text{or} \quad t_1 \ Z \ t_2 = t_3 \tag{61a}$$

Eqns (61) stand for the Type-1 relations

$$\mathcal{Q}/(\underline{t}_1 \ \underline{Z} = t_3) \quad \text{and} \quad \mathcal{Q}/(\underline{t}_1 \ \underline{Z} \ \underline{t}_2 = \underline{t}_3) \tag{61b}$$

In the next section, we shall summarize the procedures to be

adopted for obtaining such a reduced graph from a general set

of Type-1 relations forming an argument and then prove that the

reduced graph is always implementable for Type-1 statements,

using an argument analogous to that adopted for SMS statements.

5. Summary of the implementation procedure for a Type-1 argument

We assume that the logical graph of the argument is

available in terms of the set of unary, binary forward and

binary reverse relational statements between the various terms

t_j _via_ operators Z_j. We shall first describe the various steps

by which the argument can be worked out and the conclusions

corresponding to the output terms can be determined when the

input terms are given. The steps are also given briefly in

the form of a flow chart in Fig. 9 A to I. The descriptions,

in paras A to I below, follow the same order.

Fig.9. Outline of Flow Chart.

A. First we search for the input and output terms and

classify all terms as original inputs $(t_j^{(i)})$, final outputs $(t_j^{(o)})$

and intermediate (middle) input-outputs $(t_j^{(m)})$.

Fig.9. Outline of Flow Chart

$(t^{(i)}$ = input term, $t^{(m)}$ = middle term, $t^{(o)}$ = output term; $t^{(ai)}$, $t^{(am)}$, $t^{(ao)}$ = "additional" input, middle, output terms; Z = Connective operator)

A. Write down the elementary steps in the argument in any order. Check and list the terms as $t_j^{(i)}$, $t_j^{(m)}$ and $t_j^{(o)}$. Verify that the graph has no directed circuits. If it has, then go to B.

B. Check for occurrence of directed circuits. For each one present, make the "circuit removal construction" as in Fig.4(b) for each circuit. Add the vertices corresponding to new terms $t^{(i)}$, $t^{(m)}$, $t^{(o)}$ and the new operators Z_j (of type $\bigvee$) to the list in A.

C. Make the $\bigvee$-construction for multiple inputs to a term t_j, as in Eqs.(49) and (50), and do this for all such terms. Add the additional Z_j (of type $\bigvee$) and the $t_j^{(m)}$ and $t_j^{(o)}$ so generated to the list in A.

Fig.9. Outline of Flow Chart (Contd.)

D. List the binary reverse relations. If one of them,

$\underline{c}$ is not a $t^{(i)}$, carry out a $\underline{c}$-reconstruction as

in LHS of Fig.7(b). Add or replace the operators

Z_j so produced and add the additional terms $t^{(ai)}$,

$t^{(ao)}$ and also replace the relevant $t^{(m)}$ in the list in A.

E. Input the QPL states of $t^{(i)}$ as provided. For $t^{(ai)}$

choose one set of possible inputs. Call this as

Set 1, and for the later trials Set 2, Set 3, and so on.

F. Make the modified $\underline{A}$-construction, as in Fig.6(b), for

all unary relations of the A-type. Add the $t^{(am)}$ and

$t^{(ao)}$ so produced to the list of terms in A. Add also

the Z_j (of type $\bigvee$) that are introduced, to the list in A.

G. Reduce all the multiple paths in unary bouquets to

single step unary paths, as in Fig.8. Eliminate the

$t_j^{(m)}$ and Z_j so removed from the list in A, and add

the new Z_j that are introduced to obtain the "reduced"

graph.

H. Implement the "reduced graph" using the list of $t^{(i)}$

and $t^{(ai)}$ that are now available, employing only unary

and binary forward elementary relations (as in Fig.

6(a) and (c)), and obtain the list of QPL states of

$t^{(o)}$ and $t^{(ao)}$. (Rearrange the sequence of the

relations as required for this purpose.)

I. Repeat the steps from E to H, for Set 2, Set 3,... of

$t^{(ai)}$ and list the $t^{(o)}$ and $t^{(ao)}$ for each set, as

desired.

B) It is assumed that there are no directed circuits

in the graph. We check for the occurrence of any such circuits.

If there is one, as is shown in Fig. 4(a), it is removed by

adding a $\bigvee$-operator at one of the terms (say $\underline{a}$) for comparing

the two newly created terms $\underline{a1}$ and $\underline{a2}$, which lead to

$\underline{a1} \bigvee \underline{a2} = \underline{a}$ (as in Fig. 4(b). The term $\underline{a1}$, initiating the

(formerly circuitous) path becomes an input term. (Type $t^{(i)}$)

and $\underline{a2}$ becomes a middle term (Type $t^{(m)}$) while $\underline{a}$ will be an

output term (Type $t^{(o)}$. This is done until all directed circuits

are removed. (We have not yet proved that the graphs obtained

by breaking a directed circuit at different terms (like $\underline{a}$ above)

are equivalent. Probably, they are not. Hence, the point

(term $\underline{a}$) at which the circuit is cut to provide the "circuit

removal construction" must be specified for each circuit before

the logic in the graph is algebraically implemented).

C) We then check for the occurrence of any multiple inputs to a term (which may be either of the type $t^{(m)}$ or $t^{(o)}$) and, if such multiple inputs are present for any term, we make the $\bigvee$-construction, as in Eqs (49) and (50), in each case. The $t^{(m)}$ or $t^{(o)}$ that are thus produced are added to the original list in A, as well as the operators $\bigvee$ that are used. The output of such a $\bigvee$-operator can be $\emptyset$ for a middle term $t^{(m)}$, and carried forward to later steps of the argument.

D) We then list the reverse binary relations of the type $\mathcal{Q}_k(\underline{c}_j \, \underline{z}_k \, \underline{a}_j = \underline{b}_j)$ and find out if $\underline{c}_j$ corresponds to a term which is an original input $t_j^{(i)}$ or an intermediate term $t_j^{(m)}$. If c_j is __not__ a original input, we make the $\underline{c}$-reconstruction as in Fig. 7(b), so that it becomes one of the original inputs. Replace $\mathcal{Q}_c(\underline{c})$ by $\mathcal{Q}_{c1}(\underline{c1})$ and introduce the input term $\mathcal{Q}_{c2}(\underline{c2})$ and the output term $\mathcal{Q}_c(\underline{c})$. The additional inputs

$t_j^{(ai)}$ and final outputs $t_j^{(ao)}$ thus generated are added to

the list in A , but listed separately with the additional

superscript (a), as in (ai), (am), (ao). The operator $\mathbb{V}$ is

added to the list Z_j, as also the Type 1 unary operator

$\mathcal{V}'(\underline{a}\ \underline{Z} = \underline{b})$ or $\mathcal{V}'(\underline{a}\ \underline{Z}^c = \underline{b})$ which replaces the binary

reverse operator $\mathcal{V}(\underline{c}\ \underline{Z}\ \underline{a} = \underline{b})$. The output of such a $\mathbb{V}$-operator

in this process can be $\emptyset$ only for a $t^{(ao)}$, an output term.

E) Taking the set of $t^{(i)}$, $t^{(m)}$ and $t^{(o)}$ that are obtained

at this stage, we input the QPL states of those $t_j^{(i)}$ which are

provided for the argument. List the $t^{(ai)}$ arising from the

$\underline{c}$-reconstruction process separately. We choose one set of

possible inputs for these (and later modify these over all

possible QPL states that could be put into such reconstructed

$t_j^{(ai)}$). We consider one such complete set of $t_j^{(i)}$ for implementing

the graph of the argument below. When we talk of $t_j^{(i)}$ hereafter,

they include both $t^{(ai)}$ and $t^{(i)}$, and similarly for $t_j^{(m)}$ and $t_j^{(o)}$.

F) We now check the unary relations to be of the E-type,
I-type and/or A-type. For each of the unary relations of the
A-type, we make the modified A-construction as in Fig. 6(b).
This also leads to additional $t^{(am)}$ and $t^{(ao)}$, but do not add
to the list of the terms $t^{(ai)}$. These are added to the list
in A, for implementation, but they will not be changed, as they
are the consequences of the process of implementing the unary
A-type connectives. A consistency check operator $\mathbb{W}$ is also
introduced in each case, which could lead to an output $\emptyset$ for a $t^{(ao)}$

G) Take each of the unary paths in the various unary
bouquets that are present in the graph produced at this stage
and reduce them to the equivalent single unary relations, as in
Figs.6(a) and (b). Do this for all the unary bouquets that are
present. This reduces the number of $t_j^{(m)}$, by eliminating all the

$t_j^{(m)}$ in the bouquets that are neither the starting term, nor

the ending term, in each unary path. Remove the Z_j in the

original bouquets and introduce the new equivalent Z_j in the

single step unary paths.

H) At this stage we obtain a "reduced" graph which contains

only (single) unary relations and binary forward relations, and

this is the logical graph to be worked out from the $t^{(i)}$ and

$t^{(ai)}$, _via_ the Z_j and $t^{(m)}$ to the $t^{(o)}$ and $t^{(ao)}$. Renumber

the list of the terms t_j if necessary, and classify them into

the types $t^{(i)}$, $t^{(m)}$ and $t^{(o)}$ and write down all the unary and

binary forward relations between these terms, which may be in

an arbitrary order. (Distinguish between $t^{(i)}$ and $t^{(ai)}$).

Rearrange the sequence of the logical relations thus

obtained, so that they can be implemented in serial order. (The

algorithm for this is reserved for a later part of this series.

The proof that this is possible is given in Section 5(a) below).

Using the set of original inputs $t_j^{(i)}$ for the terms $t_j^{(i)}$,

implement the equations as newly listed, and obtain the final

outputs $t_j^{(o)}$. This can always be done, although some of the

middle and/or final outputs may be having the contradictory

state $\emptyset$ or the indefinite state $\triangle$.

I) If all the $t_j^{(o)}$ and $t_j^{(ao)}$ are non-$\emptyset$, then we say

that the inputs put in the step E are consistent with the

chosen argument, and the conclusions can be read out of these

outputs. (See Section 5(b) for a justification of this).

If there are contradictory states among the outputs, choose

another set of inputs for the additional inputs $t_j^{(ai)}$, taking care

that the set of original inputs $t_j^{(o)}$ given for the argument

are unchanged, and repeat the process from E to H for

this set of inputs. Once again check for absence of the

contradictory state $\emptyset$ in the outputs $t^{(o)}$. Repeat for all

possible combinations of initial inputs $(t_j^{(ao)})$ other than

those that are given for the argument.

Depending upon the nature of the argument, there may be

only one set of final outputs, or there may be more than one

set of outputs, that are non-contradictory. In the latter case,

the additional inputs, and the intermediate inputs which have

been converted into original inputs in one of the steps, may

have one or the other of different possibilities. We stop the

discussion at this stage, and the analysis of the various types

of arguments, depending on the contradictory and non-contradictory

nature of the outputs is reserved for a future communication.

(a) Proof that a reduced graph consisting of Type-1 statements
    is implementable.

The proof of this follows the same lines as those adopted

in Section 2(e) for the graph of an SNS argument. In fact, the

reduced graph for a QPL Type-1 argument is even simpler in

structure, since it consist only of unary and binary forward

statements. For each of these elementary statements of Type-1,

we have already seen that the output term can be worked out

in terms of the QPL state of the input for the unary relation,

and in terms of the QPL states of the two inputs for a binary

statement (see Section 3(c) (i) and (ii)). Although the QPL

state of a term t has two parameters representing it, namely

the quantifier $\mathcal{Q}$ and the SNS state $\underline{t}$ in the form of $\mathcal{Q}(\underline{t})$,

the rules for the determination of the output state of a QFL

relation is such that the graphical connection between the

input(s) and output follows exactly the same rules as in SNS.

The quantifier(s) of the input(s) and the output, however,

require the statement of the relation and the natures of the

input quantifiers. However, the rule for this is straightforward

and can be written so that they are algorithmically implementable

from input to output. Hence for both unary and binary forward

relations, statements of Type-1 follow the rules of SMS

statements in so far as the structure of the logical graph is

concerned and the associated relations between the quantifiers

can always be written down for application.

In view of this, we can take over the proof that a graph

$\mathscr{G}_{n+1}$ can be implemented for QFL Type-1 statements if we are given

that all graphs $\mathscr{G}_m$ (m = 3 to   n   ) are implementable. The

proof is the same as what is contained in Eqs 22(a,b) for SMS

graphs, since all QPL graphs containing a small number (say 10)

of vertices are all implementable. In fact, as mentioned above,

the proof of this for QPL graphs is unnecessary; because the

structure of the logical graph from the graph theoretical

point of view is identical for SNS and QPL, in that they consist of

unary and binary relations represented by elementary graphs

of the type shown in Fig. 1(a) and (b) for SNS graphs which are

taken over to Fig. 6(a) and (c) for QPL Type-1 statements. In view

of the isomorphism of the graph-theoretical structure of unary

and binary forward statements of SNS and QPL, we shall not give

the proof of the above statement, since it is identical with

what has been given for SNS graphs in Section 2(e).

As a matter of fact, the graph considered for SNS statements

include reverse binary statements, which are avoided in QPL

graphs of Type-1 by using the $\underline{c}$-reconstruction process, whereby

the reverse binary statement is converted into statements of

one or the other of the other two types. Hence the QPL Type-1

raph is always implementable using the results given in Figs

6(a) and (b) and Eqs. (30), (32), (33), (34) for the formulae

connecting the terms t_j on the one hand and the quantifiers

q_j on the other hand, using the operator z_k .

b) Processing of the argument through the reduced graph.

As already stated, the reduced graph has only unary and

binary forward QPL statements of Type-1. Among the former,

$\forall(\underline{a} \underline{A} = \underline{b})$ is not employed in the unary path, but is converted

in a suitable way for checking the truth of this statement. Hence,

with QPL inputs, the unary paths can only give a QPL output or

the state $\triangle$. In the latter, for matrix connectives also, only

QPL outputs occur if the inputs are also one of the four QPL

states; but if the input is $\triangle$ then the output is also $\triangle$.

In the forward direction, $\emptyset$ leads to $\emptyset$ for all connectives

(both unary and binary) except for $\lor$ as already mentioned earlier.

Hence it follows that the particular set of inputs is contradictory

if the $\emptyset$ state occurs anywhere during the working out of the

argument. So also $\triangle$ leads to $\triangle$ for all connectives, except

for $\vee$, but it does not indicate anything like contradiction.

Instead, this state is of no information value for the resulting

final output having the state $\triangle$. This situation is quite

common, and can be the consequence of an implication, as discussed

in Part III, where such arguments were given the name "invalid".

Actually there is no invalidity in the argument, but only the

result that the conclusion is of no value, just like in the SNS

implication $(\underline{a} \Rightarrow \underline{b}, \neg \underline{a}) \cong (\underline{b} = D = (1 \quad 1))$.

As is obvious, the occurrence of a null set as a result of

$\underline{a1} \vee \underline{a2} = \emptyset$, indicates that the two inputs $\underline{a1}$ and $\underline{a2}$ into the

elementary statement are mutually contradictory and therefore

either $\underline{a1}$ should be changed to $\underline{a1}^c$ or an EPL vector contained

in it, or $\underline{a2}$ must be changed to $\underline{a2}^c$ or an EPL vector contained

in it. Thus, suppose $\underline{a1} = \vee \cong (1 \quad 0 \quad 0)$ and $\underline{a2} = \wedge \cong (0 \quad 1$

which leads to $\underline{a1} \vee \underline{a2} = \emptyset$. Then we conclude that $\underline{a2}$ should

be changed to $(1 \quad 0 \quad 0) \cong \vee$, or $\underline{a1}$ must be changed to either

(0 1 1), or (0 1 0), or (0 0 1), corresponding to

$\bigwedge$, $\lessgtr$ or Φ. as the case may be. Beyond this, the treatment

of the consequences of the contradiction will depend upon the

particular argument with regard to retracing of the logic

backwards along the paths in the graph. This is not discussed

here and is reserved for a future communication.

c) Conclusion

In the next Part V, we shall consider QPL arguments having

both Type-1 and Type-2 statements which necessarily also lead

to non-quantified SIS statements. A general logical argument

containing statements belonging both to propositional calculus

and predicate calculus (Type-1 and Type-2 statements) is possible

and leads to no more difficulty in the graph-theoretical

implementation of it than what we have discussed so far. This

is because the flow of logic in the graph is modulated by the

occurrence of terms (t) and unary and binary connectives (Z) in

the same way as we have considered so far. The only new

feature that will require consideration is the fact that

there will be more than one variable for the different quantifiers

that are introduced in the argument. This is being examined

and the results/reported in due course.

FIGURES

.100.

TABLES

LIST OF REFERENCES

1. Ramachandran, G.N., Vector-Matrix Representation of Boolean
   Algebras and Application to Extended Predicate Logic (EPL)
   Part III   (Submitted for publication)

2.   -do-   Part I, Current Science, 1983, 52, 292-302

3.   -do-   Part II, Current Science, 1983, 52, 335-341.

4. Ramachandran, G.N., Syad-Nyaya System (SNS) — A New Formulation
   of Sentential Logic and its Isomorphism with Boolean Algebra
   of Genus 2, Current Science, 1982, 51, 625-636.

Vector-Matrix Representation of Boolean Algebra and Application

to Extended Predicate logic (EPL)

Part V - Theory for general argument in logic including SNS, QPL

and QSN statements[*]

G.N. Ramachandran
Gita, 5, 10A Main Road
Malleswaram West
Bangalore 560 055

<u>Vector-Matrix Representation of Boolean Algebras and Application</u>

<u>to Extended Predicate Logic (EPL)</u>

<u>Part V — Theory for general argument in logic including SNS, QPL</u>
<u>and QSN statements</u>*

<u>G.N. Ramachandran</u>
Gita, 5, 10A Main Road
Malleswarem West
Bangalore 560055

PREFACE AND SUMMARY

A brief account of the framework of this report is given

in Section 1 entitled "Introduction". Therefore, only a very

brief summary is given here.

In this report it is shown that the method of writing a

flow chart for a logical graph consisting of statements in QPL-1

which was given in Part IV, can be completely taken over for the

most general type of argument in logic containing terms of the

type of SNS, QPL-1 and QPL-2 and relations between any two of

these. However, only terms which are functions of one variable

are discussed in this report, although <u>relations</u> and the

connectives representing the relations may be functions of two or

three variables also. While doing this, the algorithm for the

working out of a reduced logical graph is given in greater detail

than in Part III and is readily seem to be computerizable.

Examples of arguments containing various graphical constructions

such as <u>c</u>-reconstruction and <u>A</u>-construction as also examples of

equations which are in pure SNS or QFL-1 or QFL-2 and also

any relations connecting one type with another, are given in

the two or three arguments whose logical graphs are given in

good detail. It is believed that all the processes described

in this report are capable of being converted into steps in

a computer program.

As in the case of Part IV, this report has not been checked

by anyone other than the author and therefore it cannot be

assured that there are no incidental errors or omissions.

On the other hand, the main principles and arguments are believed

to be correct and properly formulated. Minor lapses may kindly

be excused.

CONTENTS   Page No.

Vector-Matrix Representation of Boolean Algebras and Application

to Extended Predicate Logic (EPL)

Part V — Theory for general argument in logic including SNS, QPL and

QSN statements

1. Introduction

This report contains the extension of the general theory

for an argument composed of statements of Type-1 in QPL (hereafter

designated as QPL-1), which includes also the theory for an SNS

argument, to the case of any general argument which can have

statements in SNS (propositional calculus) and in QPL-1 and QPL-2

(which stands for statements of Type-2 in QPL and EPL, which has

been discussed extensively in Parts I and II) of predicate

calculus. The discussion in this report is made very brief since

the proof of almost all the theorems required for the above

purpose are contained in the earlier four parts. The essential

idea is to show that Section 5 of Part IV in general, and, in

particular, the outline of the flow chart of implementing any

general argument in QPL-1 (Fig.9 of Part IV), are both readily

extendable to a completely general argument in logic, which can

contain statements in SNS, QPL-1, QPL-2 and interconnections

between these as will be described below.

The framework of this report may be briefly given here.

In Section 2, the three forms of elementary statements — namely

unary, binary forward, and binary reverse — will be describe

for each of the types SNS, QPL-1, QPL-2 and mixed SNS-QPL or QPL-SN

(designated hereafter by QSN). It will be seen from this, that

the topological structure of the graph of an argument depends

only on the unary and binary (forward or reverse) nature of the

relations in it, and is independent of whether it is of one or

the other of the above four types. Therefore, <u>in the treatment</u>

<u>of the graph in the flow chart,the outline, as given in Fig.9</u>

<u>of Part IV, can be completely taken over</u>. Therefore, in Section 3

of this Part V, each of the steps A to I of this flow chart

will be taken and its implementation for each of the above four

types, namely SNS, QPL-1, QPL-2 and QSN will be explained for

each of the blocks A to I. As will be seen from this Section,

every step of the algorithm in the flow chart is implementable

according to well defined rules and formulae for each of the

four types of logical relationships. Thereafter, in Section 4,

the proof readily follows that any general argument in logic is

implementable by using our vector-matrix formalism.

In doing all these, the use of brackets (wherein the fact,

that at any stage, a term that is employed is itself the result

of an argument consisting of a series of interconnected

statements, as indicated by putting them inside a bracket) will

not be gone into, except for the recognition of this feature

here. This is a well-known principle in mathematics whenever

a function of a function is calculated and the same technique

can be adopted here also in the algebra representing such a

feature. Thus, the statements inside a bracket are taken to form

an independent argument and its graph worked out, by the

techniques developed here, to obtain the required term. Then,

the output term will be an SNS term, or a QPL term with a

quantifier, a variable and an SNS term, in the standard notation

for such a QPL term. (The term QPL will be taken to include also

EPL terms wherever relevant). Similarly, the way in which

different variables are collected and taken care of will not be

considered (as this is also a well recognised procedure in

mathematics and computer science), except as they occur in the

theory of a pair of elementary statements.

The emphasis is to show that a single argument in logic

containing a mixture of all possible types of statements that can

be put in SNS and QPL, and mixed relations in QSN using both of

these, and employing both matrix and Boolean logical connectives,

can be implemented, and the outputs calculated, making use of the

inputs. In doing this, it will be shown that the rather complicate

sequence of operations (Blocks A to I) given in the flow chart in

Fig.9 of Part IV can be followed, and each block replaced by a

corresponding one, for each of the four different types of

logical relations — namely SNS, QPL-1, QPL-2 and QSN.

2. The four types and three forms of elementary statements.

As mentioned in the previous section, we shall give a brief

summary of /the three forms of elementary statements — namely
       how

unary, binary forward and binary reverse /can have the four types

of logic namely SNS, QPL-1, QPL-2 and mixed QSN. The formulae

given below will/ be proved, as all these have been thoroughly
            not

discussed in the previous four parts. The reference is made to

these parts [1 — 4] and to the earlier publication on SNS logic

and theory of relations [5] . Also, as mentioned in the last

section, the logical graph indicating the flow of information

from term to term is identical for all of these four different

types and therefore we shall use a general notation of

a , b, c etc., for a term (in general t), and A, O, U, V, etc.,

for a relational connective ($\underline{Z}$ in general), to represent the

terms and operators of the argument in general. $\underline{t}$ may be thus

a SNS term or a QPL term etc., and $\underline{Z}$ may be a SNS-SNS relation,

SNS-QPL-2 relation etc. The structure of the graph for the

three elementary logical equations are:

Unary : $\quad \underline{a} \, \underline{Z} = \underline{b}$ (1)

Binary forward : $\underline{a} \, \underline{Z} \, \underline{b} = \underline{c}$ ($\underline{c}$ is an SNS term)   (2)

Binary reverse : $\underline{c} \, \overleftarrow{\underline{Z}} \, \underline{a} = \underline{b}$ (3a)

If $\underline{c} = T$, $\quad \underline{a} \, \underline{Z} = \underline{b}$ (3b)

If $\underline{c} = F$, $\quad \underline{a} \, \underline{Z}^C = \underline{b}$ (3c)

If $\underline{c} = D$, $\quad \underline{a} \, (\underline{Z} \oplus \underline{Z}^C) = \underline{b}$ (3d)

If $\underline{c} = X$, $\quad \underline{a} \, (\underline{Z} \otimes \underline{Z}^C) = \underline{b}$ (3e)

where $\underline{Z}^C$ stands for the complement of the relation $\underline{Z}$ (see Part I).
For the Eqn (3a) the solutions when the state of $\underline{c}$ is given are

outlined in (3b,c,d,e), so that the binary reverse is always

equivalent to a unary relation when the state of $\underline{c}$ is known,

~~so that~~ and it is only necessary to give the formula for unary and

binary forward relations, subject to the relevant conditions

for binary reverse in special cases.

The treatment below in this Section 2 gives in outline the

relevant formulae for all the four types of logical relations

and they in effect give the notation that we are adopting, which

for the most part is identical with that used earlier in the

previous parts. It is found desirable to use different fonts of

letters for the four different types of terms and operators so

that an equation itself will suggest the nature of the logic

concerned, without having to specifically state that the relevant

term is a QPL term, or the relevant relation is an SNS-QPL-2

relation, and so on.

In algebra, an operator $\underline{Z}$ connecting $\underline{a}$ to $\underline{b}$ can be associated

with an "inverse" operator (usually designated $\underline{Z}^{-1}$) connecting

$\underline{b}$ to $\underline{a}$. Thus Eq (1) has the inverse relation $\underline{b}\,\underline{Z}^{-1} = \underline{a}$. However,

we shall use the symbol $\underline{Z}^{t}$ for this operator, since from the

General Theory of Relations discussed in Part I [1], the matrix

$|Z'|$ connecting the vector $|b\rangle$ to the vector $|a\rangle$ is the

transpose $\left(|Z^t|\right)$of the matrix $|Z|$ connecting the vector $|a\rangle$ to

the vector $|b\rangle$. It then follows that the inverse unary and

binary relations corresponding to Eqs (1), (2), (3) have the form.

$$\text{Unary} \qquad : \quad \underline{b}\,\underline{Z}^t = \underline{a} \qquad\qquad (1')$$

$$\text{Binary forward—} : \quad \underline{b}\,\underline{Z}^t\,\underline{a} = \underline{c} \qquad\qquad (2')$$

$$\text{Binary reverse} \quad : \quad \underline{c}\,\overleftarrow{\underline{Z}^t}\,\underline{a} = \underline{b} \qquad\qquad (3'a)$$

$$\text{If } \underline{c} = T, \quad \underline{b}\,\underline{Z}^t = \underline{a} \qquad\qquad (3'b)$$

$$\text{If } \underline{c} = F, \quad \underline{b}\,\underline{Z}^{tc} = \underline{a} \qquad\qquad (3'c)$$

These inverse relations find particular application in

QPL-1, and hence their properties are discussed in Section 2(d)(i)

where their application to SNS, QPL-2 and QSN are also discussed.

(a) SNS relations

An SNS term is denoted by $\underline{t}$ and an SNS-SNS relational

operator denoted by $\underline{Z}$, so that the three types of relations take

the form

$$\text{Unary} \quad : \quad \underline{a}\ \underset{\sim}{Z} = \underline{b} \tag{4a}$$

$$\text{Binary forward} : \quad \underline{a}\ \underset{\sim}{Z}\ \underline{b} = \underline{c} \tag{4b}$$

$$\text{Binary reverse} : \quad \underline{c}\ \overset{\leftarrow}{\underset{\sim}{Z}}\ \underline{a} = \underline{b} \tag{4c}$$

As has been discussed in [5], Part I [1] and Part IV [4], the

SNS logic, which is the logical extension of propositional

calculus, can be represented by Boolean 2-vectors and 2X2 matrices

of the type,

$$(t_\alpha \quad t_\beta) \quad ; \quad \begin{pmatrix} z_{\alpha\alpha} & z_{\alpha\beta} \\ z_{\beta\alpha} & z_{\beta\beta} \end{pmatrix} \tag{5a,b}$$

Then the formulae giving the output in terms of the input term

and the matrix of the relation for the above three types are as

follows:

$$\text{Unary} \quad : \quad \langle a \mid Z \mid = \langle b \mid \tag{6a}$$

$$\text{Binary forward} : \quad \langle a \mid Z \mid b \rangle = \langle c \mid \tag{6b}$$

where

$$\langle a \mid Z \mid b \rangle = c_\alpha \ , \quad \langle a \mid Z^c \mid b \rangle = c_\beta \ , \quad (c_\alpha \quad c_\beta) = \langle c \mid \tag{6c}$$

This is for matrix operators $\underset{\sim}{Z}$.

For Boolean operators $\underset{\sim}{U}$ and $\underset{\sim}{V}$:

$$\langle a \mid \oplus \ \langle b \mid = \langle c \mid \ \text{for} \ \underset{\sim}{U} \ , \quad \langle a \mid \otimes \ \langle b \mid = \langle c \mid \ \text{for} \ \underset{\sim}{V} \tag{6d,e}$$

No formula is given for the binary reverse because by Eqs (3b to e)

it is converted into a unary relation when the SNS state of $\underline{c}$

is given. ⫽The four states are T,F,D,X, standing for

$$T = (1 \quad 0) = \text{True} ; \quad F = (0 \quad 1) = \text{False} ; \qquad (7a,b)$$

$$D = (1 \quad 1) = \text{Doubtful} ; \quad X = (0 \quad 0) = \text{Impossible} \quad (7c,d)$$

There are 16 possible matrices $|Z|$ corresponding to each

of the four components having the Boolean values 1 or 0. Of

these, 10 are in general use in current literature in logic;

and, in our notation, they are classified as follows:

$$\underset{\sim}{E}, \quad \underset{\sim}{N} \qquad - \quad \underset{\sim}{E}\text{-type} \qquad (8a)$$

$$\underset{\sim}{A}, \quad \underset{\sim}{Q}^{C}, \quad \underset{\sim}{I}^{C}, \quad \underset{\sim}{J}^{C} \quad - \quad \underset{\sim}{A}\text{-type} \qquad (8b)$$

$$\underset{\sim}{Q}, \quad \underset{\sim}{A}^{C}, \quad \underset{\sim}{I}, \quad \underset{\sim}{J} \quad - \quad \underset{\sim}{Q}\text{-type or } \underset{\sim}{I}\text{-type} \qquad (8c)$$

The above operators have, for their matrices, the 2X2 truth

table for the logical connective represented by them (see Ref [5]).

In addition, the other two operators found to be of common use

are $\underset{\sim}{D}$, $\underset{\sim}{X}$, given by

$$|D| = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \qquad |X| = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \qquad (8d,e)$$

where $\underline{a} \underline{D} \underline{b} = \underline{c}$ leads to $\underline{c} = D$ for all states of $\underline{a}$ and $\underline{b}$

(except X's) and $\underline{a} \underline{X} \underline{b} = X$ for all $\underline{a}$ and $\underline{b}$. The remaining

four matrices are not simple connectives, but have properties

similar to D and X — for example, $\underline{c}$ is T for all $\underline{a}$, independent

of the state of $\underline{b}$, and so on. They are representable as sums

of elementary matrices of the $\underline{A}$-type and are not discussed

further. This idea of having a general matrix $|Z|$ as a sum of

$\underline{A}$-type matrices is of great use for writing complicated logical

relations which are not of the standard type, in SNS, QPL and QSN.

or three
They are also representable as a product of two/implications,

which are simultaneously true. (See Section 5(b)).

(b) QPL-2 relations

These have a structure identical with the SNS relations,

except that Boolean 3-vectors and 3X3 Boolean matrices are employed.

The notation adopted for relations are as in Eqs. (1), (2) and (3).

They are:

$$\text{Unary} \qquad : \underline{a} \underline{Z} = \underline{b} \longmapsto \langle a| Z| = \langle b| \qquad (9a)$$

Binary forward : $\underset{\equiv}{a} \underset{\approx}{Z} \underset{\equiv}{b} = \underset{\equiv}{c} \longmapsto \underset{\equiv}{c} = (c_\alpha \quad c_\beta),$ (9b,c)

where

$$\langle a \mid Z \mid b \rangle = c_\alpha \quad , \quad \langle a \mid Z^c \mid b \rangle = c_\beta$$ (9d,e)

Binary reverse : $\underset{\equiv}{c} \, Z \, \underset{\equiv}{a} = \underset{\equiv}{b}$ (10a)

If $\underset{\equiv}{c} = T$, $\underset{\equiv}{a} Z = \underset{\equiv}{b}$ (10b)

If $\underset{\equiv}{c} = F$, $\underset{\equiv}{a} Z^c = \underset{\equiv}{b}$ (10c)

The results for $\underset{\equiv}{c} = D$ and X follow from Eqs. (3d) and (3e).

The above are for matrix connectives. For the Boolean connectives $\mathbb{U}$ and $\mathbb{V}$, the equations follow the pattern of (6d and e), and are

$$\langle a \mid \oplus \langle b \mid = \langle c \mid \quad \text{for} \quad \mathbb{U}$$ (11a)

$$\langle a \mid \otimes \langle b \mid = \langle c \mid \quad \text{for} \quad \mathbb{V}$$ (11b)

The state vectors and matrix connectives of EPL have the forms

$$(t_\gamma \quad t_\delta \quad t_\epsilon) \quad ; \quad \begin{pmatrix} Z_{\alpha\alpha} & Z_{\alpha\beta} & Z_{\alpha\gamma} \\ Z_{\beta\alpha} & Z_{\beta\beta} & Z_{\beta\gamma} \\ Z_{\gamma\alpha} & Z_{\gamma\beta} & Z_{\gamma\gamma} \end{pmatrix}$$ (12a,b)

Thus, in addition to the basic states

$$(1 \quad 0 \quad 0) = \forall = \text{For all}$$ (13a)

$$(0 \quad 1 \quad 0) = \lessgtr = \text{For some, but}$$ (13b)
$$\text{not for all, nor for none}$$

$$(0 \quad 0 \quad 1) = \bar{\phi} = \text{For none}$$ (13c)

five more EPL states are recognizable, since there are $2^3 = 8$

possible states for Boolean 3-vectors. These are,

$(1 \quad 1 \quad 0) = \beth =$ There exists $\qquad\qquad$ (14a)

$(0 \quad 1 \quad 1) = \Lambda =$ Not for all $\qquad\qquad$ (14b)

$(1 \quad 0 \quad 1) = \ominus =$ For all or none $\qquad\qquad$ (14c)

$(1 \quad 1 \quad 1) = \triangle =$ Indefinite $\qquad\qquad$ (14d)

$(0 \quad 0 \quad 0) = \emptyset =$ Impossible (Called "Null Set",
to distinguish it from the
"Impossible" in SNS) $\qquad\qquad$ (14e)

For ready reference and use, these eight EPL states are designated

by q(1) to q(8) and stand for the following:

$q(1) = (1 \ 0 \ 0)$, $q(2) = (0 \ 1 \ 1)$, $q(3) = (0 \ 1 \ 0)$, $q(4) = (1 \ 0 \ 1)$,

$q(5) = (0 \ 0 \ 1)$, $q(6) = (1 \ 1 \ 0)$, $q(7) = (1 \ 1 \ 1)$, $q(8) = (0 \ 0 \ 0)$ $\qquad$ (15)

They have the property that $q^c(i) = q(i^c)$, where

$$q^c(2j - 1) = q(2j) \; ; \quad q^c(2j) = q(2j - 1) \qquad\qquad (16)$$

These eight 3-vectors are called the "canonical" representations

of the logical states and they are extensively used in QPL-2.

The four QPL states alone are represented as follows in

the standard literature and they are used as such in QPL-1

(see below).

$$(1 \quad 0 \quad 0) \equiv (\forall x) \, (\underline{a}(x)) \; ; \quad (0 \quad 0 \quad 1) \equiv (\forall x) \, (\neg \underline{a}(x)) \quad (17a,b)$$

$$(1 \quad 1 \quad 0) \equiv (\exists x) \, (\underline{a}(x)) \; ; \quad (0 \quad 1 \quad 1) \equiv (\exists x) \, (\neg \underline{a}(x)) \quad (17c,d)$$

The representation of QPL-2 connective is a 3X3 Boolean

matrix and therefore there can be $512(2^9)$ different matrices.

A way of building all these matrices as sums of only three

EPL implications   , is indicated in Section 5(b).   Here

we shall consider only those matrices which are relevant for

use in QPL logical formulae commonly used. Thus, the $\underset{\approx}{A}$-type

matrix can have 64 different possibilities and they stand for the

relations of the type $\forall \, \& \, \forall$, $\forall \, \& \, \exists$, etc., where the first

term can have anyone of the eight states and the second one can

have also anyone of the eight states. As shown in Part II [2]

the matrix representation for this has the form

$$\underset{\approx}{A}(i, \, j) \longmapsto \left| A \, (i, \, j) \right| = \left| q(i) \right\rangle \otimes \left\langle q(j) \right| \, , \quad i, \, j = 1 \text{ to } 8 \quad (18a)$$

Similarly there are 64 relations of the type $\forall \wedge \forall$, $\forall \wedge \exists$, etc.,

which have for their representation matrices

$$\underset{\sim}{O}(i,\ j) \longmapsto |O(i,\ j)| = |q(i)\rangle \oplus \langle q(j)| \ , \ i,\ j = 1 \text{ to } 8 \qquad (18b)$$

In addition to the above, four 3X3 matrices are widely applicable

in QPL-2 logic. These are

$$\underset{\approx}{E} \longmapsto \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} ; \qquad \underset{\approx}{N} \longmapsto \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \qquad (19a,b)$$

$$\underset{\approx}{D} \longmapsto \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \qquad \underset{\approx}{X} \longmapsto \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad (19c,d)$$

The formulae connecting $\underset{\approx}{A}$ and $\underset{\approx}{O}{}^{c}$, $\underset{\approx}{O}$ with $\underset{\approx}{A}{}^{c}$, $\underset{\approx}{I}$ with O, $\underset{\approx}{I}$ with $\underset{\approx}{J}$

etc., are not given here and may be obtained from Part II.

However, just as in the case of SNS, it is necessary to recognise

the matrices to be of three types: $\underset{\approx}{E}$ and $\underset{\approx}{N}$ to be of the $\underset{\approx}{E}$-type,

$\underset{\approx}{A}$, $\underset{\approx}{O}{}^{c}$, $\underset{\approx}{I}{}^{c}$, $\underset{\approx}{J}{}^{c}$ for all (i, j) to be of the $\underset{\approx}{A}$-type and $\underset{\approx}{O}$, $\underset{\approx}{A}{}^{c}$, $\underset{\approx}{I}$, $\underset{\approx}{J}$,

for all (i, j) to be of the $\underset{\approx}{O}$(or $\underset{\approx}{I}$)-type.

As is seen from Eqs. (9) and (10), the algebra of the 3X3

matrices in QPL-2 closely follow the pattern of the 2X2 matrices

in SNS; only they are more in number and the equations contain

more parameters. These 3X3 matrices relate one or two QPL terms

to another QPL term and, in general, QPL relation in terms of

quantifier variables has the following form for unary and binary

relations $(20a$ and $b)$, and for one example of binary reverse relation

$(20c)$:

$$\underline{a}(x) \ \underset{\approx}{Z} \ (x, \ y) \ = \ \underline{b}(y) \tag{20a}$$

and

$$\underline{a}(x) \ \underset{\approx}{Z} \ (x, \ y) \ \underline{b}(y) \ = \ \underline{c} \tag{20b}$$

$$\underline{c} = T, \ \underline{c} \ \overset{\leftarrow}{\underset{\approx}{Z}}(x, \ y) \ \underline{a}(x) \ = \ \underline{b}(y) \longmapsto \underline{a}(x) \ \underset{\approx}{Z} \ (x, \ y) \ = \ \underline{b}(y) \tag{20c}$$

These forms are the most general ones that can happen; but it

is also possible to have relations between terms having less

number of variables or having the same variable throughout.

This can happen, for example, by having $x = y$ in (20a), (20b)

or (20c). These are particular cases of the general result and

do not require any special attention. In what is discussed both

so far, and later, with reference to QPL-2, we shall use only

the general structure /as given for Eqs (20a to c).

$\lfloor$ for variables $\rfloor$

(c) QSN relations

We shall consider the mixed SNS-QPL and QPL-SNS types of

the

relations before considering QPL-1 type of relation, because these

have an algebraic structure closely resembling what we have

discussed already in the subsections (a) and (b) for SNS-SNS and

QPL-2 - QPL-2 relations.

The mixed relations QSN, of the types SNS-QPL and QPL-SNS

are closely similar and therefore they are considered together.

Both have the same form for unary, binary forward and binary

reverse relations as in Eq. (6) for SNS and in Eqs (9) and (10)

for QPL-2. In the corresponding logical equations for mixed

relations, we indicate SNS vectors by $\underline{a}$, $\underline{b}$ etc., and QPL vectors

in the canonical form by $\underline{\underline{a}}$, $\underline{\underline{b}}$ etc. To distinguish the inter-

of

connecting matrix to be the mixed type which may be a 2X3 matrix

for SNS-QPL or a 3X2 matrix for QPL-SNS, we designate it by $\underline{\underline{z}}$.

Thus, for the mixed case, the three standard forms of

elementary relations are as follows:

SNS-QPL

$$\text{Unary} \quad : \quad \underline{\underline{a}} \; \underline{\underline{Z}} = \underline{b} \quad\quad\quad (21a)$$

$$\text{Binary forward} \quad : \quad \underline{\underline{a}} \; \underline{\underline{Z}} \; \underline{\underline{b}} = \underline{c} \quad\quad\quad (21b)$$

$$\text{Binary reverse} \quad : \quad \underline{c} \; \overset{\leftarrow}{\underline{\underline{Z}}} \; \underline{\underline{a}} = \underline{b} \quad\quad\quad (21c)$$

QPL-SNS

$$\text{Unary} \quad : \quad \underline{a} \; \underline{\underline{Z}} = \underline{\underline{b}} \quad\quad\quad (22a)$$

$$\text{Binary foward} \quad : \quad \underline{a} \; \underline{\underline{Z}} \; \underline{b} = \underline{\underline{c}} \quad\quad\quad (22b)$$

$$\text{Binary reverse} \quad : \quad \underline{c} \; \overset{\leftarrow}{\underline{\underline{Z}}} \; \underline{\underline{a}} = \underline{b} \quad\quad\quad (22c)$$

The corresponding vectors for $\underline{\underline{a}}$ and $\underline{\underline{a}}$ are $(a_\alpha \; a_\beta)$ and $(a_\gamma \; a_\delta \; a_\epsilon)$

Only the matrix for the connective operator has a new form and

it is as follows:

SNS-QPL: $\quad\quad \underline{\underline{Z}} \longmapsto$ 2X3 matrix $\quad \begin{pmatrix} z_{\alpha\gamma} & z_{\alpha\delta} & z_{\alpha\epsilon} \\ z_{\beta\gamma} & z_{\beta\delta} & z_{\beta\epsilon} \end{pmatrix} \quad\quad (23a)$

QPL-SNS: $\quad\quad \underline{\underline{Z}} \longmapsto$ 3X2 matrix $\quad \begin{pmatrix} z_{\gamma\alpha} & z_{\gamma\beta} \\ z_{\delta\alpha} & z_{\delta\beta} \\ z_{\epsilon\alpha} & z_{\epsilon\beta} \end{pmatrix} \quad\quad (23b)$

The matrices of the $\underline{A}$-type and $\underline{O}$-type can be constructed in the same manner as in Eqs. (18a,b) for QPL-2. For this purpose, the four SNS states can be designated as in (24a):

$$s(1) = (1 \ \ 0), \quad s(2) = (0 \ \ 1), \quad s(3) = (1 \ \ 1), \quad s(4) = (0 \ \ 0) \quad (24a)$$

with formulae for complementing the vectors similar to those in (16), namely

$$s^c(2j - 1) = s(2j) \ ; \quad s^c(2j) = s(2j - 1) \tag{24b}$$

Then, we have, for SNS-QPL matrices of the $\underline{A}$ and $\underline{O}$-types, the formulae (25), in which the SNS index is underlined:

$$\underline{A}(i, j) \longmapsto |A(\underline{i}, j)| = |s(i) > \otimes < q(j)| \tag{25a}$$

$$\underline{O}(i, j) \longmapsto |O(\underline{i}, j)| = |s(i) > \oplus < q(j)| \tag{25b}$$

For the QPL-SNS matrices of the same two types, we have similarly

$$\underline{A}'(i, j) \longmapsto |A'(i, \underline{j})| = |q(i) > \otimes < s(j)| \tag{26a}$$

$$\underline{O}'(i, j) \longmapsto |O'(i, \underline{j})| = |q(i) > \oplus < s(j)|$$

It follows from the definitions in (25) and (26) that

$$|A'(i, \underline{j})| = |A^t(\underline{j}, i)| \quad ; \quad |O'(i, \underline{j})| = |O^t(\underline{j}, i)| \qquad (27a,b)$$

so that we may write for the operators also, the symbolic equations

$$\underline{A}'(i, \underline{j}) = \underline{A}^t(\underline{j}, i) \quad ; \quad \underline{O}'(i, \underline{j}) = \underline{O}^t(\underline{j}, i) \qquad (28a,b)$$

and it is sufficient to define the matrices of the $\underline{A}$-type and

$\underline{O}$-type for the SNS-QPL connectives, and their transposes give the

corresponding matrices for QPL-SNS relations.

The $\underline{E}$-type connectives are not relevant for 2X3 and 3X2

matrices However, the Boolean binary operators $\bigcup$ and $\bigvee$ are

relevant. For using these, the 2-vector $(s_\alpha \quad s_\beta)$ can be extended

to form a three-vector $(s_\gamma' \quad s_\delta' \quad s_\epsilon')$ by the following rules:

$$\underline{s} = (1 \quad 0) \longmapsto \underline{s}' = (1\ 0\ 0), \quad \underline{s} = (0\ 1) \longmapsto \underline{s}' = (0\ 0\ 1) \qquad (29a,b)$$

$$\underline{s} = (1 \quad 1) \longmapsto \underline{s}' = (1\ 1\ 1), \quad \underline{s} = (0\ 0) \longmapsto \underline{s}' = (0\ 0\ 0) \qquad (29c,d)$$

It is readily verified that they are logically reasonable. Then

for the logical operators "upon" and "vidya" we use the following

equations,

$$\underline{\underline{s}} \ \underline{U} \ \underline{q} = \underline{q} \ \underline{U} \ \underline{\underline{s}} \ \longmapsto \ \underline{\underline{s}}' \ \underline{U} \ \underline{q} \longmapsto \langle s'| \oplus \langle q| \tag{30}$$

$$\underline{\underline{s}} \ \underline{V} \ \underline{q} = \underline{q} \ \underline{V} \ \underline{\underline{s}} \ \longmapsto \ \underline{\underline{s}}' \ \underline{V} \ \underline{q} \longmapsto \langle s'| \otimes \langle q| \tag{31}$$

For the binary reverses of the QSN type in (21c) and (22c),

the following hold (with algebra identical with SNS and QFL-2

statements) as in (32) and (33) below:

$$(21c) \ : \ \underline{c} = T \longmapsto \underline{a} \ \underline{Z} = \underline{b} \ ; \ \underline{c} = F \longmapsto \underline{a} \ \underline{Z}^c = \underline{b} \tag{32a,b}$$

$$(22c) \ : \ \underline{c} = T \longmapsto \underline{\underline{a}} \ \underline{Z} = \underline{b} \ ; \ \underline{c} = F \longmapsto \underline{\underline{a}} \ \underline{Z}^c = \underline{b} \tag{33a,b}$$

(i) An illustration of the use of QSN relations

The following simple statement connecting three QPL terms,

as in a binary relation, requires the use of an intermediate SNS

term and a QSN relation of the type SNS-QPL.

"If all categories of food are available and some beverages
are also available, then all persons present are not unhappy" (34)

We use x as variable and $\underline{f}$ to represent "food available"; y as

variable and $\underline{b}$ to represent "beverage available" and z as variable

for persons and $\underline{u}$ for "unhappy". Then (34) can be symbolized as

follows:

$$(\forall x)\ (\underline{f}(x))\ \&\ (\exists y)\ (\underline{b}(y))\ \Rightarrow\ (\forall z)\ (\neg\underline{u}(z)) \tag{35}$$

However, to work this out by the vector-matrix representation,

we have to use an intermediate SNS term ($\underline{g}$ say) and write it as

follows:

$$(\forall x)\ (\underline{f}(x))\ \&\ (\exists y)\ (\underline{b}(y))\ =\ \underline{g}\quad \text{(QPL-2 binary forward)} \tag{36a}$$

$$\underline{g}\ \Rightarrow\ (\mathbb{\Phi} z)\ (\neg\underline{u}(z))\,\text{(SNS-QPL unary)} \tag{36b}$$

Just to make the story complete, we shall give below the matrices

to be employed in (36a) and (36b):

$$(36a)\ :\ \underline{Z}_1\ \longmapsto\ |Z_1|\ =\ |q(1) > \otimes < q(6)|\ =\ \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{37a}$$

$$(36b)\ :\ \underline{Z}_2\ \longmapsto\ |Z_2|\ =\ |s(2) > \oplus < q(5)|\quad \begin{pmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix} \tag{37b}$$

An interesting variation of the practical statement in (34)

can be written, having only a single variable, as follows (see

comments following Eqs. (20a,b,c)).

"If all persons have food and some have wine, then
all are happy" (38a)

The relevant logical equation, similar to (35), using an obvious

notation, is the following.

$$(\forall z)\ (\underline{f}(z))\ \&\ (\exists z)\ (\underline{w}(z))\ \Longrightarrow\ (\forall z)\ (\underline{h}(z)) \tag{38b}$$

and this also leads to two equations as in (36), which need two

matrices, one 3X3 and the other 2X3, for their practical implementa-

tion.

The generalization of this to $\mathcal{V}_a(\underline{a})\ \underset{\approx}{z}\ \mathcal{V}_b(\underline{b})\ \Longrightarrow\ \mathcal{V}_c(\underline{c})$

is obvious, and it can be worked out, using two equations

$$\mathcal{V}_a(\underline{a})\ \underset{\approx}{z}\ \mathcal{V}_b(\underline{b})\ \equiv\ \underline{a}\ \underset{\approx}{z}\ \underline{b}\ =\ \underline{g} \tag{39a}$$

$$\underline{g}\ \underline{I}(1,\ 1)\ =\ \underline{c}\ \equiv\ \mathcal{V}_c(\underline{c}) \tag{39b}$$

See Eq.(46) below for the matrix representation of $\underline{a}\ \underset{\approx}{z}\ \underline{b}\ \equiv\ \underline{c}$.

(ii) Use of $\underline{E}$ ($\underline{i}$, j) in QSN - another illustration

Although we had said earlier that the diagonal QPL matrices

$|E|$ and $|N|$ have no counterpart in QSN, since the QSN matrices

are rectangular (being 2X3 or 3X2), equivalence relations between

SNS terms and QPL terms can be written, and represented via QSN

matrices $\underline{E}(\underline{i}, j)$, as is shown below.

We first take an example, for $\underline{a} \underline{Z} = \underline{b}$ as in (40).

If $\underline{a}$ = T, then $\underline{b} = \bigvee$; if $\underline{a}$ = F, then $\underline{b} = \neg \bigvee = \bigwedge$ (40)

In order to write the matrices, we first need the matrix for

$\underline{I}(\underline{i}, j)$ and $\underline{I}'(\underline{i}, j)$, which are readily written down from the

logical formulae

$$\underline{a} \Rightarrow \underline{b} \equiv \neg \underline{a} \vee \underline{b} \quad ; \quad \underline{a} \Rightarrow \underline{b} \equiv \neg \underline{a} \vee \underline{b} \qquad (41a,$$

Hence, the matrices $|I|$ and $|I'|$ can be written in terms of

$|O|$ and $|O'|$ as

$$|I(\underline{i}, j)| = |O(\underline{i}^c, j)| \quad ; \quad |I'(\underline{i}, j)| = |O'(\underline{i}^c, j)| \qquad (42a,$$

Now, considering the relation (40), the following are simultaneously

true:

$$\langle a \mid I(1, 1) \mid = \langle b \mid \quad ; \quad \langle a^c \mid I(2, 2) \mid = \langle b \mid \qquad (43a,b)$$

Hence, using the principle of the vidya operator, the matrix

that will represent the two combined relations in (40) is $E(1, 1)$

in the relation

$$\langle a \mid E(1, 1) \mid = \langle b \mid \quad , \text{ where } \mid E(1, 1) \mid = \mid I(1,1) \mid \otimes \mid I(2,2) \mid \qquad (44a,b)$$

~~and~~ Hence,

$$\mid E(1, 1) \mid = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix} \qquad (44c)$$

It is readily verified that the matrix so obtained in (44c) has

the required properties.

More generally, the combined relation

$$\underline{a}(= s(i)) \implies \underline{b}(= q(j)) \quad ; \quad \neg \underline{a}(= s(i^c)) \implies \neg \underline{b}(= q(j^c)) \qquad (45a,b)$$

has the form $\langle a \mid E(i, j) \mid = \langle b \mid$, with

$$\mid E(i, j) \mid = \mid I(i, j) \mid \otimes \mid I(i, j^c) \mid \qquad (45c)$$

The formula (45) is extremely useful in dealing with QPL

relations of the type $\underline{a} \underset{\approx}{Z} \underline{b} \equiv \underline{c}$, for this equation is

implementable <u>via</u> the following matrices

$$\langle \underline{a} \mid Z \mid \underline{b} \rangle = g_{\alpha} \; ; \quad \langle \underline{a} \mid Z^{c} \mid \underline{b} \rangle = g_{\beta} ; \quad \langle \underline{g} \mid E \mid = \langle \underline{c} \mid \quad (46a,$$

where $\mid E \mid$ is a 2X3 matrix of the form (45c). The details are

omitted.

So also, the corresponding formulae for QPL-SNS relations

of the type $\underline{a} \underline{\underline{E}}'(\underline{i}, j) = \underline{b}$ are obvious, and are not considered

in detail. It may be mentioned that more specialised relations

of the form as in (40) can be written down. For example, consider

the relation

If $\underline{a}$, then $\underline{b} = \bigvee$; if $\neg \underline{a}$, then $\underline{b} = \oint$ \hfill (47a)

This can be represented by $\underline{a} \underline{\underline{Z}} = \underline{b}$, with the QSN-matrix

$$\mid Z \mid = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \hfill (47b)$$

This was, in fact, used in the problem discussed in Section 7(c)

of Part II[2] . The general principles of examples of this type

are briefly discussed later in sections 4 and 5, including the analo

of the QSN-equivalence operator $\underline{\underline{E}}(i, j)$ in QPL, namely $\underline{\underline{E}}(i, j)$.

(d) QPL-1 relations

Since QPL-1 algebra has been extensively discussed in Parts

III and IV $\left[3, 4\right]$, only the essentials will be mentioned here,

so as to correlate it to QPL-2, SNS and QSN. To distinguish them

from QPL-2 relations, the QPL terms in QPL-1 relations are

indicated by the symbols $\mathcal{Q}$, b, etc., and the relational

operators are indicated by $\mathbb{Z}$, A, O etc. In terms of these,

the "formal" relations can be expressed in the form of Eqns (1),

(2), (3), t replacing $\underline{t}$ and $\mathbb{Z}$ replacing $\underline{Z}$, as in QPL-2; but, for

practical application, the relations obey an algebra that is

quite different.

Thus, a term $\mathcal{Q}$ is written in terms of the quantifier as

$(\mathcal{G}/x)$ $(\underline{a}(x))$, simplified to $\mathcal{G}/(\underline{a})$. We shall call it the standard

form, to distinguish it from the canonical form $\underline{\underline{a}}$ of $\mathcal{Q}$. The

two forms are interconvertible, using the canonizer for going

from $\mathcal{Q}$ to $\underline{a}$, and the standardizer for going from $\underline{\underline{a}}$ to $\mathcal{Q}$.

(The standard form used in the general theory of this Part. V, following the treatment in Part III, is slightly different from that used in Part II; but there will be no difficulty in redefining the standardizer in Table 7 of Part II, using the notation of Parts III and IV. The essential difference is that the four standard states $(\mathcal{Q})$ of QPL are taken to be those in (17a,b,c,d), and their equivalent canonical forms $\underline{\underline{a}}$ are also given in (17)).

In the unary and binary relations that are special to QPL-1, the relational operator $\mathbb{Z}$ has two parts $\mathcal{Q}$ and $\underset{\sim}{Z}$ and may be represented formally as $\mathbb{Z} = (\mathcal{Q}, \underset{\sim}{Z})$. Then, we give below the format of the three forms of relations as in Eqs (1), (2), (3):

Unary

$$\mathcal{Q}_a(\underline{\underline{a}}'), \quad \mathcal{Q}(\underline{\underline{a}} \underset{\sim}{Z} = \underline{\underline{b}}) \longmapsto \mathcal{Q}_b(\underline{\underline{b}}') \tag{48a}$$

has the solution

$$\underline{\underline{a}}' \underset{\sim}{Z} = \underline{\underline{b}}' \; , \; (\mathcal{Q} \otimes \mathcal{Q}_a) \otimes \mathcal{Q}_Z = \mathcal{Q}_b \tag{48b,c}$$

The new Boolean connective "into" ($\otimes$) is defined in Section 3(b)

of Part IV, and $\mathcal{Q}_Z$ is also defined therein. However, for

ready reference, the table for "into", as operative in QPL-1,

is given below in (49). The operation $\circledast$ is both cummutative

and associative.

$$
\begin{aligned}
&\forall \otimes \forall = \forall \; ; \quad \forall \otimes \exists = \exists \; ; \quad \forall \otimes \triangle = \triangle \; ; \\
&\exists \otimes \exists = \triangle \; ; \quad \exists \otimes \triangle = \triangle \; ; \quad \triangle \otimes \triangle = \triangle \; .
\end{aligned}
\Bigg\} \tag{49}
$$

Binary forward

$$
\mathcal{Q}_a(\underline{a}'), \quad \mathcal{Q}_b(\underline{b}'), \quad \mathcal{Q}(\underline{a} \, \underline{Z} \, \underline{b} = \underline{c}) \; \longmapsto \; \mathcal{Q}_c(\underline{c}') \tag{50a}
$$

has the solution

$$
\underline{a}' \, \underline{Z} \, \underline{b}' = \underline{c}' \; , \quad \text{and} \quad f \, (\, \mathcal{Q}_a, \, \mathcal{Q}_b, \, \mathcal{Q}_c, \, \underline{z}) = \mathcal{Q}_c \tag{50b,c}
$$

(See Part IV for the nature of the function f in (50c).) The

above is for a matrix connective $\underline{Z}$ in $\mathbb{Z} = (\mathcal{Q}, \, z)$. For $\mathbb{Z} = \sqcup$

and $\sqcup\!\!\!\vee$, see below, after (51).

Binary reverse

As given in Part IV, this has the formulae:

$$
\mathcal{Q}_c(\underline{c}'), \quad \mathcal{Q}_a(\underline{a}') \; , \quad \mathcal{Q}(\underline{c} \, \overset{\leftarrow}{Z} \, \underline{a} = \underline{b}) \; \longmapsto \; \mathcal{Q}_b(\underline{b}')
$$

where

$$
\underline{a}' \, \underline{Z}\!/\!\underline{b}' \; \text{if} \; \underline{c}' = T; \quad \underline{a}' \, \underline{Z}^c = \underline{b}' \; \text{if} \; \underline{c}' = F \tag{51 a,b}
$$

and

$$(\mathcal{O}\!\!/ \otimes \mathcal{O}\!\!/_c \otimes \mathcal{O}\!\!/_a) \oplus (\ \mathcal{O}\!\!/_Z \ \text{or} \ \mathcal{O}\!\!/_Z c) = \mathcal{O}\!\!/_b, \ \text{for} \ \underline{c} = T \ \text{or} \ F \quad (51c,$$

However, in the spirit of Eq.(3) of converting the binary

reverse into a unary relation, we may rewrite (51) below, as in

(52).

$$\mathcal{O}\!\!/_c(\underline{c}'), \ \mathcal{O}\!\!/(\underline{c} \overset{\leftarrow}{Z} \underline{a} = \underline{b}) \longmapsto \mathcal{O}\!\!/'(\underline{a} \underset{\sim}{Z} = \underline{c}), \ \text{if} \ \underline{c} = T; \qquad (52a)$$

$$\longmapsto \mathcal{O}\!\!/'(\underline{a} \underset{\sim}{Z}^c = \underline{c}) \ \text{if} \ \underline{c} = F; \qquad (52b)$$

and, in both cases,

$$\mathcal{O}\!\!/ \otimes \mathcal{O}\!\!/_c = \mathcal{O}\!\!/' \qquad (52c)$$

The formulae for $Z = \sqcup$ and $\vee$ are identical with Eqs.(11a,b)

for QPL-2 relations, and the relations take the form

$$a \sqcup b = \underline{a} \oplus \underline{b} \ ; \ a \vee b = \underline{a} \otimes \underline{b} \qquad (53a,b)$$

Thus, if a relation $\sqcup$ or $\vee$ is encountered in a QPL-1 argument,

then the QPL-1 input terms involved in it are converted into

their canonical forms and (53a or b) is applied to get the output

in the canonical form, which is then converted into the standard

form, for further QPL-1 manipulations. To avoid these complicated

manipulations, short tables for $\bigcup (\oplus)$ and $\bigvee (\otimes)$ for QPL-1

terms (analogous to (49) for $\oplus\hspace{-0.3em}\otimes$) are given below. (Both are

commutative and associative).

$$\forall \oplus \forall = \forall ; \quad \forall \oplus \exists = \exists ; \quad \forall \oplus \triangle = \triangle \left.\begin{array}{c}\\\end{array}\right\}$$
$$\exists \oplus \exists = \exists ; \quad \exists \oplus \triangle = \triangle ; \quad \triangle \oplus \triangle = \triangle \left.\begin{array}{c}\\\end{array}\right\} \qquad (54a)$$

$$\forall \otimes \forall = \forall ; \quad \forall \otimes \exists = \forall ; \quad \forall \otimes \triangle = \forall \left.\begin{array}{c}\\\end{array}\right\}$$
$$\exists \otimes \exists = \exists ; \quad \exists \otimes \triangle = \exists ; \quad \triangle \otimes \triangle = \triangle \left.\begin{array}{c}\\\end{array}\right\} \qquad (54b)$$

So also, if a QPL-2 output is used as QPL-1 input, or

vice versa, the change-over occurs only at a term ($\underline{t}$), and

hence it can be carried out using the transformation of the

standardizer or canonizer for QPL terms.

(i) Use of "inverse" relations, in general

As mentioned in the beginning of this Section 2, equivalent

inverse relations (1'), (2'), (3') can be written for the corresponding

original relations (1), (2), (3). In each case, the relation

from $\underline{a}$ to $\underline{b}$ is inverted so that it is from $\underline{b}$ to $\underline{a}$. This finds

the greatest application for ~~the compoun~~ during of two QPL-1 relations

in sequence; but its beauty is best appreciated in SNS (and in

QPL-2 and QSN). In all the~~se~~ (latter three) ~~cases,~~ the vector-matrix formulae

that occur are identical, and we shall first write them for

SNS elementary relations. Thus Eqs (1) and (1') have their

representations as in (55a,b), and they are equivalent (see Theory

of Relations in Part I, Section 2(a) — the word "reverse" used

therein is to be replaced by "inverse").

$$(\ \langle a | \ z \ | = \ \langle b | \ ) \ \equiv \ (\ \langle b \ | \ z^t | \ = \ \langle a | \ ) \qquad (55a,b)$$

As a simple example, we have

$$(\underline{a} \Rightarrow \underline{b}) \equiv (\neg \underline{b} \Rightarrow \neg \underline{a}) \qquad (56a)$$

or $\quad (\underline{a} \ \underset{\sim}{I} = \underline{b}) \equiv (\underline{b} \ \underset{\sim}{J} = \underline{a}), \quad \underset{\sim}{J} = I^t \equiv \underset{\sim}{N} \ \underset{\sim}{I} \ \underset{\sim}{N} \qquad (56b,c)$

The counterparts for (2) and (2') are, for this example,

$$(\underline{a} \ \underset{\sim}{I} \ \underline{b}) \equiv (\underline{b} \ \underset{\sim}{J} \ \underline{a}) = \underline{c} \qquad (57)$$

whose matrix representations are

$$\langle a \mid I \mid b \rangle = c_\alpha \,, \quad \langle a \mid I^c \mid b \rangle = c_\beta \tag{58a}$$

$$\langle b \mid I^t \mid a \rangle = \langle b \mid J \mid a \rangle = c_\alpha \,, \quad \langle b \mid I^{tc} \mid a \rangle = \langle b \mid J^c \mid a \rangle = c_\beta \tag{58b}$$

The generalization of this follows from the identities in the

Theory of Relations [1] $\overline{\text{given}}^{|and \; et \; al/}$ in (59a,b), namely

$$\langle a \mid Z \mid b \rangle \equiv \langle b \mid Z^t \mid a \rangle \;\; ; \;\; \langle a \mid Z^c \mid b \rangle \equiv \langle b \mid Z^{tc} \mid a \rangle \tag{59a,b}$$

The inter-relation between (3a) and (3'a) is similar in that

they go over into the forms of the l.h.s and r.h.s. of (55a,b).

Hence, these are not discussed in detail.

The most interesting consequence of the Boolean vector-matrix

representation is that the equivalences in (55) and in (59a,b)

can be completely taken over for QPL-2 and QSN also. The only

differences are that the nature of the matrices is different in

each case, as in (60):

$$|Z| \text{ is } 2X2 \;\; ; \;\; |Z^t| \text{ is } 2X2 \quad \text{for SNS-SNS} \tag{60a}$$

$$|Z| \text{ is } 3X3 \;\; ; \;\; |Z^t| \text{ is } 3X3 \quad \text{for QPL-QPL} \tag{60b}$$

$|Z|$ is 2X3 ; $|Z^t|$ is 3X2   for SNS-QPL   (60c)

$|Z|$ is 3X2 : $|Z^t|$ is 2X3   for QPL-SNS   (60d)

An important consequence is that, for all these four types of

relations,

$$(\; |Z_1| \; Z_2| \; \ldots \; |Z_k| \;)^t = |Z_k^t| \; \ldots \; |Z_2^t| \; Z_1^t| \qquad (61)$$

leading to the fact that two unary relations can be combined

either in the direct or in the inverse forms; and they lead to

equivalent resultants (see Section 3 below).

(ii) Use of inverse relations in QPL-1

In general, the formula giving the equivalences of inverse

relations in QPL-1 is the same as mentioned above, namely,

$$\mathcal{Q}/(a \; Z = b) \equiv \mathcal{Q}/(b \; Z^t = a), \quad \text{for } Z = \text{A-type, O-type and E-type} \qquad (62)$$

However, there is an exception for I-type relations inside the

bracket that is quantified. This feature has been discussed

in great detail in Section 3(c)(ii) of Part III [3] , and so only

the results are given here. Thus, for $\underset{\sim}{I}$-type relations,

namely with $\underset{\sim}{Z} = \underset{\sim}{N}, \underset{\sim}{N}\underset{\sim}{I}, \underset{\sim}{I}\underset{\sim}{N}, \underset{\sim}{N}\underset{\sim}{I}\underset{\sim}{N}(= \underset{\sim}{J})$,

$$\forall (\underline{a} \underset{\sim}{I} = \underline{b}) \equiv \forall (\neg \underline{b} \underset{\sim}{I} = \neg \underline{a}) \tag{63a}$$

but

$$\exists (\underline{a} \underset{\sim}{I} = \underline{b}) \equiv \exists (\underline{b} \underset{\sim}{I} = \underline{a}) \tag{63b}$$

The latter arises from the well-known relation "For some, if

$\underline{a}$ then $\underline{b}$" is to be symbolized as $(\exists x) (\underline{a}(x) \& \underline{b}(x))$, (see

for example p. 50 of the book [6] by Suppes). Now, since

$\underline{a} \& \underline{b} \equiv \underline{b} \& \underline{a}$,

$$(\exists x)(\underline{a}(x) \& \underline{b}(x)) \equiv (\exists x)(\underline{b}(x) \& \underline{a}(x)) \tag{64a}$$

Hence

"For some, if $\underline{a}$ then $\underline{b}$" $\equiv$ "For some if $\underline{b}$, then $\underline{a}$" (64b)

In fact, Eq (64b) can be deduced also straightaway from the Venn

diagram showing the relation between the sets $\underline{a}(x)$ and $\underline{b}(x)$, as

shown in Fig.2 of Part III. Hence, symbolically, we can write

(64b) in the form of (63b), and we obtain the important fact that

the inverting of the relation from $\underline{a}$ to $\underline{b}$ in (63a) and (63b) have

different algebraic forms. In particular, (63a) obeys the

general rule (62), since $\underset{\sim}{N}\,\underset{\sim}{I}\,\underset{\sim}{N}$ which occurs on the r.h.s. of it

is the same as $\underset{\sim}{J} \equiv \underset{\sim}{I}^t$. However, (63b) is quite different.

Consequently, the compounding of two QPL-1 implications reqires

new rules, as discussed in great detail in Part III and briefly

again in Section 4(f)(i) of Part IV. The essentially new result

that emerges is that if two unary implications in QPL-1 have

quantifiers $\exists$ and $\forall$, or $\forall$ and $\exists$, respectively, they can be

combined to lead to a single implication, as in (65) below

(see also Table 5 of Part IV):

$$\exists(\underline{a}\,\underset{\sim}{I} = \underline{b}),\quad \forall(\underline{b}\,\underset{\sim}{I} = \underline{c}) \ \equiv\ \exists(\underline{a}\,I = \underline{c}) \tag{65a}$$

$$\forall(\underline{a}\,\underset{\sim}{I} = \neg\,\underline{b}),\quad \exists(\underline{b}\,\underset{\sim}{I} = \underline{c})$$

$$\equiv \text{Inverse of } \Big(\exists(\underline{c}\,\underset{\sim}{I} = \underline{b}),\quad \forall(\underline{b}\,\underset{\sim}{I} = \neg\underline{a})\Big)$$

$$\equiv \text{Inverse of } \exists(\underline{c}\,\underset{\sim}{I} = \neg\,\underline{a}) \equiv \exists(\neg\,\underline{a}\,\underset{\sim}{I} = \underline{c}) \tag{65b}$$

These are the categorical syllogisms <u>Darii</u> and <u>Dimension</u>,
in mood 1, considered in Part III, of which the latter is not
contained in the classical list of 18 valid syllogisms, but is
one of the new ones derived by us in that report.

3. Implementation of the steps of an argument for elementary statements of different types.

We shall show in Section 4 that the different steps of the Flow Chart in Fig.9 of Part IV [4] (for arguments of QPL-1) are implementable for SNS, QPL-1, QPL-2 and QSN statements. In order to facilitate the understanding of its capabilities, we shall consider in this Section 3, the main features of elementary statements, a pair of unary statements, of a chain of such statements forming a unary path, and of a unary bouquet, for all these four types.

(a) Implementation of binary forward statements

A binary statement connecting $\underline{a}$ and $\underline{b}$ with $\underline{c}$, namely

$$\underline{a} \; \underline{Z} \; \underline{b} = \underline{c} \tag{66}$$

is always implementable given the inputs $\underline{a}'$ and $\underline{b}'$, for all four types SNS, QPL-2, QSN and QPL-1. For the first three, the algebra is identical, as in (6) and (9) carried over to QSN, as

mentioned in Section 2(c). It follows that, in (66), the term

$\underline{a}$ can be any of the three types and $\underline{b}$ also can be one of the

three types, and it can be implemented, using the right mxn matrix.

For QPL-1 also, given $\underline{\underline{a}} \equiv \mathcal{O}_{\!a}(\underline{a})$ and $\underline{\underline{b}} \equiv \mathcal{O}_{\!b}(\underline{\underline{b}})$ (note

that QPL-1 and QPL-2 forms are interchangeable) and the relation

$\mathcal{O}(\underline{\underline{a}} \, Z \, \underline{\underline{b}})$, the state of $\underline{\underline{c}}$ in Eq.(66) can be worked out uniquely,

as in Section 3(c)(i) of Part IV . Hence, for all possible types,

the output $\underline{\underline{c}}$ of a binary forward statement can be worked out,

when we are given the inputs.

(b) Binary reverse and unary statements

The binary reverse form always leads to a unary relation,

as shown in Eqs.3(a) to (e), for all four types, and this creates

no problem, since the forward relations from which they are

derived are very similar. So also, for a single unary statement

of a general type, given the relation $\underline{a} \, Z = \underline{b}$ and the input

$\underline{a}'$, the output $\underline{b}'$ in $\underline{a}' \, Z = \underline{b}'$ is calculable from straightforward

rules. For SNS-SNS, QPL-2-QPL-2, SNS-QPL-2 and QPL-2-SNS

relations, the matrix relations have the same form, $\langle a' \mid Z \mid = \langle b' \mid$,

and create no problem. If the relation is of the QPL-1 type,

$\mathcal{Q}(\underline{a}\ \underline{Z} = \underline{b})$, then Eq.(48) is applicable, and here again, the

output $\mathcal{Q}_b(\underline{b}')$ follows, when we are given $\mathcal{Q}_a(\underline{a}')$ and the relation

$\mathcal{Q}(\underline{a}\ \underline{Z} = \underline{b})$. This is particularly so when $\underline{Z}$ is an $\underline{I}$-type or

$\underline{E}$-type connective operator. However when $\underline{Z}$ is one of the four

$\underline{A}$-type connectives in QPL-1, we have to go back to the original

relation $\mathcal{Q}(\underline{a}\ \underline{Z}\ \underline{b} = T)$ from which it was derived, and the

$\underline{A}$-construction (as in Fig.6(b) of Part IV) has to be made for

implemeting it. For uniformity in the proof, the same $\underline{A}$-construction

is made for SNS, QPL-2 and QSN relations also.

Thus, we are left only with the implementation of unary

relations in sequence containing the $\underline{I}$-type and $\underline{E}$-type operators

in general. The general graph-theoretical structure of these unary

relations occurring in sequence, is a bouquet (see Section 4(f)(iv)

of Part IV). In such a case, some special considerations have

to be taken into account. Each unary path contained in a unary

bouquet (from input to a final output of the bouquet), as shown

in Fig.8 of Part IV, may have any number of relations of the

four types SNS, QPL-2, QSN and QPL-1, in any sequence. Since

the algebra of SNS, QPL-2 and QSN are identical (as seen above)

except for the nature of the matrix (i.e. 2X2, 3X3, 2X3 or 3X2),

we shall club these together, and call them as Type-2, and call QPL-1

statements as Type-1, for the discussion below in the next

subsection (c).

(c) Implementation of a unary path

Suppose we have a sequence of unary relations in a unary

of the nature given below in (67 a to g),

$$\underline{\underline{a}} \ \underline{\underline{Z1}} = \underline{\underline{b}} \tag{67a}$$

$$\underline{\underline{b}} \ \underline{\underline{Z2}} = \underline{\underline{c}} \tag{67b}$$

$$\underline{\underline{c}} \ \underline{\underline{Z3}} = \underline{\underline{d}} \tag{67c}$$

$$Q/1(\underline{\underline{d}} \ \underline{\underline{Z4}} \ \underline{\underline{e}} = \underline{\underline{f}}) \tag{67d}$$

$$\mathcal{Q}/2 \; (\underline{\underline{f}} \; \underline{\underline{Z5}} \; \underline{\underline{g}} \; = \; \underline{\underline{h}}) \tag{67e}$$

$$\underline{\underline{h}} \; \underline{\underline{Z6}} \; = \; \underline{\underline{k}} \tag{67f}$$

$$\underline{\underline{k}} \; \underline{\underline{Z7}} \; = \; \underline{\underline{\ell}} \tag{67g}$$

We consider first a practical example like this to illustrate

the problems involved and to discuss them. We shall give a

general procedure for all such cases, after discussing the

particular example.

In the sequence, putting in the SNS input $\underline{\underline{a}}'$ in the SNS-QPL

relation $\underline{\underline{Z1}}$, the QPL output $\underline{\underline{b}}'$ can be worked out from (67a);

which, on being put in the QPL-2 relation (67b), gives the QPL

output $\underline{\underline{c}}'$. This again, on being input into the QPL-SNS relation

(67c), gives the SNS output $\underline{\underline{d}}'$.

In the next QPL-1 equation, $\underline{\underline{d}}'$ which is an SNS term can be

put as an input and the output $\mathcal{Q}/1'(\underline{\underline{f}}') \equiv \underline{\underline{f}}'$ can be obtained

from (67d). (This can be done, either by inputting $\underline{\underline{d}}'$ inside the

bracket on the l.h.s. quantified by $\mathcal{Q}/1$, or as a QPL-2 input

$\underline{\underline{d}}$', by converting $\underline{d}$' to $\underline{\underline{d}}$' as in Eq.(29a to d), and then

converting $\underline{\underline{d}}$' of QPL-2 to $\mathcal{d}$ of QPL-1 using the standardizer.

Either procedure gives the same result, as can be checked out

in detail.) However, as discussed in Part IV, there is no

guarantee that this will go through the next QPL-1 equation in

(67e). For two unary QPL-1 relations in succession to be

implemented in general, they have to be <u>first</u> combined theoretically

to form a single QPL-1 relation, which is <u>then</u> implemented by

putting an input and taking out the output (as shown in Part IV).

Hence the correct procedure is not to input $\underline{\underline{d}}$' in (67d), but to

combine (67d) and (67e) to get one equation — namely

$$\mathcal{Q}/3 \; (\underline{\underline{d}} \; \underset{\sim}{Z8} \; \underline{g} \; = \; \underline{\underline{h}}) \tag{68a}$$

In this, if we input $\underline{\underline{d}}$', or the corresponding $\underline{\underline{d}}$' , then we get

the output

$$\mathcal{Q}/3'(\underline{\underline{h}}') \equiv \underline{\underline{h}}' \tag{68b}$$

Thereafter $\underline{h}'$ can be input into the QSN relation (67f) to

obtain $\underline{k}'$, and then again be input to the SNS relation (67g)

to get finally $\underline{\ell}'$.

The above description may look very complicated, but the

essential principle to be grasped is very simple — namely that

the Type-1 unary relations cannot be implemented one after the

other, and may lead to the indefinite state, even though the

combined statement corresponding to two statements in succession

may lead to a definite state by theory. Therefore, the Type-1

statements should be combined <u>theoretically</u> if they occur in

succession (2, 3 or any number of relations in continuous

succession) and <u>the equivalent single QPL-1 relation thereby</u>

<u>obtained should be put in place of the number of successive</u>

<u>QPL-1 relations</u>.

On the other hand, QFL-2, SNS, QSN relations can be implemented

one after the other by putting in the input and finding out the

output for one relation after the other. Therefore, the right

procedure is first to examine the unary statements in the unary

path, and check whether there occur two or more successive QPL-1

statements at any place. If so, they are first theoretically

combined to make them into a single QPL-1 statement. We then

replace the two or more successive statements of Type-1 by a

single statement of Type-1. Thereafter, all the statements in

the path can be implemented one after the other, by putting in

the input state of each statement (starting from the first) and

putting the output of this as input to the next successive

statement, and carrying out this procedure in a serial order

upto the end of the particular unary path.

Therefore, in summary, the only caution that is necessary

in the implementation of a unary path is to note that the obvious

method, of putting in the input in a relation and taking the

output and fitting it to the next relation, is valid only for

Type-2 unary statements, and not for Type-1 unary statements.

Therefore, Type-1 statements in succession must be combined first

using the theoretical procedure given in Part IV, Section 4(f)(i)

(which may involve inverting them as in (65)), and the equivalent

single QPL-1 statement obtained thereby substituted in the

sequence, before the path is implemented in a practical example,

by putting in the actual state of the input of the first statement.

(d) Review of subsections (a), (b) and (c) and implementation of
    unary bouquets.

Thus, we have obtained the procedure for implementing a

binary forward statement (which can always be done by putting in

the inputs and taking out the output) for all four types, SNS,

QPL-2, QSN and QPL-1. We have also seen that binary reverse

statements can be converted to unary statements, in general.

While doing this conversion, if the term $\underline{c}$ in $\underline{c} \overleftarrow{Z} \underline{a} = \underline{b}$ is not

an original input, then a procedure analogous to the $\underline{\underline{c}}$-reconstruction

in Fig.6 of Part IV will have to be made. On doing this, we obtain

a unary statement from the binary reverse statement, which may be

of the E-type, A-type or I-type. Of these, the A-type is fed in

as a reconstructed binary forward A-relation and hence only the

E-type and I-type unary relations occur along unary paths in the

form of bouquets.

For an indication of the nature of the problem that arises

in implementing a unary bouquet, we shall take the example given

in Fig.8(a) and (b) of Part IV. In this, all the unary paths

originate from the same term t_1 and terminate in t_4, t_6, t_7, t_8

and t_9. As mentioned in Part IV, the path from t_1 to each terminus

t_j is a unique one. (This is best noted by tracing the unary path

in the inverse direction from the terminus to the origin.) We

shall therefore show how each of these paths can be implemented

and the output t_j' obtained, when we are given the input t_1' for

the origin term t_1.

It must be mentioned that the procedure that we are indicating

is only one of several ways in which the process of implementing

the bouquets can be done. Our attempt is to give a theoretical

indication that the implementation can be carried out; but not

necessarily to indicate the most effective, or the least

time-consuming, process for a practical algorithm or computer

program. In other words, we only try to give an existence proof

that the unary bouquets can be worked out; but our method of

proof is also practical.

Along each of the paths from t_1 to t_j in a bouquet, there

can be unary relations of all different types, as in Eqs. (67a to g).

As mentioned in that connection, the Type-2 relations can all be

processed one after the other in a practical way, so that it is

only necessary to combine the Type-1 (QPL-1) relations in order

to make the path completely implementable. Hence the path is

divided into a sequence of Type-1 and Type-2 statements — say as below:

t_1— (Type-2 sequence) —(Type-1 sequence) —(Type-2 sequence)...

$$\text{(Type-1 sequence)} - t_j \qquad (69)$$

In these, each Type-1 sequence is examined and if it has more than one Type-1 elementary relation, then all the relations in the sequence are combined <u>theoretically</u> by the method indicated in Section 4(f) of Part IV and the single resultant Type-1 relation is written. This is done for each one of the Type-1 sequences. Thereafter, we will have a sequence from $\underline{t}_1$ to $\underline{t}_j$ in (69) in which every elementary relation is implementable from input $\underline{t}_1$ to the output term $\underline{t}_j$ in continuous sequence. Hence, given the input $\underline{t}_1'$ to the origin of the path, the final output $\underline{t}_j'$ along that particular path can be obtained. This can be repeated for each one of the terminal terms $\underline{t}_j$ (e.g. $\underline{t}_4$, $\underline{t}_6$, $\underline{t}_7$, $\underline{t}_8$ and $\underline{t}_9$ of Fig.8 of Part IV) of the bouquet. Hence the whole bouquet can be implemented from the given input state $\underline{t}_1'$ of the term $\underline{t}_1$.

4. Flow chart for the implementation of a general argument containing all four types of elementary statements.

In this section, we shall show that the flow chart shown in

Fig.9 of Part IV for an argument containing QPL-1 statements

can be generalized for an argument containing all four types

of statements —namely SKS, QPL-1, QPL-2 and QSN. In the last

Section 3, we have shown that binary forward statements are

implementable for all four types. We have also shown that unary

bouquets are implementable, under all conditions. Therefore,

it is only necessary to show that an argument in general can be

converted into a form in which it contains only these two types

of statements and that circular paths can be eliminated, multiple

inputs to a term can be converted to a form with only two inputs

at a time, contradictions checked for, and so on. We give below,

in .Fig. 1, the diagram of the flow chart for this in a form

similar to Fig.6 of Part IV. There are only minor differences

between the earlier flow chart for QPL-1 statements, and the

present one. However, in order to make the chart quite general,

terms and connective operators are given the general symbols

$\underline{t}$, $\underline{a}$, etc, and $\underline{Z}$, $\underline{A}$, etc in the steps A to I of the chart.

The application to each of the four types of statements is

indicated in the text that follows the flow chart under the

headings A to I.

Fig.1   Flow Chart for the Implementation of a General Argument
        in Logic (for one variable).

(a) Notes and comments on the flow chart

   A. In the flow chart we give only a procedure for working

out the outputs $t_{-j}^{(o)}$ and the additional outputs $t_{-j}^{(ao)}$ for a

given logical graph corresponding to given $t_{-j}^{(i)}$ and $t_{-j}^{(ai)}$ as in

H below. We do not consider the process of converting the logic

of an argument given in words into the symbolic form we need.

This is not quite straightforward and requires some technical

considerations. These are, however, omitted here and will be

                              (See, however, Section 4 for examples).
considered in a future report/ As mentioned above, this flow

chart only gives the procedure of working out a good logical

graph from the inputs that are provided. As indicated in the

introductory Section 1 and in the title of Fig.1, the QPL-1

Fig.1 Flow Chart for the Implementation of
a General Argument in Logic (for one variable)

($\underline{t}^{(i)}$ = input term, $\underline{t}^{(m)}$ = middle term, $\underline{t}^{(o)}$ = output term; $t^{(ai)}$, $t^{(am)}$, $t^{(ao)}$ = "additional" input, middle, output terms; $\underline{Z}$ = Connective operator; $\underline{V}$ = vidya operator in general (may be $\underline{V}$ or $\bigvee$) for consistency check.)

A. Write down the elementary steps of the argument as provided. Draw the corresponding logical graph, and from its structure, list the terms as $\underline{t}_j^{(i)}$, $\underline{t}_j^{(m)}$ and $\underline{t}_j^{(o)}$, and list all $\underline{Z}_j$. Call this List A.

$\downarrow$

B. Check for the occurrence of directed circuits. For each one present, make the "circuit removal construction" as in Fig.4(b) of Part IV for each circuit. Add the vertices corresponding to the new terms $\underline{t}^{(i)}$, $\underline{t}^{(m)}$, $\underline{t}^{(o)}$ and the new operators $\underline{Z}_j$ (of the type $\underline{V}$) to List A. Call the new list as List B.

$\downarrow$

C. Make the $\underline{V}$-construction for multiple inputs to a term $\underline{t}_j$, as in Eqs (49) and (50) of Part IV, and do this for all such terms. Add the additional $\underline{Z}_j$ (of the type $\underline{V}$) and the $\underline{t}_j^{(m)}$ and $\underline{t}_j^{(o)}$ so generated to List B. Call the new list as List C.

$\downarrow$

Fig.1 Flow Chart (Contd.)

D. List the binary reverse relations. If any of these does not have $\underline{c}$ as a $t^{(i)}$, carry out the $\underline{c}$-reconstruction (see text). Add, or replace the operators $\underline{Z}_j$ so produced and add the additional terms $\underline{t}^{(ai)}$, $\underline{t}^{(ao)}$ and also replace the relevant $\underline{t}^{(m)}$ in the list C. Call the new list as List D.

E. Input the states given for $\underline{t}^{(i)}$. For $\underline{t}^{(ai)}$ choose one set of possible inputs, using one of the basic states — T, F or $\forall, \Sigma, \Phi$. Call this as Set 1, and for the later trials Set 2, Set 3, and so on, for List D.

F. Make the modified $\underline{A}$-construction, as in Fig.6(b) of Part IV, for all unary relations of the $\underline{A}$-type. Add the $\underline{t}^{(am)}$ and $\underline{t}^{(ao)}$ so produced to list D. Add also the operators $\underline{Z}_j$ (of type $\underline{V}$) that are introduced, to List D. Call the new list as List F.

G. List out the unary paths from origin to each terminus in the unary bouquets and reduce the QPL-1 sequences to single QPL-1 relations. Eliminate the $\underline{t}_j^{(m)}$ and $\underline{Z}_j$ so removed and add the new $\underline{Z}_j$ so introduced to List F. Call this as List G. (Obtain thus the List of terms and operators of the "reduced graph", containing only binary forward relations and unary paths that are implementable sequentially.)

H. Implement the List G of the reduced graph, rearranging the sequence in the list if needed, and obtain the logical states of all outputs $\underline{t}^{(o)}$ and $\underline{t}^{(ao)}$. This is for Set 1 of the inputs $\underline{t}^{(i)}$ and $\underline{t}^{(ai)}$ as in E, for List D.

I. Repeat the steps E, F, G, H, for Set 2, Set 3,..., of $\underline{t}^{(ai)}$ and list the $\underline{t}^{(o)}$ and $\underline{t}^{(ao)}$ for each set, as desired.

statements are all supposed to be formulated for one variable x.

As indicated in Eq (35), a relation in QFL-2 of the type,

$$(\mathcal{Q}/x) \ (\underline{a}(x)) \ \& \ \mathcal{Q}/(x) \ (\underline{b}(x)) \implies (\mathcal{Q}/x) \ (\underline{c}(x)) \tag{69a}$$

has, in so far as its logic is concerned, the same format as for

$$(\mathcal{Q}/x) \ (\underline{a}(x)) \ \& \ \mathcal{Q}/(y) \ (\underline{b}(y)) \implies (\mathcal{Q}/z) \ (\underline{c}(z)) \tag{69b}$$

So also,

$$(\exists x) \ (\underline{a}(x)) \implies (\exists x) \ (\underline{b}(x)) \ ; \ (\exists x)(\underline{a}(x)) \implies (\exists y)(\underline{b}(y)) \tag{70a,b}$$

have also the same formulae as far as the matrix logic is concerned.

Thus, many of the equations in the argument could also be

implemented for functions (terms) having the same, or different

variables, but with each being only a function of one variable.

Thus, in this report, we only assure that all the theory that is

formulated is valid for terms which are functions of a single

variable for QFL statements, in association with any number of

SNS statements having no variables attached to them. The theory

of this report may be considered to be restricted to logical

terms which are functions of one variable.

b. As mentioned in connection with the flow chart in

Part IV, we should mention that the exact position where the

break in a directed circuit is made, has to be specified

beforehand. We have not proved that the properties of a circuit

will remain the same when the check for consistency is made at

one term instead of another term in the path. This again is a

matter that has to be studied further (see, however, Section 4(b)).

C. In removing multiple inputs to a term $\underline{t}_j$ and replacing

them by a series of binary inputs and checking them for consistency,

we follow the same procedure as in Section 4(c) of Part IV, where

this was described for QPL terms, using the QPL vidya operator $\mathbb{W}$

It is obvious that, if the term $\underline{t}_j$ is an SNS term, the same

equations will hold good except that the consistency check

operator will be the SNS vidya operator, namely $\underset{\approx}{V}$.

D. The $\underline{c}$-reconstruction process is quite similar for all types of relations and we shall write them for three particular equations that require this process, to illustrate its variations. The reconstruction in the logical graph is similar in all cases.

SNS:

$$\underline{c} \overset{\leftarrow}{\underset{\sim}{Z}} \underline{a} = \underline{b} \longmapsto \underline{a} \underset{\sim}{Z} = \underline{b} \text{ for } \underline{c}' = T; \longmapsto \underline{a} \underset{\sim}{Z}^c = \underline{b} \text{ for } \underline{c}' = F \qquad (71a)$$

QPL-1:

$$\mathcal{Q}(\underline{c} \overset{\leftarrow}{\underset{\sim}{Z}} \underline{a} = \underline{b}) \longmapsto \mathcal{Q}'(\underline{a} \underset{\sim}{Z} = \underline{b}) \text{ for } \mathbb{C}' = \mathcal{Q}_c(T); \mathcal{Q}' = \mathcal{Q}_c \otimes \mathcal{Q}$$
$$\longmapsto \mathcal{Q}(\underline{a} \underset{\sim}{Z}^c = \underline{b}) \text{ for } \mathbb{C}' = \mathcal{Q}_c(F); \mathcal{Q}' = \mathcal{Q}_c \otimes \mathcal{Q} \qquad (71b)$$

QPL-2:

$$\underline{c} \overset{\leftarrow}{\underset{\sim}{Z}} \underline{a} = \underline{b} \longmapsto \underline{a} \underset{\approx}{Z} = \underline{b} \text{ for } \underline{c}' = T; \longmapsto \underline{a} \underset{\approx}{Z}^c = \underline{b} \text{ for } \underline{c}'' = F \qquad (71c)$$

For SNS-QPL and QPL-SNS, the equations are closely similar to (71a) and (71c), and are obvious.

E. Note that the input states $\underline{t}^{(ai)}$ need only be one or the other of the basic states, namely $T(1 \ 0)$ and $F(0 \ 1)$ for SNS, and $\bigvee(1 \ 0 \ 0)$, $\big899(0 \ 1 \ 0)$, $\bigphi(0 \ 0 \ 1)$ for QPL-2.

Similarly, for QPL-1, only the four QPL states namely $\forall(T)$,

$\forall(F)$, $\exists(T)$ and $\exists(F)$ need be tried. We will finally obtain

the list of outputs and additional outputs corresponding to all

of these. Only those for which there are no contradictions in the

outputs, or additional outputs need be considered; and from a list

of these $\underline{t}^{(co)}$ and $\underline{t}^{(o)}$, it would be possible to work out which

sets of outputs are permissible for a given argument. It has to

be noted that for a given set of $\underline{t}^{(i)}$, there may be more than

one set of $\underline{t}^{(o)}$ that are permissible depending on the choice of

$\underline{t}^{(ai)}$, which becomes necessary because of the procedures described

in B, C and D. (See below under H).

F. No special notes need be made except that the added

operators of type $\underline{V}$ may be either $\underline{V}$ or $\underset{\approx}{V}$, depending upon the

nature of the term which is involved in the $\underline{A}$-construction.

G. The reduced graph is always implementable by the method

of proof that was given in Part IV. However, in practice, if

necessary, the sequence of the listing of elementary relations

may have to be rearranged so that the implementation can go on

in continuous sequence from one equation to the next. The nature

of a practical algorithm for this purpose is indicated in

Section 4(c) by taking an example and the principles involved

in generalizing it is indicated there.

H and I. The rearrangement of the list has been commented

upon under G. The way of interpreting the outputs and additional

outputs has also been commented upon under E. The only new comment

that is needed is the following: We first implement List G for

Set 1 chosen under the step E, and the outputs are printed, along

with the additional outputs. The procedure from E to H is repeated

for Set 2, Set 3 and so on and the list of $\underline{t}^{(ai)}$'s that were

chosen, and the corresponding $\underline{t}^{(o)}$'s and $\underline{t}^{(ao)}$'s, are printed

in a tabular form. We shall indicate in Section 4(c) how such

a table can be interpreted for the nature of the outputs of an

argument, for the given set of inputs.

(b) Setting up of the flow chart of an argument

As mentioned in the introduction and in the comments

relating to A of the flow chart, the flow chart only assures

that given a set of elementary statements forming an argument

or the logical graph of the argument, we can always implement

these statements so as to obtain the states of the outputs in

the flow chart, given the input states into the argument.

however, there are some problems involved in writing down the

set of elementary statements that describe the logic of a given

argument in the most efficient manner. We shall illustrate this

by a simple example of an argument consisting of two unary

statements, namely

$$\underline{a} \Longrightarrow \underline{b} \ , \quad \neg \underline{c} \Longrightarrow \neg \underline{b} \qquad (72a,b)$$

The logical graph of this is shown in Fig. 2(a).

Fig.2   Logical graph of a simple argument consisting of two steps
   (a) As given in Eq (72 a & b).
   (b) The same after V-construction to remove multiple inputs.
   (c) The same, after inverting the second Eq (72b) to
       obtain the pair of Eqs (73a, b).
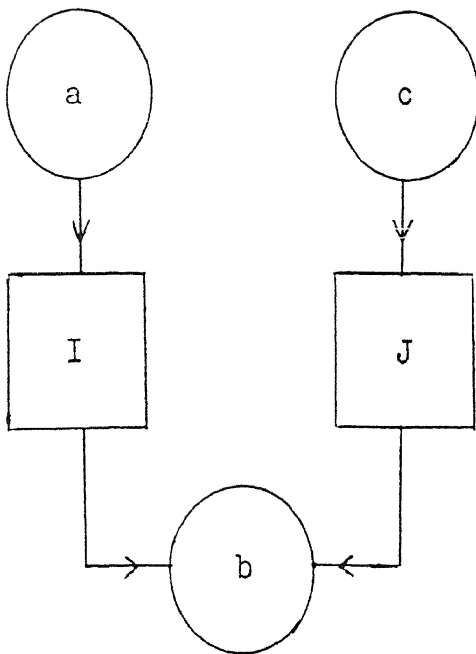
(a) As given in Eq (72 a, b)
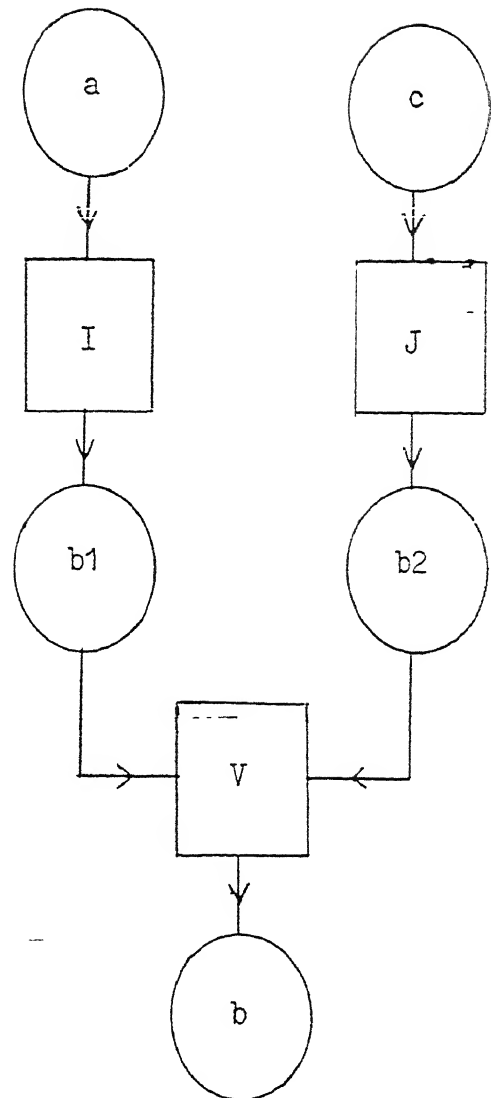
(b) The same after V-construction to remove multiple inputs
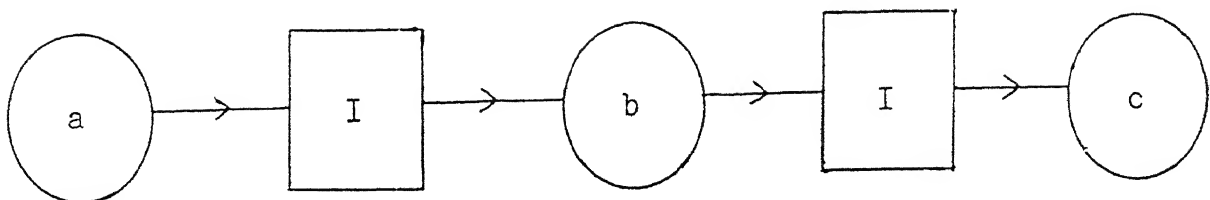
(c) The same, after inverting the second Eq (72b) to obtain the pair of Eqs (73a, b).

Since there are two inputs to the term $\underset{\sim}{b}$, under the

instructions in Step C of the flow chart, a $\underset{\sim}{V}$-construction

will have to be made and we obtain the graph in Fig. 1(b).

Suppose that we are given the input $\underset{\sim}{a}$ = T , and we are

required to find the output state of $\underset{\sim}{c}$. We can do this by inputting

the states T and F for $\underset{\sim}{c}$ in Fig. 2(b) and finding out which

(^)
state lead to a non-contradictory state for $\underset{\sim}{b}$. This is shown

schematically in Table 1.

Table 1. Checking for non-contradictory inputs for $\underset{\sim}{c}$ in Fig.2(b).

On examining Table 1, it will be seen that $\underset{\sim}{c}$ = F leads

to the contradictory state X for $\underset{\sim}{b}$ and is hence disallowed.

Consequently, if only the states T and F are allowed for $\underset{\sim}{c}$ (as

mentioned in Step E of Flow Chart for the states to be tried)

it follows that $\underset{\sim}{c}$ can only have the state T. Thus, the logical

graph of Fig.2 leads to the result that $\underset{\sim}{c}$ can only be T..

Table 1. Checking for non-contradictory inputs for $\underline{c}$
in Fig. 2(b)

(a) For Equations (72a,b), with $\underline{a} = T \longmapsto \underline{b1} = T$			
$\underline{c}$	T	F	D
$\underline{b2}$	D	F	D
$\underline{b}^*$	T	X	T
Conclusion	$\underline{c}$ = F leads to $\underline{b}$ = X and is forbidden, so that $\underline{c}$ = T.		

(b) For Equations (74a,b) with $\underline{a} = T \longmapsto \underline{b1} = F$			
$\underline{c}$	T	F	D
$\underline{b2}$	D	F	D
$\underline{b}^*$	F	F	F
Conclusion	Both $\underline{c}$ = T and $\underline{c}$ = F are possible, so that $\underline{c}$ = D.		

* $\underline{b1} \underset{\sim}{V} \underline{b2} = \underline{b}$

However, in SMS logic, we can also have the input $\underline{c}$ = D in

Fig. 2(b), when we get the result shown in column 3 of Table 1.

We find that there is no contradiction and therefore the state D

for $\underline{c}$ is also permissible. However, this is reasonable, since

D = T $\oplus$ F, and since (72b) is permissible for $\underline{c}$ = T, it will

also give a non-contradictory state for $\underline{b}$, for $\underline{c}$ = T or F = D,

and nothing new is obtained from column 3.

Now, we shall re-write Eq. (72a,b) by inverting the second

equation, when we obtain the two logical equations

$$\underline{a} \Longrightarrow \underline{b} \ , \quad \underline{b} \Longrightarrow \underline{c} \tag{73a,b}$$

corresponding to the logical graph shown in Fig. 2(c). If the

input $\underline{a}$ = T is put in, then by the first implication $\underline{L}$, we

obtain $\underline{b}$ = T, and from it, by the second implication $\underline{L}$, we obtain

$\underline{c}$ = T. Thus, we find that, when the second equation (72b) is

inverted and worked out, then the state of the output $\underline{c}$ so

obtained (namely T) is the same as what was obtained from (72a,b),

$\underline{\underline{via}}$ the $\underset{\sim}{V}$-construction for consistency check. In other words,

the result obtained is the same, whether the second equation in

(72) and (73) is taken in the direct or inverted forms.

We shall now make a similar check for the pair of equations

in (74a,b) below, with the second equation being

inverted in (75b), but with the input to it being $\neg \underline{\underline{b}}$(or $\underline{\underline{b}}$ = F).

$$\underline{\underline{a}} \implies \neg \underline{\underline{b}} \; , \quad \neg \underline{\underline{c}} \implies \neg \underline{\underline{b}} \qquad\qquad (74a,b)$$

$$\underline{\underline{a}} \implies \neg \underline{\underline{b}} \; , \quad \underline{\underline{b}} \implies \underline{\underline{c}} \qquad\qquad (75a,b)$$

By making a $\underset{\sim}{V}$-construction as in Fig. 2(b), we get the results

shown in Table 1(b), for the possible inputs of $\underline{\underline{c}}$. In this case,

with $\underline{\underline{b}}1$ = F, both the pure states T and F of $\underline{\underline{c}}$ do not lead to a

contradiction and are therefore permissible for $\underline{\underline{c}}$. Hence, the

output for the pair of equations (74a,b) is:

$$\underline{\underline{c}} = T \underset{\sim}{U} F \equiv T \oplus F = D \qquad\qquad (74)$$

Obviously, as shown in the third row of Table 1(b), $\underline{\underline{c}}$ = D is

itself consistent for this pair of equations.

Now, if we consider the pair (75a,b), we again obtain

$\underline{\underline{c}}$ = D as follows (in agreement with (74)):

$$\underline{\underline{a}} = T, \quad \underline{\underline{a}} \underset{\sim}{\underline{I}} \underset{\sim}{\underline{N}} = \underline{\underline{b}} \longmapsto \underline{\underline{b}} = F; \quad \underline{\underline{b}} = F, \quad \underline{\underline{b}} \underset{\sim}{\underline{I}} = \underline{\underline{c}} \longmapsto \underline{\underline{c}} = D \qquad (75a,b)$$

Thus, the unary relation of SNS implication

$$\underline{\underline{b}}', \quad \underline{\underline{b}} \underset{\sim}{\underline{I}} = \underline{\underline{c}} \longmapsto \underline{\underline{c}}' \qquad (76)$$

leads to the same result for the state of $\underline{\underline{c}}'$, for both $\underline{\underline{b}}' = T(1 \ 0)$

and $F(0 \ 1)$, irrespective of the sense in which the relation is

worked out (from $\underline{\underline{b}}$ to $\underline{\underline{c}}$ in (76), or equivalently as $\underline{\underline{c}} \ I^t = \underline{\underline{b}}$,

from $\underline{\underline{c}}$ to $\underline{\underline{b}}$). We have proved this above in detail, for $\underline{\underline{b}}' = T$

or $\underline{\underline{b}}' = F$ and hence it is true also for $\underline{\underline{b}}' = D$.

In the same way, any unary $\underset{\sim}{\underline{I}}$-type relation in QPL-2, in the

forward direction, namely

$$\underline{\underline{b}}', \quad \underline{\underline{b}} \underset{\approx}{\underline{I}} = \underline{\underline{c}} \longmapsto \underline{\underline{c}}' \qquad (77)$$

will give the same result for $\underline{\underline{c}}'$ as the Eqs (78a,b,c) below, with

the relation written in the inverted form from $\underline{\underline{c}}$ to $\underline{\underline{b}}$:

$$\underline{\underline{b}}' = \underline{\underline{b1}}, \quad \underline{\underline{c}}' \underset{\approx}{\underline{I}}^t = \underline{\underline{b2}}, \quad \underline{\underline{b1}} \bigvee \underline{\underline{b2}} \neq \emptyset \qquad (78a,b,c)$$

We shall give an example and indicate the nature of a general

proof of this in Section 5(a), as well as its extension to QSN

relations.

The extension to the $\underline{E}$-type relations of the identity of

the output, when $\underline{b} \, \underline{E} = \underline{c}$ (or $\underline{b} \, \underline{N} = \underline{c}$) is applied in the forward,

or reverse, sense, is quite straightforward. Also, as indicated

in Section 5 ., the extension to $\underline{E}(i, j)$ and $\underline{E}^c(i, j)$ in QPL

and similar operators in QSN is readily proved. The $\underline{A}$-type

relations are always worked out only using the $\underline{A}$-construction

in unary relations (see Step F of flow chart), so that the above

proof is unnecessary in this case.

This study thus gives a confirmation of the general principle

proved in Part I [1] , namely that the inverse of a relation $\underline{Z}$,

with matrix $|Z|$, has the matrix $|Z^t|$, since the above treatment

in SNS for 2X2 matrices is readily generalizable to $m \times n$ matrices.

Therefore, if we have an argument in which a final output

(for example $\underline{x}$) is not the output of a statement such as $\underline{g}\ \underline{I} = \underline{x}$

or $\underline{g}\ \underline{A}\ \underline{b} = \underline{x}$, but is of the form $\underline{x}\ \underline{J} = \underline{y}$ or $\underline{x}\ \underline{A}\ \underline{b} = \underline{g}$, then

$\underline{x}$ is taken as an additional input $(\underline{t}_j^{(ai)})$, and the permissible

states of $\underline{x}$ are worked out, as indicated in the flow chart.

Putting in $\underline{x}' = T(1\ \ 0)$ and $F(0\ \ 1)$ for SNS terms, and $\underset{\equiv}{x}' =$

$\mathcal{Y}(1\ \ 0\ \ 0)$, $\lessgtr(0\ \ 1\ \ 0)$ and $\phi(0\ \ 0\ \ 1)$ for QPL terms, the

permissible pure states of $\underline{x}$ can be worked out as per the procedure

indicated in the flow chart. Then, the output state for $\underline{x}'$ is the

Boolean sum of the permissible pure states thus obtained, using

the logical operator $\underset{\approx}{U}$ or $|U$ as the case may be. (Note: The

adjective "pure" is used for the basic states of BA-2 or BA-3,

to distinguish them from "mixed" states, which are Boolean sums

of two or more pure states. This follows the terminology for

vectors representing pure and mixed states in quantum mechanics.)

The above considerations also give a justification of the

procedure stated in Step ⋅ of the Flow Chart for all additional

inputs $\underline{t}^{(ai)}$.

(c) Rearrangement of the logical equations of a reduced graph
to be sequentially implementable

We shall/illustrate the principles of writing down the
[*new* written above "shall"]

logical equations of an argument in sequential order, so as to

be implementable straightaway one after the other. For this,

we take an argument in SNS, as the principles of graph construc-

tion depend only on the unary or binary nature of each elementary

equation, and does not depend on whether it is in SNS, QPL or

QSN. We shall show how the reduced graph in Step F of the

Flow Chart is converted into such a set of implementable equations.

We have also chosen an argument without any binary reverse

equations, as this requires the examination of two possibilities,

namely $\underline{c} = \underline{T}$, and F, in $\underline{c} \overset{\leftarrow}{Z} \underline{a} = \underline{b}$. So also, the $\underline{A}$-construction

is avoided. (Later, we shall indicate how both these specialities

can be built in and worked out in an extension of this argument.)

The set of equations of this small argument, which has, however,

multiple inputs to a term, is given in (79). We then indicate

how two $\underline{V}$-constructions added to it, as in (80), make it into

a reduced graph. Thereafter, we shall describe the algorithmic

process of writing the logical equations for this reduced graph,

so as to be sequentially implementable.

(i) Simple example of an argument without circularity

The argument has the logical equations as in (79a to f):

$$\underline{a} \ \underline{A} \ \underline{b} = \underline{g} \quad (79a) \qquad ; \qquad \underline{z} \ \underline{NI} = \underline{g} \quad (79b)$$

$$\underline{c} \ \underline{I} = \underline{h} \quad (79c) \qquad ; \qquad \underline{x} \ \underline{J} = \underline{g} \quad (79d)$$

$$\underline{g} \ \underline{O} \ \underline{h} = \underline{k} \quad (79e) \qquad ; \qquad \underline{g} \ \underline{ON} \ \underline{k} = \underline{y} \quad (79f)$$

We are further given that $\underline{a}$, $\underline{b}$, $\underline{c}$ are the inputs, and have the

states T, F, T respectively, and we wish to find the states of

the outputs $\underline{x}$, $\underline{y}$, $\underline{z}$. Since there are multiple inputs to $\underline{g}$, we

rewrite the equations (79a), (79b) and (79d) with appropriate

vidya operators, as below:

$$\underline{a} \ \underline{A} \ \underline{b} = \underline{g1} \quad (80a) \qquad ; \qquad \underline{z} \ \underline{N} \ \underline{I} = \underline{g2} \quad (80b)$$

$$\underline{g1} \ \underline{V} \ \underline{g2} = \underline{g2}' \quad (80c) \qquad ; \qquad \underline{x} \ \underline{J} = \underline{g3} \quad (80d)$$

$$\underline{g3} \ \underline{V} \ \underline{g2}' = \underline{g} \quad (80e)$$

where (80a), (80b), (80d) replace (79a), (79b), (79d), and (80c)

and (80e) are added to the set of equations describing the

argument. We then obtain a "reduced graph" of the argument,

with only unary and binary forward operators, and this is shown

in Fig. 3. It is a directed graph, and is a tree.

Fig.3. Reduced graph of the argument composed of Eqs. (79),
modified as in (80).

The algorithm for writing the equations in sequence is as

follows:

We first examine the graph for inputs (i), additional

inputs (ai), final outputs (o), and middle terms (m), by the

following procedure: The input terms $\underline{t}^{(i)}$ and $\underline{t}^{(ai)}$ are those

that have no edges leading to them, and output terms $\underline{t}^{(o)}$ are

those that have no edges going out of them. The middle terms

have edges going into them and also coming out of them.

If this check is made on the graph in Fig.3 we obtain the

following list:
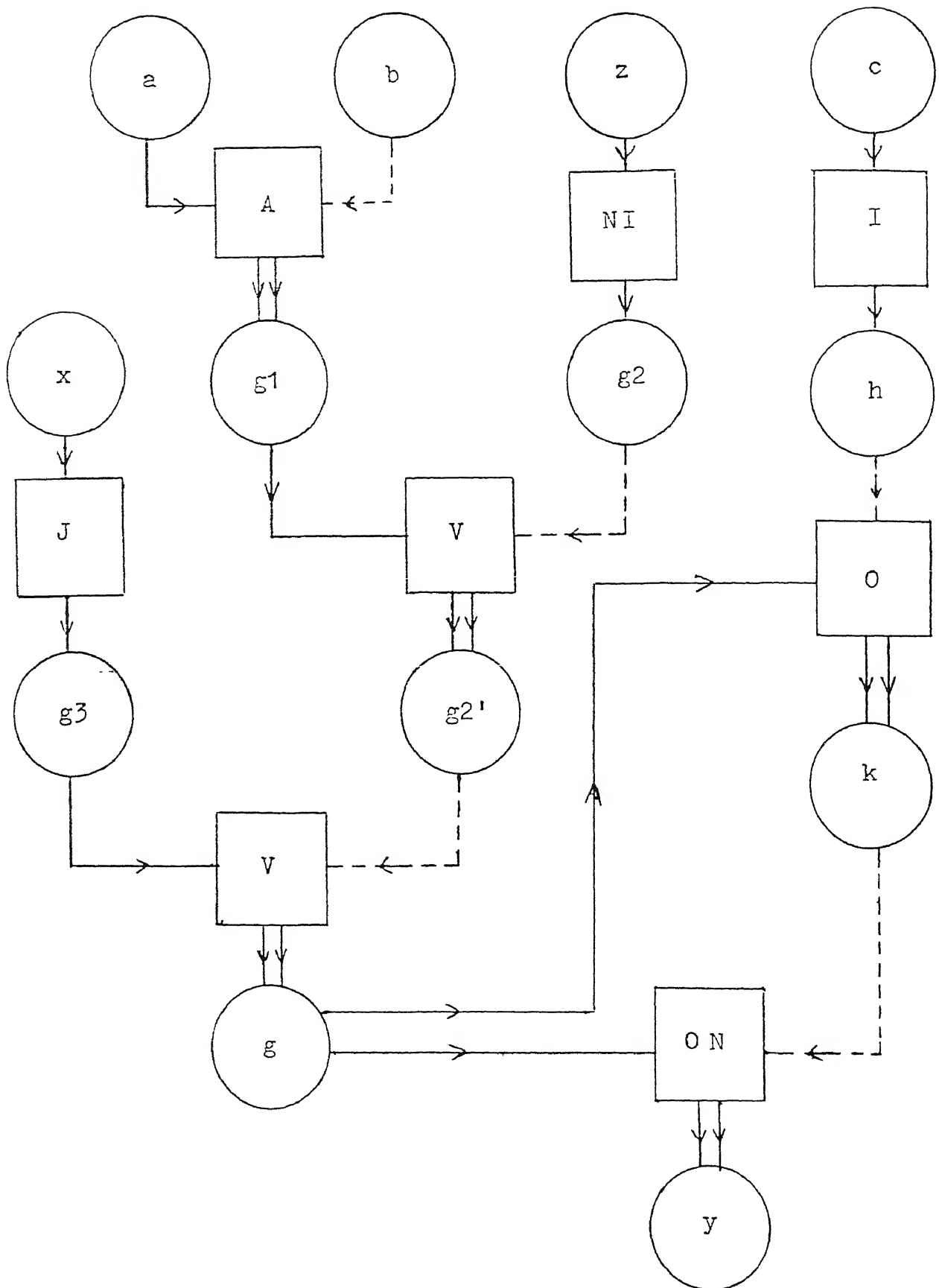
Fig.3 Reduced graph of the argument composed of Eqs. (79)
       modified as in (80).

$$\text{Inputs } (\underline{t}^{(i)}) \quad : \quad \underline{a}, \ \underline{b}, \ \underline{c}$$

$$\text{Additional inputs } (\underline{t}^{(ai)}) \quad : \quad \underline{x}, \ \underline{z}$$

$$\text{Outputs } (\underline{t}^{(o)}) \quad : \quad \underline{y}$$

$$\text{Middle terms } (\underline{t}^{(m)}) \quad : \quad \underline{g1}, \ \underline{g2}, \ \underline{g2}', \ \underline{g3}, \ \underline{g}, \ \underline{h}, \ \underline{k}$$

We then proceed to write the logical equations in sequential

order in a series of steps, numbered Step 1, Step 2, etc.

Step 1

Write down all the elementary equations with $\underline{t}^{(i)}$ and $\underline{t}^{(ai)}$

as inputs and write down the list of intermediate outputs of

Step 1 (which may be called $\underline{t}^{(1)}$).

$$\underline{a} \ \underline{A} \ \underline{b} = \underline{g1} \quad (81a) \quad ; \quad \underline{c} \ \underline{I} = \underline{h} \quad (81b)$$

$$\underline{x} \ \underline{J} = \underline{g3} \quad (81c) \quad ; \quad \underline{z} \ \underline{NI} = \underline{g2} \quad (81d)$$

List of $\underline{t}^{(1)}$ for Step 1 : $\underline{g1}, \ \underline{g2}, \ \underline{g3}, \ \underline{h}$.

Step 2

Take each $\underline{t}^{(1)}$ and write down the elementary equations

containing them as follows:

$$\underline{g1} \; \underline{V} \; \underline{g2} \; = \; \underline{g2}' \qquad (81e)$$

$$(\underline{g}) \; \underline{O} \; \underline{h} \; = \; --- \qquad (\text{Hold})$$

$$\underline{g3} \; \underline{V} \; (\underline{g2}') = \; --- \qquad (\text{Hold})$$

List of $\underline{t}^{(2)}$ for Step 2 : $\underline{g2}'$

It will be noticed that the terms enclosed by brackets in the equations tested for Step 2 are not available either as original or additional inputs, or as intermediate outputs $(\underline{t}^{(1)})$, of Step 1. Hence these equations are marked as "Hold" and are carried over to Step 3.

Step 3

Take the held over equations in the previous steps, as well as equations involving $\underline{t}^{(2)}$ and write them as below.

$$\underline{g3} \; \underline{V} \; \underline{g2}' \; = \; \underline{g} \qquad (81f)$$

$$(\underline{g}) \; \underline{O} \; \underline{h} \; = \; --- \qquad (\text{Hold})$$

List of $\underline{t}^{(3)}$ for Step 3 : $\underline{g}$

In the same way as in Step 2, one equation has to be put in "Hold" in Step 3 also, since one of the terms in the equation is not contained in the list of inputs, or of $\underline{t}^{(1)}$, $\underline{t}^{(2)}$, of the previous steps.

Step 4

Similar to Step 3, we obtain

$$\underline{\underline{g}} \ \underline{\underline{O}} \ \underline{\underline{h}} = \underline{\underline{k}} \qquad (81g)$$

$$\underline{\underline{g}} \ \underline{\underline{ON}} \ (\underline{\underline{k}}) = - \qquad (Hold)$$

List of $\underline{t}^{(4)}$ for Step 4 : $\underline{\underline{k}}$

Step 5

Doing the same as in Steps 3 and 4, we obtain

$$\underline{\underline{g}} \ \underline{\underline{ON}} \ \underline{\underline{k}} = \underline{\underline{y}} \qquad (81h)$$

List of $\underline{t}^{(5)}$ for Step 5 : $\underline{\underline{y}}$.

Thus, all the equations leading down to the final output

$\underline{\underline{y}}$ are obtained. The sequentially implementable equations are

given by (81a) to (81h) under the various steps discussed above.

It is veriried that the number of equations (namely 8) is equal

to the number of operators contained in the graph in Fig. 3.

Although the number of steps (namely 5) seems to be too

large for this simple problem, it becomes reasonable for larger

problems, and the principle of this algorithm is very general

and the process can be used for any logical graph containing

unary and binary elementary statements (not necessarily a

reduced graph but one without circular arguments). If an unary

bouquet connecting $\underline{a}$ say to $\underline{b}$, $\underline{c}$, ..., $\underline{k}$ occurs, then all

the unary paths in it originating from $\underline{a}$ are taken to be

implementable in a single "step" of the algorithmic process

described above.

The essence of the algorithm is that, at each step (say

Step n) only <u>the input terms</u> ($t^{(i)}$ and $t^{(ai)}$) and <u>the output</u>

<u>terms of the previous steps</u> ($t^{(1)}$, $t^{(2)}$,..., $t^{(n-1)}$) are

employed for testing the implementability of equations in that

Step n, consisting of the held over equations of the previous

step ($\underline{n}$-1), and elementary equations containing the outputs $\underline{t}^{(n-1)}$.

Hence, every elementary statement in every step is implementable

when the sequential processing of the statements reaches it.

(ii) <u>Example with circularity</u>, <u>c-reconstruction</u> and <u>A-construction</u>

The results of the implementation of the reduced graph in

Fig.3 and its interpretation for the states of the outputs $\underline{x}, \underline{y}, \underline{z}$

are discussed in the next Section 4(d). Here we shall now

consider an amplification of the graph of Fig. 3, in order to

illustrate the Step B (circuit removal), Step D ($\underline{c}$-reconstruction)

and Step F ($\underline{A}$-construction) of the Flow Chart. All these

processes become necessary on changing one equation, namely

(79b), as in (82) below.

$$\text{Replace (79b)} \quad \text{by} \quad \underline{y} \;\; \underset{\sim}{\underleftarrow{Q}}{}^{c} \;\; \underline{\underline{z}} \;=\; \underline{g} \qquad (82a)$$

$$\text{where} \qquad \underset{\sim}{Q}{}^{c} \;\equiv\; \underset{\sim\sim\sim}{NAN} \qquad (82b)$$

This brings in a circuit composed of the following terms and

operators:

$$-\underline{y}\,-\underset{\sim}{Q}{}^{c}\,-\underline{g}\,-\underset{\sim\sim}{ON}\,-\underline{y}\,- \qquad (83)$$

As stated in the explanation of Step B in Fig. 1, the exact point

at which the circuit is broken has to be stated, and we choose

it to be at the term $\underline{\underline{y}}$, and write the relevant equations as

$$\underline{\underline{y1}} \;\; \underset{\sim}{\underleftarrow{Q}}{}^{c} \;\; \underline{\underline{z}} \;=\; \underline{g} \quad (84a) \;\; ; \;\; \underline{g} \;\; \underset{\sim\sim}{ON} \;\; \underline{\underline{k}} \;=\; \underline{\underline{y2}} \qquad (84b)$$

$$\underline{\underline{y1}} \;\; \underset{\sim}{V} \;\; \underline{\underline{y2}} \;=\; \underline{\underline{y}} \qquad (84c)$$

We then remove the multiple inputs to $\underline{g}$ (as per Step C

of the Flow Chart) by introducing two vidya operators (exactly

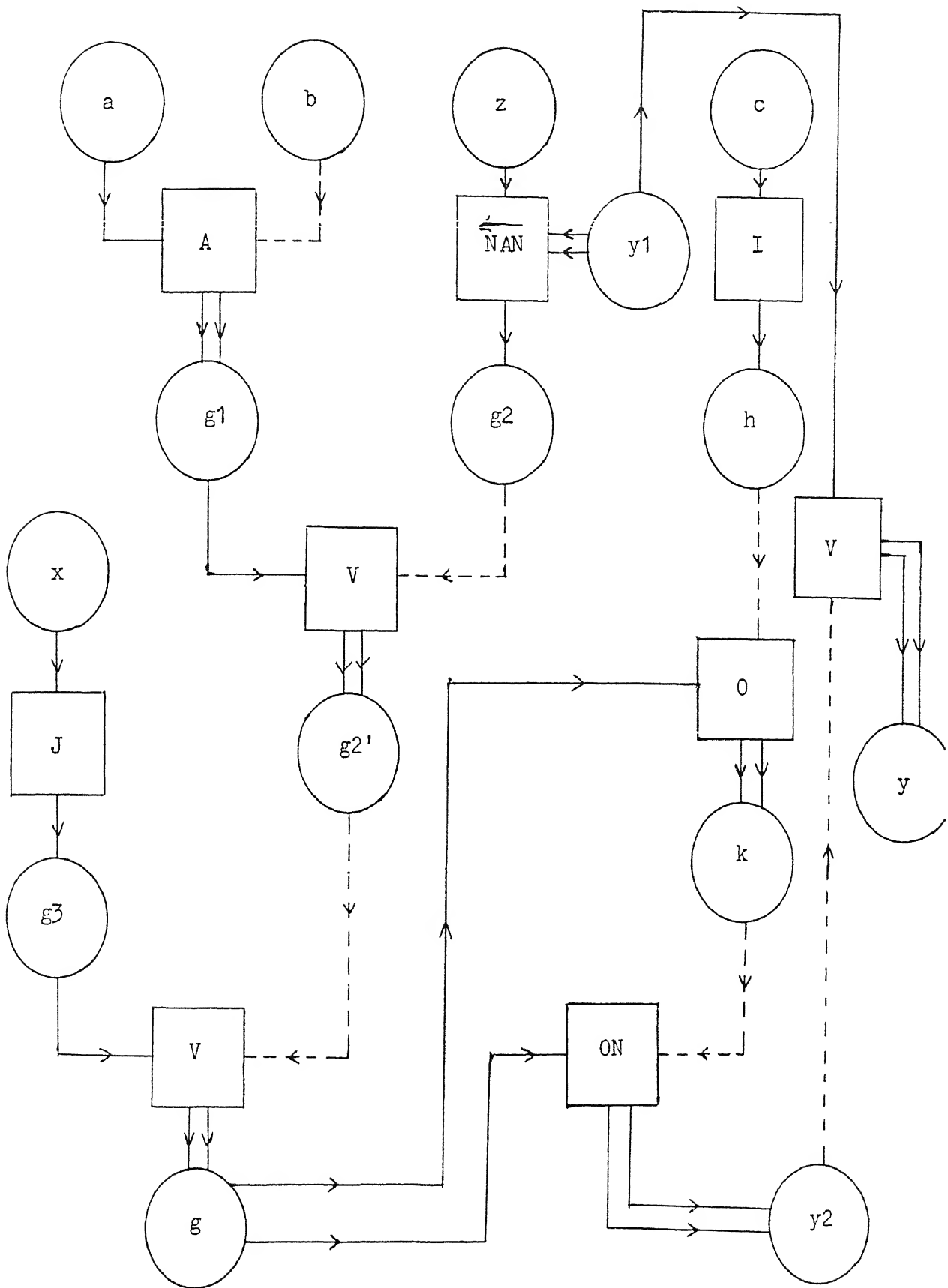as in (80c) and (80e)), and then obtain the graph shown in Fig.4(a).

Fig.4(a) Graph of the argument composed of Eqs. (79), (80)
and (82), after the "circuit removal construction" in
Step E, and the "V-construction" for removing multiple
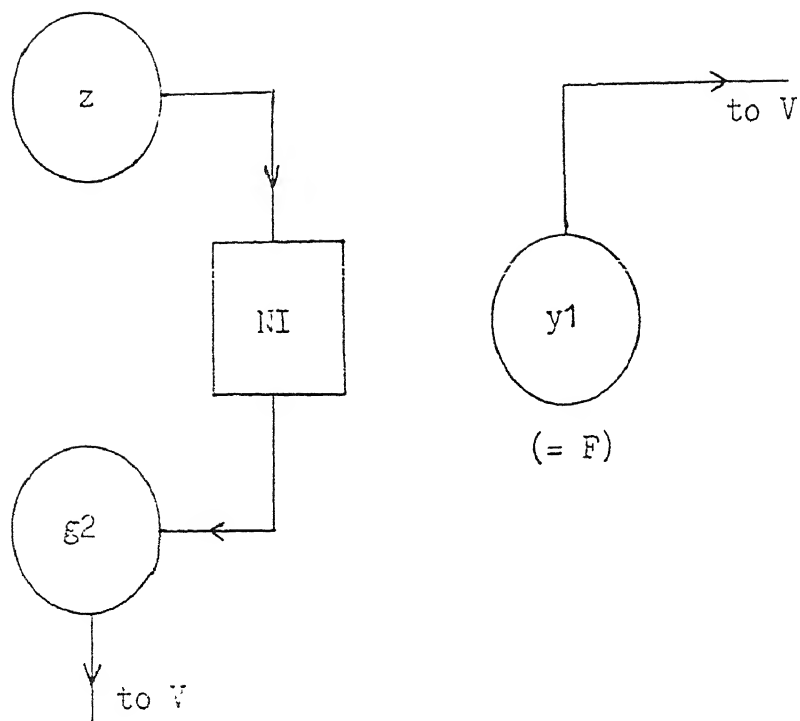inputs in Step C, of the Flow Chart.

(b) $\underline{c}$-reconstruction (Step D of Flow Ch rt) when $\underline{\underline{y1}}$ = F.
(See also Fig. 7(b) of Part IV.)

(c) $\underline{c}$-reconstruction (as in Step D of Flow Chart) when
$\underline{\underline{y1}}$ = T and A-construction for $\underset{\sim}{\text{NAN}}$ (as in Step F)
to $\underline{z1}$ = F and $\underline{\underline{g2}}$ = F. (See also Fig.6(b) of Part IV.)

It will be noticed that, in this graph, the circuit in (83) is

converted into a directed path from $\underline{\underline{y1}}$ to $\underline{\underline{y2}}$, namely

$$\underline{\underline{y1}} \;-\; \text{NAN} \;-\; \underline{\underline{g2}} \;-\; \underset{\sim}{V} \;-\; \underline{\underline{g2}}' \;-\; \underset{\sim}{V} \;-\; \underline{g} \;-\; \underset{\sim}{\text{ON}} \;-\; \underline{\underline{y2}} \;,\text{ and a}$$

consistency check $\underline{\underline{y1}} \; \underset{\sim}{V} \; \underline{\underline{y2}}$ = $\underline{y}$ between the origin $\underline{\underline{y1}}$ and

the terminus $\underline{\underline{y2}}$ of this path. Since the equations pertaining

to this graph are obtained by combining (79a-h), (80a-e, and

(84a-c), we shall rewrite them in (85) for ready reference:

Fig.4(a) Graph of the argument composed of Eqs.(79), and (82), after
the "circuit removal construction" in Step B, and the
"V-construction" for removing multiple inputs in Step C, of
the Flow Chart.

Fig.4(b). c-reconstruction (Step D of Flow Chart) when $\underline{y1}$ = F.
(See also Fig. 7(b) of Part IV.)
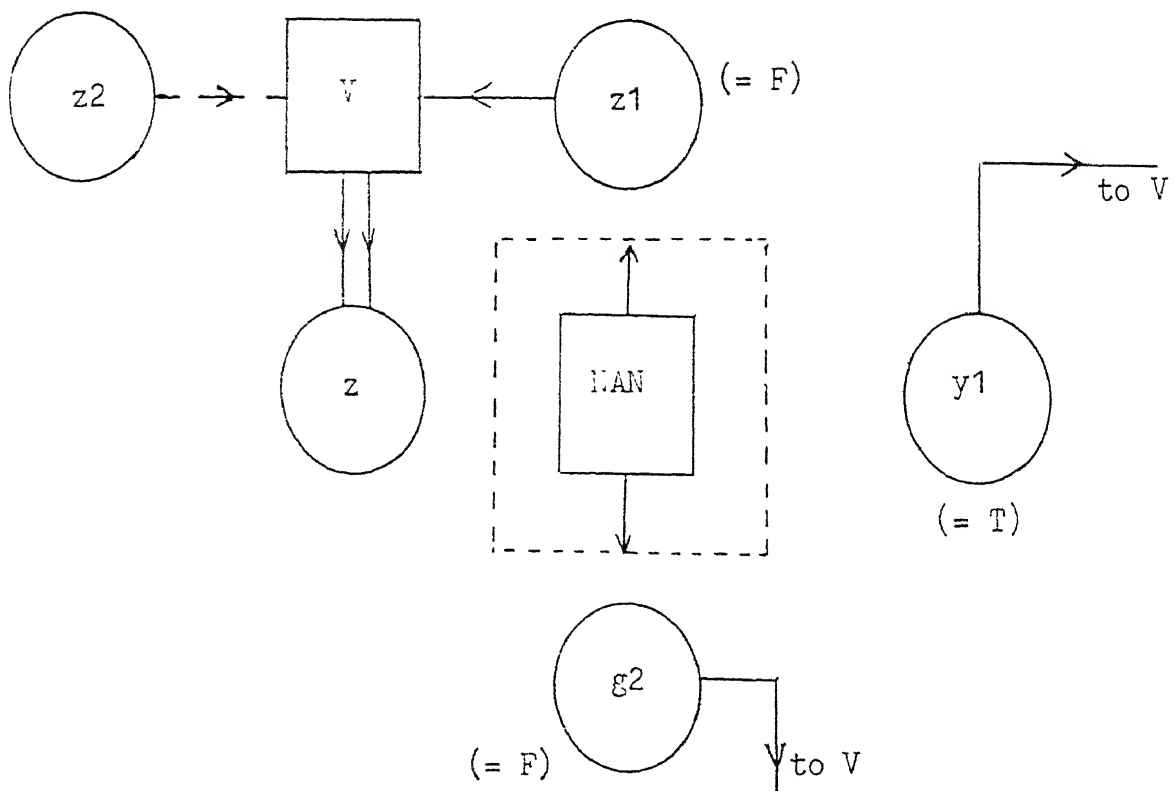
Fig.4(c). c-reconstruction (as in Step D of Flow Chart) when
$\underline{y1}$ = T and $\underline{A}$-construction for MAN (as in Step F) leading
to $\underline{z1}$ = F and $\underline{g2}$ = F. (See also Fig.6(b) of Part IV.)

$$\underline{a} \ \underset{\sim}{A} \ \underline{b} \ = \ \underline{\underline{g1}} \qquad (85a) \ ; \qquad \underline{\underline{c}} \ \underset{\sim}{I} \ = \ \underline{h} \qquad (85b)$$

$$\underline{\underline{y1}} \ \underset{\sim\sim\sim}{\overset{\leftarrow}{NAN}} \ \underline{z} \ = \ \underline{\underline{g2}} \qquad (85c) \ ; \qquad \underline{\underline{g1}} \ \underset{\sim}{V} \ \underline{\underline{g2}} \ = \ \underline{\underline{g2'}} \qquad (85d)$$

$$\underline{x} \ \underset{\sim}{J} \ = \ \underline{\underline{g3}} \qquad (85e) \ ; \qquad \underline{\underline{g3}} \ \underset{\sim}{V} \ \underline{\underline{g2'}} \ = \ \underline{\underline{g}} \qquad (85f)$$

$$\underline{\underline{g}} \ \underset{\sim}{O} \ \underline{h} \ = \ \underline{k} \qquad (85g) \ ; \qquad \underline{g} \ \underset{\sim\sim}{ON} \ \underline{k} \ = \ \underline{\underline{y2}} \qquad (85h)$$

$$\underline{\underline{y1}} \ \underset{\sim}{V} \ \underline{\underline{y2}} \ = \ \underline{y} \qquad (85i)$$

It is to be noted that the Eqns (85a-i) are those that are
obtained after performing Steps B and C of the Flow Chart in
Fig.1. Now, we perform the Steps D, E,F of the Flow Chart.

Step D: c-reconstruction of binary reverse relation

There is only one binary reverse relation in the argument
under study —namely (85c). If we put $\underline{\underline{y1}}$ = T in this, the
c-reconstruction yields the equations:

$$\underline{\underline{y1}} \ = \ T \qquad (86a) \ ; \qquad \underline{\underline{z}} \ \underset{\sim\sim\sim}{NAN} \ \underline{\underline{g2}} \ = \ T \qquad (86b)$$

If, on the other hand, we take the case of $\underline{\underline{y1}}$ = F, the
c-reconstruction yields the equations:

$$\underline{\underline{y1}} \ = \ F \qquad (87a) \ ; \qquad \underline{\underline{z}} \ \underset{\sim\sim}{NI} \ = \ \underline{\underline{g2}} \qquad (87b)$$

Of these, (o7b) corresponds to (80b) of the simple argument

discussed in Subsection 4(c)(i) above and therefore we consider

this first. It is illustrated in Fig. 4(b), where the change

from Fig. 4(a) produced by the $\underline{c}$-reconstruction is indicated.

(Note that the matrix $|NI| = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \equiv |0|$ and its complement

$|0^c| = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \equiv |N \mid A \mid N|$, an $\underline{A}$-type matrix with one 1 and

three zeros.)

If $\underline{y1}$ is taken to be T and the corresponding $\underline{c}$-reconstruction

is made, as in Eqn. (86b), then an A-construction has also to be

made. Both these together are shown in Fig. 4(c). Since the

$\underline{A}$-type operator involved is NAN, namely "nand", we have the

consequences $\underline{z} = F$ and $\underline{g2} = F$. Of these, $\underline{g2} = F$ is shown

straightaway in Fig. 4(c). We could also say that $\underline{z} = F$ is the

only possible state of $\underline{z}$, as it is an input $(\underline{t}^{(ai)})$, with no

edge leading ot it. However, the $\underline{A}$-construction is fully shown

in Fig. 4(c) to take care of the general case where $\underline{z}$ may be

a middle term $\underline{t}^{(m)}$. The relevant equations for $\underline{y1} = T$ in Fig.4(a)

are therefore (putting $z = z2$) :

$$y1 = T \quad (88a) \quad ; \quad z1 = F \quad (88b)$$

$$z1 \underset{\sim}{V} z2 = z \quad (88c) \quad ; \quad g2 = F \quad (88d)$$

In these, $z = X$ if $z2 = T$, so that we are led back to the

possibility of z(of (88c)) being only F, as in (86b), if $y1 = T$.

In the subsection 4(d)(iii), we shall show how Steps E and F

of the Flow Chart are implemented for the two graphs we have

obtained, namely those corresponding to Figs. 4(a) and (b), and

to Figs. 4(a) and (c) respectively. In the following subsection

4(d) (i), we consider the solution of the equations corresponding

to the simpler argument delineated in Fig. 3.

(d) Working out the outputs of logical graphs

(i) Solution of the reduced graph of the simpler problem.

As already mentioned, we are given that the original

inputs $t^{(i)}$ are $a = T$, $b = F$, $c = T$, in the reduced

graph (Fig. 3). We also have, as additional inputs

$t^{(ai)}$, the terms $\underline{x}$ and $\underline{z}$, which are sought to be determined.

Therefore, we solve Eqns (80a-f), with the assumed states T

and F for both $\underline{x}$ and $\underline{z}$ (four possibilities), and, in each case,

determine the state of $\underline{y}$. When this is done with Eqns. (80a)

to (80f), we obtain the results shown in Table 2.

Table 2: Solution of Equations (80a-h) pertaining to Fig.3

It will be seen from Table 2 that the combinations

($\underline{x}$ = T, $\underline{z}$ = F) and ($\underline{x}$ = F, $\underline{z}$ = F) as additional inputs, lead

to contradictions. Hence, the argument cannot lead to $\underline{z}$ = F.

Taking $\underline{z}$ = T, the "additional input" $\underline{x}$ may be T or F.

For both possibilities, there are no contradictions, and both lead

to $\underline{y}$ = F. Hence, $\underline{x}$ is indefinite (D), $\underline{y}$ = F, $\underline{z}$ = T for

this argument, with $\underline{a}$ = T, $\underline{b}$ = F, $\underline{c}$ = T. No definite conclusion —

namely whether x = T or F, can be deduced, with these inputs,

for this argument.

Table 2 : Solution of Equations (80a-h) pertaining to Fig.3

Inputs $\underline{t}^{(i)}$: $\underline{a}$ = T, $\underline{b}$ = F, $\underline{c}$ = T

$\underline{t}^{(ai)}$		$\underline{t}^{(1)}$				$\underline{t}^{(2)}$	$\underline{t}^{(3)}$	$\underline{t}^{(4)}$	$\underline{t}^{(5)}$
$\underline{x}$	$\underline{z}$	$\underline{g1}$	$\underline{g2}$	$\underline{g3}$	$\underline{h}$	$\underline{g2'}$	$\underline{g}$	$\underline{k}$	$\underline{y}$
T	T	F	D	D	T	F	F	T	F
T	F	F	T	D	F	X	X	X	X
F	T	F	D	F	T	F	F	T	F
F	F	F	T	F	D	X	X	X	X

In fact, this can be shown very simply by inverting the

two equations (79b) and (79d) and rewriting the six equations

in (79a) to (79f) as in (89a) to (89f) below. (Note that

$z \text{ NI} = g$ is written as $g \text{ NI} = z$ and $x \text{ J} = c$ is written

as $g \text{ I} = x$).

$$a \,\&\, b = g \quad ; \quad a = T, \, b = F, \text{ yields } g = F \tag{89a}$$

$$g \Rightarrow x \quad ; \quad g = F, \text{ yields } x = D \tag{89b}$$

$$\neg g \Rightarrow z \quad ; \quad g = F, \text{ yields } z = T \tag{89c}$$

$$c \Rightarrow h \quad ; \quad c = T, \text{ yields } h = T \tag{89d}$$

$$g \lor h = k \quad ; \quad g = F, \, h = T, \text{ yields } k = T \tag{89e}$$

$$g \lor \neg k = y \quad ; \quad g = F, \, k = T, \text{ yields } y = F \tag{89f}$$

The conclusion is that this argument, with inputs $a = T$, $b = F$,

$c = F$ leads to $x = D$, $y = F$, $z = T$, exactly /as deduced above, from

Fig.3, via the steps in the Flow Chart.

Although such rewriting of the argument by the person

solving it can appreciably simplify it, our purpose here is

mainly to show that there is a computerizable algorithmic

procedure (as shown in the Flow Chart of Fig. 1) that can be

given, and that it can solve for the states of the required

(output) terms of any logical graph without human interference.

We have shown this for the problem composed of Eqs (79a-f).

We shall now obtain the reduced graphs and its equations for

the two possibilities $\underline{y1}$ = T and F of the larger argument of

of Fig.4(a).

(ii) Solution of the argument of the larger graph in Fig. 4(a)

The inputs $(\underline{t}^{(i)})$ to this graph are $\underline{a}$, $\underline{b}$, $\underline{c}$; additional

inputs are $\underline{x}$, $\underline{z}$, $\underline{y1}$ and the output $\underline{y}$. Suppose, as in Fig.3,

we are given that $\underline{a}$ = T, $\underline{b}$ = F, $\underline{c}$ = T. For the three additional

inputs, we choose T and F for each of $\underline{x}$, $\underline{z}$ and $\underline{y1}$ and get

eight possibilities. We shall divide these into two sets,

namely with $\underline{y1}$ = F and $\underline{y1}$ = T, and consider them separately.

<u>Case</u> α : $\underline{y1} = F$ corresponds to Figs 4(a) and 4(b).

85(a-h) with 87(a,b)

In this case, the implementable equations/are identical

with those of the simpler argument in Fig.3, namely Eqs. (81a)

to (81h), except for the following change and addition at the

end of that list.

Change (81h)   to $\underline{\underline{g}}$ $\underset{\sim}{ON}$ $\underline{\underline{k}}$ = $\underline{\underline{y2}}$ (90)

$\underline{\underline{y1}}$ = F (91a) ; $\underline{\underline{y1}}$ $\underset{\sim}{V}$ $\underline{\underline{y2}}$ = $\underline{y}$ (91b)

Hence, Table 2 holds good, with $\underline{y}(\underline{t}^{(5)})$ replaced by $\underline{\underline{y2}}$. We

found therein that $\underline{y}$ can only be F, so that, in the present

case, $\underline{\underline{y2}}$ = F. Putting this and the value $\underline{\underline{y1}}$ = F as in (91a)

and (91b), we obtain that $\underline{y}$ = F. Hence, for case (α), the

solution of the larger graph in Fig. 4(a) is

$\underline{\underline{x}}$ = D, $\underline{y}$ = F, $\underline{\underline{z}}$ = T   (92)

the same as for the simpler graph of Fig. 3.

We shall now consider the case when $\underline{\underline{y1}}$ = T in Fig. 4(a).

Case β : $\underline{y1}$ = T, corresponding to Figs. 4(a) and 4(c)

In this case, we have chosen $\underline{y1}$ = T, which makes (85c) become

$\underline{z1}$ $\underline{\text{UAN}}$ $\underline{g2}$ = T, so that $\underline{z1}$ = F and $\underline{g2}$ = F, as shown in

Fig.4(c). There is an independent disconnected graph $\underline{z1}$ $\underline{V}$ $\underline{z2}$ = $\underline{z}$

with $\underline{z1}$ = F, so that the only non-contradictory solution for

$\underline{z}$ is F, when $\underline{z1}$ = F. Hence, we obtain for the equations for

the main reduced graph the same list as in (81a) to (81h), but

with the following changes and additions:

Change (81d) to $\underline{g2}$ = F   (93a)

Change (81h) to $\underline{g}$ $\underline{\text{ON}}$ $\underline{b}$ = $\underline{y2}$ (93b)

Add the equations

$$\underline{y1} = T \quad (93c); \quad \underline{y1} \underline{V} \underline{y2} = \underline{y} \quad (93d)$$

Add $\underline{z}$ = F from the disconnected graph   (93e)

Hence, the only additional input is $\underline{x}$, which is put T and F

for the two possibilities. The consequences, on solving Eqs.

(81) and (93), are given in Table 3 below. It will be seen from

Table 3. Solution of Equations (80a-h) and (93a-e) of the
         larger graph-Case β.

Table 3: Solution of Equations (80a-h) and (93a-e) of the larger graph — Case β

Inputs $t^{(i)}$: $a = T$, $b = F$, $c = T$

$t^{(ai)}$				$t^{(1)}$				$t^{(2)}$	$t^{(3)}$	$t^{(4)}$	$t^{(5)}$
x	z	$g2$	$y1$	$g1$	$g2'$	$g3$	h	g	k	$y2$	y
T	F	F	T	F	F	D	T	— F	T	F	X
F	F	F	T	F	F	F	T	F	T	F	X

the table that the input state T of $\underline{y1}$ leads to the output

$\underline{y2}$ = T, so that $\underline{y1} \underset{\sim}{V} \underline{y2}$ = $\underline{y}$ leads to a contradiction. This

happens for both the possibilities, $\underline{x}$ = T and $\underline{x}$ = F. Hence,

there is no valid solution of the equations for this case

(corresponding to $\underline{y1}$ = T) and the choice $\underline{y1}$ = T is impermissible.

Hence, for the graph in Fig. 4(a), $\underline{y1}$ can only be F,

corresponding to case β , and this leads the solution in Eqs.(92),

namely $\underline{x}$ = D, $\underline{y}$ = F, $\underline{z}$ = T, which are the same as those for

the simpler graph of Fig. 3.

Thus, we have considered the various types of constructions

for obtaining the solution of logical graphs — such as $\underline{c}$-construction

$\underline{A}$-reconstruction and $\underline{V}$-construction for an SHS argument. The

principles of these are unchanged when applied to QPL-1, QPL-2

and QSN statements forming parts of logical arguments, and are not

particularly worked out. Instead, we consider an argument

containing all these types of statements (but without the above

requirements) in order to illustrate how these are tackled $\underline{via}$

our vector-matrix formalism.

(e) Example of an argument containing three types of terms

In this subsection, we shall give an example of the

implementation of a logical graph (argument) containing QFL-1,

QFL-2 and QSM relations. Since the main purpose is to illustrate

the way in which these are worked out using the formalism developed

by us, complexities of graph construction (as in Fig.3 and Fig.4

of the previous subsection) are avoided and no c-construction or

A-reconstruction is required for this graph. On the other hand,

interconversions of terms from QPL-1 to QFL-2 form and vice-versa,

formulation and use of 3X3 and 3X2 matrices, theoretical

combination of two QFL-1 statements to obtain a single resultant

QPL-1 relation, working out of a unary bouquet etc., are

illustrated. The argument, given in terms of logical equations

in standard terminology, is converted into a sequentially

implementable set of equations in our formalism, which is then

implemented and the outputs worked out. The argument, which is

rather complicated in order to illustrate all these possible

intricacies / given in words as follows. (This argument, written
                    may be

mainly to illustrate the above logical points, may not be a

satisfactory paragraph in sociology.)

(a) If there are strong leaders, then all people are restrained.

(b) For all people, restraint and safety mean contentment.

(c) For all people, discontent means dissatisfaction.

(d) For some people, satisfied implies happy.

(e) If some people are happy, then there is prosperity.

(f) If all people are not happy, then there is trouble in all places, and <u>vice-versa</u> (i.e. if some people are happy, then there is no trouble in some places.)

(g) If there exist leaders who are not corrupt, or there are people who fight for a pure administration, then, in all places, there will be no injustice.

(h) If there exist criminal leaders, there will be injustice in all places.

(i) If there is no trouble in all places, or there is no injustice in all places, then there is quiet peace; otherwise no quietude.

We give the logical equations for each of the statements

(a) to (i) in (94a) to (94i) at the beginning of the next

subsection (i). We employ three variables:

(x) for leaders, (y) for people and (z) for places.

We are given the <u>inputs</u>:

All leaders are strong $\quad\quad$: $\mathcal{Q}$ (x) = $(\forall x)$ (a_T), $\quad$ in (a)

For all people there is safety: $\mathcal{b}$ (y) = $(\forall y)$ (b_T), $\quad$ in (b)

All leaders are not corrupt $\quad$: $\mathbb{C}$ (x) = $(\forall x)$ (c_F), $\quad$ in (g)

There exist people who fight $\quad$: $\mathcal{d}$(y) = $(\exists y)$ (d_T), $\quad$ in (g)
for a pure administration

We are required to find out the logical states of the

following <u>outputs</u>:

Prosperity $\quad\quad\quad\quad\quad$: $\quad \underline{p}$ (SNS term) $\quad$ in $\quad$ (e)

Quietude $\quad\quad\quad\quad\quad\quad$: $\quad \underline{q}$ (SNS term) $\quad$ in $\quad$ (i)

Criminality of leaders: $\quad \Upsilon$ (QPL-1 term) in $\quad$ (h)

It is to be noted that the output Υ in (94h) is put in as a QPL

additional input $\underline{r}(\underline{t}^{(ai)})$, which is clear from Fig. 5 (See

subsection (ii) below).

The other terms are denoted by the following symbols:

Restraint, of people $\quad\quad$: $\quad \mathbb{C}$ (y) $\quad\quad$ in $\quad$ (a)

Contentment, of people $\quad$: $\quad \mathcal{f}$ (y) $\quad\quad$ in $\quad$ (b)

Satisfaction, of people : $\quad \mathcal{g}$ (y) $\quad\quad$ in $\quad$ (c)

Happiness, of people $\quad$: $\quad \mathcal{h}$ (y) $\quad\quad$ in $\quad$ (d)

Trouble, in places $\quad\quad$: $\quad \mathcal{i}$ (z) $\equiv \underline{i}$(z) in $\quad$ (f)

Injustice, in places $\quad$: $\quad \mathcal{k}$ (z) $\equiv \underline{\underline{k}}$(z) in $\quad$ (g), (h).

We now proceed to write the above argument in logical

symbolism.

(i) Logical equations of the argument in standard language

The set of equations, in the standard terminology and symbology of predicate logic, is given below in (94a) to (94i).

$$(\exists x) \; (\underline{a}(x)) \; \Rightarrow \; (\forall y) \; (\underline{e}(y)) \tag{94a}$$

$$(\forall y) \; (\underline{e}(y) \; \& \; \underline{b}(y) \; = \; \underline{f}(y)) \tag{94b}$$

$$(\forall y) \; (\neg \underline{f}(y) \; \Rightarrow \; \neg \underline{g}(y)) \tag{94c}$$

$$(\exists y) \; (\underline{g}(y) \; \& \; \underline{h}(y) \;) \tag{94d}$$

$$(\exists y) \; (\underline{h}(y)) \; \Rightarrow \; \underline{p} \tag{94e}$$

$$(\forall y) \; (\neg \underline{h}(y)) \; \equiv \; (\forall z) \; (\underline{i}(z)) \tag{94f}$$

$$(\exists x) \; (\neg \underline{c}(x)) \; \vee \; (\exists y) \; (\underline{d}(y)) \; \Rightarrow \; (\forall z) \; (\neg \; \underline{\ell}(z)) \tag{94g}$$

$$(\exists x) \; (\underline{r}(x)) \; \Rightarrow \; (\forall z) \; (\; \underline{\ell}(z)) \tag{94h}$$

$$(\forall z) \; (\neg \underline{i}(z)) \; \vee \; (\forall z) \; (\neg \underline{\ell}(z)) \; = \; \underline{q} \tag{94i}$$

The inputs are a, b, c, d and the outputs are $\underline{p}$, $\underline{q}$ and Υ (or $\underline{r}$). Of the three outputs, one (namely Υ) is a QPL-2 additional input in Eqn (94h). Since QPL terms are freely being converted from QPL-1 to QPL-2 and vice versa, we note that these

can be done by using the operators $\underset{\approx}{C}$ (canonizer) and

$\underset{\approx}{S}$ (standardizer) for this purpose. In the logical graph, these

will appear as unary operators. In Fig. 5 below, these alone

are shown as small squares, while other logical connectives are

shown as large squares, with the notation in our formalism —

see subsection (ii) below.

Eqns.(94a) to (94i) are implementable, but for the following

considerations:

Firstly, QPL-1 to QPL-2 form and QPL-1 to QPL-2 form should

be expressed _via_ canonizer and standardizer, in the form (in

general) of $\underset{=}{\mathcal{Q}} \underset{\approx}{C} = \underset{=}{a}$ and $\underset{=}{a} \underset{\approx}{S} = \underset{=}{\mathcal{Q}}$ respectively.

Secondly, (1c) and (1d) are two QPL-1 relations in sequence

and they must be combined theoretically. They have the pattern

of _Dimension_ (Sl.No.4. in Table 5 of Section 4(f)(i) of Part IV),

with $\overline{}$f corresponding to $\underset{=}{a}$, g corresponding to $\underset{=}{b}$ and $\underset{=}{h}$ corresponding

to $\underline{c}$ of that Table 5, and hence, we obtain a single resultant

QPL-1 relation, as in (95c) below:

Replace (94c) and (94d) by $(\exists y) (\underline{f}(y) \implies \underline{h}(y),$ (95c)

Thirdly, as mentioned earlier in Section 2(d)(i), we must

write (94g) as two equations (95g'), (95g") as follows:

$$(\exists x) (\neg \underline{c}(x)) \lor (\exists y) (\underline{d}(y)) = \underline{k} \quad \text{(QPL-2 to SNS)} \qquad (95g')$$

$$\underline{k} \implies (\forall z) (\neg \underline{\ell}(z)) \quad \text{(SNS to QPL-2)} \qquad (95g")$$

Fourthly, Eqns (95g") and (94h) are found to have multiple

inputs for $\underline{\ell}(z)$. Hence, we call the output of (95g") as $\underline{\ell_1}(z)$

and that of (94h) as $\underline{\ell_2}(z)$, and add the equation (95h') as below:

$$\underline{\ell_1}(z) \lor \underline{\ell_2}(z) = \underline{\ell}(z) \qquad (95h')$$

and $\underline{\ell}(z)$ is then input to (94i).

Eqns (94), with the modifications in (95), are implementable

in sequence. However, while converting these to our language, we

write them in the sequence that would be obtained using the

algorithm, discussed in Section 4(c), for working out the argument

from Fig.3. In this, a unary sequence (or bouquet) is worked out

in one step.

(ii) Logical graph of the argument and/equations in our language

The logical graph of the above argument (after making the

modifications mentioned in subsection (i) above) is given in

Fig.5. It will be seen from the graph that the initial inputs

($\underline{t}^{(i)}$) are a, b, c and d, whose states are as given in

Table 4 below, and the outputs are $\underline{p}$ and $\underline{q}$. The other input $\underline{r}$

Table 4. Working out the logical graph in Fig. 5 (pages 98 to 101)

---

is really an output, put in as additional input ($\underline{t}^{(ai)}$) and its

equivalent QPL-1 form is γ , obtained by the application of the

standardizer $\underline{S}$. In view of this, we write the equation

$$\underline{r} \quad \underline{S} = \gamma \qquad\qquad (96)$$

and put it in a box in Fig.5
separately/ and input for $\underline{r}$ ($\underline{t}^{(ai)}$)   the three QPL-2 pure

states (1 0 0), (0 1 0) and (0 0 1) as the three possibilities

of this term $\underline{r}$.

For the rest of the graph, we follow the algorithmic procedure

---

Fig.5. Logical graph of the argument containing all three types
    of terms, and relations of the type SNS, QFL and QSN (in p 95a)
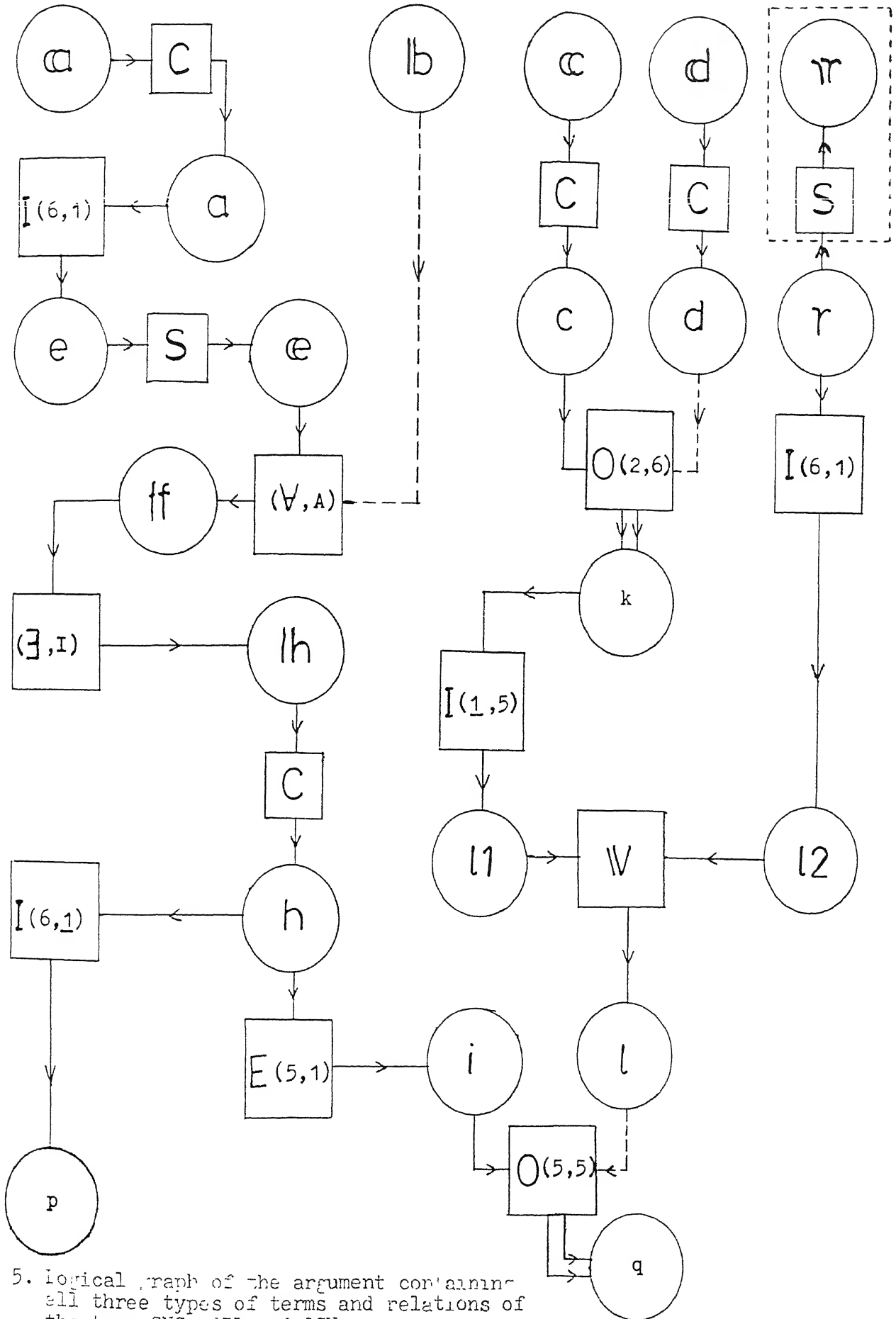
5. Logical graph of the argument containing
all three types of terms and relations of
the type SNS, QPL and QSN

of starting from the $t^{(i)}$'s and the $t^{(ai)}$ and proceeding

step by step, as described in Section (d) above. Then, we get

the results as indicated in Table 4 where the steps of the

algorithm are also indicated. Since r enters the picture

only as Eqn (97k), the equations from (97a) to (97j) are worked out

for the given states of the inputs $t^{(i)}$, and the remaining

four equations (97k) to (97o) are alone worked out for the

three possibilities of r.

As mentioned in Section 2, the symbology for a unary and binary

Type-1 operators will be written as in (98a,b,c), so that it has

the same structure as for QPL- Type-2 and SKS operators:

$$Q\ (q\!\!/,\ z) = b \quad \text{stands for} \quad q\!\!/(a\ z = b) \qquad (98a)$$

$$Q\ (q\!\!/,\ z)\ b = C \quad \text{stands for} \quad q\!\!/(a\ z\ b = c) \qquad (98b)$$

$$C\ (q\!\!/,\ \overleftarrow{z})a = b \quad \text{stands for} \quad q\!\!/(c\ \overleftarrow{z}\ a = b) \qquad (98c)$$

Also, the equivalence operator $E(i,\ j)$ stands for the following:

$$(a\ E(i,\ j) = b) \equiv (a\ I(i,\ j) = b)\ \&\ (a\ I(i^c,\ j^c) = b) \qquad (99)$$

Also, in the matrix operators for QSN relations, the index

corresponding to the SNS term is underlined, to distinguish it

from a pure QPL-2 operator — e.g.

$\underline{a}\ \underline{I}(\underline{i},\ j) = \underline{b}$, $\underline{i}$ = 1 to 4, j = 1 to 8, for SNS-QPL-2 relation (100a)

$\underline{a}\ \underline{O}(i,\ \underline{j})\ \underline{b} = \underline{c}$, i = 1 to 8, $\underline{j}$ = 1 to 4, for QPL-2-SNS relation (100b)

Coming back to the argument worked out in Table 4, we see

that it leads to the conclusions $\underline{p}$ = T, $\underline{q}$ = T, $\underline{r}$ = (0 0 1)   (F),

with the given inputs mentioned earlier. There is prosperity,

quietude, and no leaders are criminal. For obtaining the last

of these, we have used $\underline{r}$ as an additional input in $\underline{r}\ \underline{I}\ (6,\ 1) = \underline{\ell 2}$.

We can also obtain the same result by inverting this equation.

If we do this, there is no need to have $\underline{\ell 1}$, $\underline{\ell 2}$ for $\underline{\ell}$.

Instead, we obtain, from (97m), the equation $\underline{k}\ \underline{I}(\underline{1},\ 5) = \underline{\ell}$,

which, with $\underline{k}$ = (1 0), leads to $\underline{\ell}$ = (0 0 1). The inverted

equation for $\underline{r}$, obtained from (97k) is

$$\underline{\ell}\ \underline{I}(2,\ 5)\ =\ \underline{r} \qquad\qquad (101a)$$

Table 4. Working out the logical graph in Fig. 5 *)

Nature	Equation	Input(s)	Output	Eqn.No.
Initial inputs ($\underline{t}^{(i)}$) : $\mathcal{a}$ = ∀(T), $\mathcal{b}$ = ∀(T), $\mathcal{c}$ = ∀(F), $\mathcal{d}$ = ∃(T)				
Additional inputs ($\underline{t}^{(ai)}$): $\underline{r}$ = (1 0 0), (0 1 0) or (0 0 1)				
Step 1				
(QPL-1 to 2)	$\mathcal{a}\ \underline{c}$ = $\underline{a}$	∀(T)	(1 0 0)	(97a)
(QPL-2)	$\underline{a}\ \underline{I}(6,1)$ = $\underline{e}$	(1 0 0)	(1 0 0)	(97b)
(QPL-2 to 1)	$\underline{e}\ \underline{s}$ = $\mathcal{e}$	(1 0 0)	∀(T)	(97c)
(QPL-1)	$\mathcal{e}\ (∀,A)\ \mathcal{b}$ = f	∀(T),∀(T)	∀(T)	(97d)
(QPL-1)	$f\ (∃,I)$ = h	∀(T)	∃(T)	(97e)
(QPL-1 to 2)	$h\ \underline{c}$ = $\underline{h}$	∃(T)	(1 1 0)	(97f)
(QPL-2 to SNS)	$\underline{h}\ \underline{I}(6,\underline{1})$ = $\underline{p}$	(1 1 0)	(1 0)	(97g)
(QPL-2)	$\underline{h}\ \underline{E}(5,1)$ = $\underline{i}$	(1 1 0)	(0 1 1)	(97h)
(QPL-1 to 2)	$\mathcal{c}\ \underline{c}$ = $\underline{c}$	∀(T)	(1 0 0)	(97i)
(QPL-1 to 2)	$\mathcal{d}\ \underline{c}$ = $\underline{d}$	∃(T)	(1 1 0)	(97j)
(QPL-2)	$\underline{r}\ \underline{I}(6,1)$ = $\underline{l2}$	(1 0 0) (0 1 0) (0 0 1)	(1 0 0) (1 0 0) (1 1 1)	(97k)
Outputs of Step 1 : $\underline{p}$, $\underline{i}$, $\underline{c}$, $\underline{d}$, $\underline{l2}$				

Table.4 (Contd)

Nature	Equation	Input(s)	Output	Eqn.No.
Step 2 (QPL-2 to SNS) Output of Step 2 : $\underline{k}$	$\underline{c}\ \underset{\approx}{Q}(2,6)\ \underline{d} = \underline{k}$	(0 0 1),(1 1 0)	(1 0)	(97 ℓ)
Step 3 (SNS to QPL-2) Output of Step 3 : $\underline{\ell_1}$	$\underline{k}\ \underline{I}(1,\ 5) = \underline{\ell_1}$	(1 0)	(0 0 1)	(97m)
Step 4 (QPL-2) Output of Step 4 : $\underline{\ell}$	$\underline{\ell_1}\ \bigvee\ \underline{\ell_2} = \underline{\ell}$	(0 0 1),(1 0 0) (0 0 1),(1 0 0) (0 0 1),(1 1 1)	(0 0 0) (0 0 0) (0 0 1)	(97n)
Step 5 Output of Step 5 : $\underline{q}$	$\underline{i}\ \underset{\approx}{Q}(5,5)\ \underline{\ell} = \underline{q}$	(0 1 1),(0 0 0) (0 1 1),(0 0 0) (0 1 1),(0 0 1)	(0 0) (0 0) (1 0)	(97o)
Conversion of $\underline{r}$ to QPL-1 from γ (for non-contradictory $\underline{r}$) (QPL-2 to QPL-1)	$\underline{r}\ \underset{\approx}{S} = \gamma$	(0 0 1)	$\forall$(F)	(97p)

*The details of working out the equations (97a) to (97p) are given in the notes below.

(97a) : Using the canonizer in Table 7, Part II,

$$ \mathcal{Q} = \forall(\mathrm{T}) \xmapsto{C} \underline{\underline{a}} = (1\ 0\ 0) $$

(97b) : Using the theory in Section 6(f), Part II, we obtain

$$ (1\ 0\ 0) \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix} = (1\ 0\ 0) $$

(97c) : Using the standardizer in Table 7, Part II, suitably modified, we obtain

$$ (1\ \ 0\ \ 0) \xmapsto{S} \forall(\mathrm{T}) $$

(97d) : Using Eq.(32c) in Section 3(c) (i) of Part IV, we have

$$ \mathcal{Q}'_{\underline{e}} = \forall,\ \underline{\underline{c}} = \mathrm{T};\ \mathcal{Q}'_{\underline{b}} = \forall,\ \underline{\underline{b}} = \mathrm{T} \quad \text{in} \quad \forall(\underline{\underline{e}}\ \underset{\sim}{A}\ \underline{\underline{b}} = \underline{\underline{f}}), \text{ and} $$

we obtain

$$ \mathcal{Q}'_{\underline{f}} = \mathcal{Q}'_{\underline{c}} \oplus \mathcal{Q}'_{\underline{b}} = \forall,\ \underline{\underline{f}} = \underline{\underline{e}}\ \underset{\sim}{A}\ \underline{\underline{b}} = \mathrm{T}, \text{ giving } \int\!\!f = \forall(\mathrm{T}) $$

(97e) : Using Fig. 6(a) of Part IV, we have $\mathcal{Q}'_{\underline{f}} = \forall,\ \underline{\underline{f}} = \mathrm{T}$

in $\exists(\underline{\underline{f}}\ \underset{\sim}{I} = \underline{\underline{h}})$, so that $\mathcal{Q}'_{\underline{h}} = (\forall \otimes \exists) \oplus \exists = \exists$ and

$\underline{\underline{h}}' = \underline{\underline{f}}\ \underset{\sim}{I} = \mathrm{T}$, giving $\int\!\!h = \exists(\mathrm{T}).$

(97f) : As in (97a), we use Table 7, Part II to give $\exists(\mathrm{T}) \xmapsto{C} (1\ 1$

(97g) : As in (97b), we obtain, for the 3X2 matrix $\underline{\underline{I}}(6,\ \underline{1})$,

$$ (1\ 1\ 0) \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 1 \end{pmatrix} = (1\ \ 0) = \mathrm{T} $$

(97h) : We use the result $(\underline{\underline{h}}\ \underline{\underline{E}}(5,1) = \underline{\underline{i}}) \equiv ((\underline{\underline{h}}\ \underset{\sim}{I}(5,1) = \underline{\underline{i}})$ & $(\underline{\underline{h}}\ I(6,\ 2) = \underline{\underline{i}}))$, so that

$$ E(5,1) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} \otimes \begin{pmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} $$

and with $\underline{h} = (1 \ 1 \ 0)$, $\underline{i} = (1 \ 1 \ 0)\begin{pmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix} = (0 \ 1 \ 1)$

(97i), (97j) : Both these use the canonizer $\underset{\approx}{C}$

(97k) : Using Section 6(f) of Part II, we have

$$(1 \ 0 \ 0)\begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix} = (1 \ 0 \ 0) \ ; \quad (0 \ 1 \ 0)\begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix} = (1 \ 0 \ 0) \ ;$$

$$(0 \ 0 \ 1)\begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix} = (1 \ 1 \ 1)$$

(97ℓ) Using the theory in Part II, with $\underline{c} = (0 \ 0 \ 1)$

$$(0 \ 0 \ 1)\begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = (1 \ 1 \ 1)\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = 1 = k_\alpha$$

$$(0 \ 0 \ 1)\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = (0 \ 0 \ 0)\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = 0 = k_\beta$$

so that $\underline{k} = (k_\alpha \ k_\beta) = (1 \ 0) = T.$

(97m) : As in (97g), we use the 2X3 matrix for $\underline{I}(1, 5)$ and obtain

$$(1 \ 0)\begin{pmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix} = (0 \ 0 \ 1)$$

(97n) : For the three possibilities for $\underline{\underline{\ell}}2$ in (97k), we obtain

$$(0 \ 0 \ 1) \otimes (1 \ 0 \ 0) = (0 \ 0 \ 0); \quad (0 \ 0 \ 1) \otimes (0 \ 1 \ 0) = (0 \ 0 \ 0);$$

$$(0 \ 0 \ 1) \otimes (0 \ 0 \ 1) = (0 \ 0 \ 1)$$

(97o) : As in (97b), remembering that the impossible state $\phi = (0 \ 0 \ 0)$ can lead only to $X = (0 \ 0)$, we obtain for the only non-ϕ state of $\underline{\underline{\ell}}2$, namely $(0 \ 0 \ 1)$,

$$(0 \ 1 \ 1)\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = (1 \ 1 \ 1)\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = 1 = q_\alpha$$

$$(0 \ 1 \ 1)\begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = (1 \ 1 \ 0)\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = 0 = q_\beta$$

so that the output $\underline{q} = (q_\alpha \ q_\beta) = (1 \ 0) = T$.

(97p) : Using the standardiser, as in (97c), $(0 \ 0 \ 1) \overset{S}{\longmapsto} \bigvee(F)$

which yields

$$\underset{=}{r} = (0\ 0\ 1) \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} = (0\ 0\ 1) \qquad (101b)$$

This is the same as what we have obtained for $\underset{=}{r}$ in Table 4,

assuming the three basic states of LA-3 for it, and making the

consistency check in each case. As mentioned earlier, we shall

prove in Section 5, this property of the inversion of a statement,

for all implications and equivalences in QFL-2, SIS and QSM, from

the property of $\underset{\sim}{I}(i,\ j)$ that it is equivalent to $\underset{\sim}{I}(j^c, i^c)$ in

the inverted relation.

Thus we have considered practical examples of a logical

graph in SIS having various complications in the logical

interconnections occurring in the graph and also an example

of a simple logical graph, but in which all the different types

of relations such as QFL, SIS, ISI occur. By combining the

features which occur in examples considered both in Section

4(d) and 4(e) above, it should be possible to take care of

practically any logical argument with terms which are logical

functions of only one variable.

In the next section, we shall consider some more general

relations and their interpretation in our language in terms of

suitable matrices, either 3X3, 2X2, 2X3 or 3X2. We shall not

consider in this report arguments having terms which are logical

functions of more than one variable such as $\underline{a}(x, y)$. It should,

however, be recognized that the unary and binary connective

operators could be functions of one variable or two variables,

in the various examples considered above. Thus, as mentioned

in Eq.(20) of Section 2(b), the simple relation $\underline{a}(x) \underset{\approx}{\underline{I}} (i, j) = \underline{b}(y)$

really has for the implication operator the form $\underset{\approx}{\underline{I}}(i, x; j, y)$

where the first variable x is to be connected with the input

term $\underline{a}$ (x) and the second variable y of the operator is to be

associated with the variable of the output $\underline{b}(y)$. Such features

will be discussed further when we consider more general logical

relations in which the terms themselves may be functions of more than

one variable.

5. Some theorems and special considerations

First we shall consider the proof of the fact that the

inversion of an implication checked via the consistency operator

$\bigvee$ and the direct application of the implication in the forward

sense will both lead to the same result. We saw particular

examples of this in Section 4 (b) and 4 (e)(ii). We shall give a general

proof first for QPL-2 unary implications $\underset{\approx}{I}(i, j)$ and then

indicate how it is equally valid for SNS and QSN relations of

the same type.

(a) Inversion of an implication considered via consistency checks.

First we shall indicate the well-known lattice-type

relationships between q(1) to q(8) of QPL and also between

s(1) to s(4) of SNS. These are shown in Fig. 6 (a) and (b).

Fig.6. Inclusion relations shown by lines with arrows for
(a) QPL states represented by 3-element Boolean vectors
of BA-3, and (b) SNS states represented by 2-element
Boolean vectors of BA-2 .
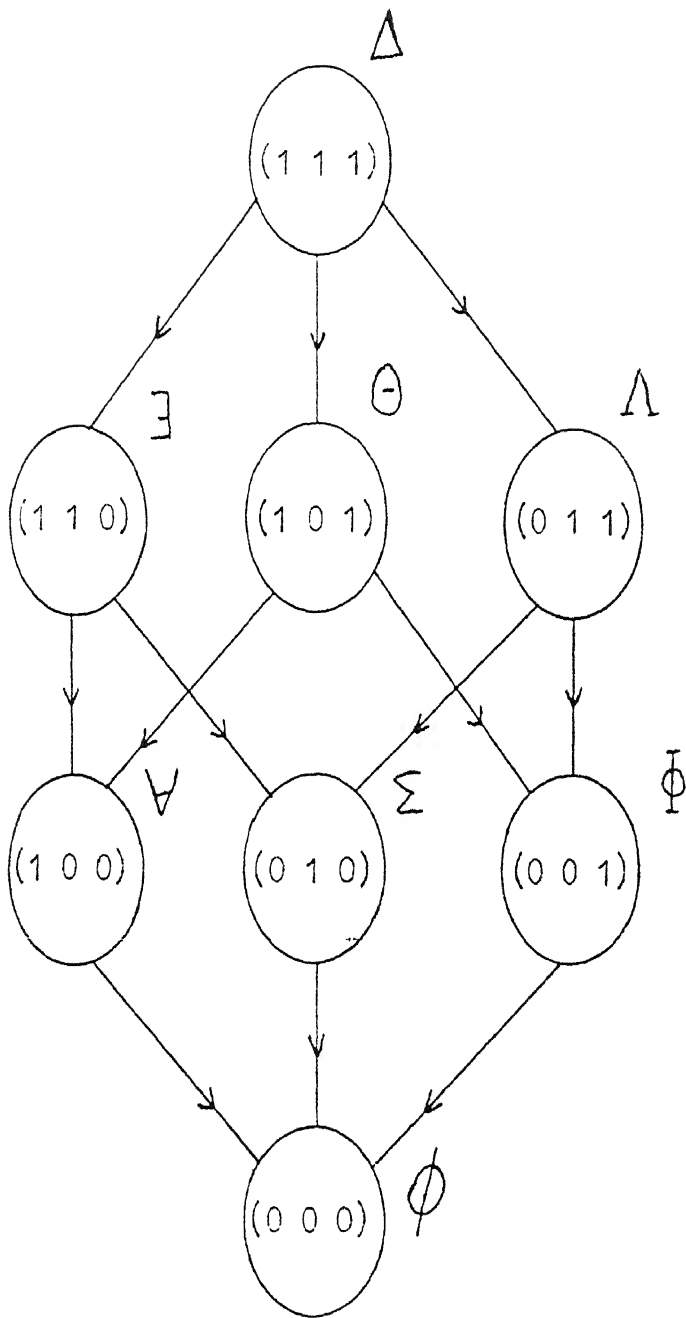
(a)

(b)
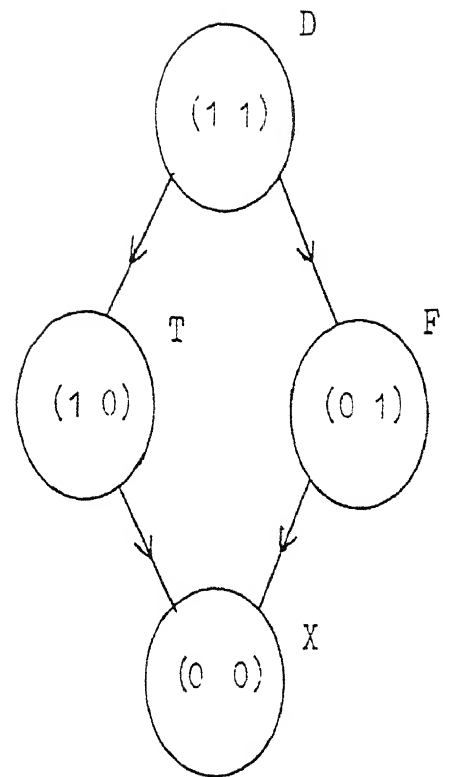
Fig.6 Inclusion relations shown by lines with arrows for
(a) QFL states represented by 3-element Boolean
vectors of BA-3, and (b) SHS states represented
by 2-element Boolean vectors of BA-2.

In this figure a state $q(j)$, connected by a directed line to

$q(k)$ below it, means that the vector (γ_j δ_j ϵ_j) representing

the former contains in it all the one's of the vector (γ_k δ_k ϵ_k)

associated with $q(k)$. We shall represent it by the symbology:

$$\text{Lattice inclusion} \quad : \quad q(j) \subset q(k) \qquad (102)$$

We wish to prove the result that if

$$\underline{\underline{a}} \; \underline{\underline{I}} \, (i, \, j) \; = \; \underline{\underline{b}} \qquad (103)$$

then, in the inverse relation between $\underline{\underline{b}}$ and $\underline{\underline{a}}$ represented by

Eqns (104a and b),

$$\underline{\underline{b}} \; \underline{\underline{I}} \, (j^c, \, i^c) \; = \; \underline{\underline{a}}' \quad , \quad \underline{\underline{a}} \; \bigvee \underline{\underline{a}}' \; \neq \; \emptyset \qquad (104a,b)$$

if $\underline{\underline{b}}$ is given successively the three basic states $q(1)$ ($\equiv (1\ 0\ 0)$),

$q(3)$ ($\equiv (0\ 1\ 0)$), and $q(5)$ ($\equiv (0\ 0\ 1)$, of QPL, and those that

do not lead to the null set $\emptyset$ in (104b) are added together,

then $\underline{\underline{b}}$ so obtained will be the same as in (103).

The proof follows straightaway from the fact that the

implication (103) means that, only if $\underline{\underline{a}} \subseteq q(i)$, will $\underline{\underline{b}}$ be equal

to $q(j)$. Then if we take $\underline{b}' \subseteq q^c(j) \equiv q(j^c)$, then, by (104a),

$$\underline{b}' \longmapsto \underline{a}' \subseteq q(i^c) \equiv q^c(i) \tag{105}$$

Consequently $\underline{a}' \otimes \underline{a} = q^c(i) \otimes q(i) = \emptyset$ for $\underline{a}'$ given by the

inverted implication. On the other hand, if $\underline{b}' \subseteq q(j)$ is input

in (104a), it leads to $\underline{a}' = (1\ 1\ 1) = \triangle$, and this yields

$\underline{a}' \bigvee \underline{a} \neq \emptyset$. Hence, the consistency check gives a non-impossible

result only if $\underline{b}' \subseteq q(j)$. This means that the state of the

term $\underline{b}$ obtained by the consistency check is $q(j)$, the same

as that obtained by the forward relation (103) when $\underline{a} \subseteq q(i)$.

This proof for QFL is equally valid for SNS without any

change except that the 3-vectors $q(j)$, $q(k)$ are replaced by

$s(j)$, $s(k)$ which are 2-vectors, and Eq. (103) is replaced by the

SNS relation $\underline{a}\ \underline{I} = \underline{b}$. We then obtain the same conclusion,

namely that this direct implication relation gives the same

result as that obtained from the inverted implication relation

$(\underline{b}\ N)\ \underline{I} = (\underline{a}\ N)$ by checking for consistency. (Note that, in

SNS, $(\underline{a}\ \underline{N}) = \underline{a}^c$, for the basic states T and F).

By combining the properties of 3-vectors in QPL and 2-vectors

in SMS, we can write a proof in an exactly similar manner for the

LSM type of implications, namely, QFL-2 to SMS and SMS to QFL-2.

The essential equation used is $(\underline{a}\ \underline{I}(i,\ j) = \underline{b}) \equiv (\underline{b}\ \underline{I}(j^c,\ i^c) = \underline{a})$

for the former, and $(\underline{a}\ \underline{I}(i,\ j) = \underline{b}) \equiv (\underline{b}\ \underline{I}(j^c,\ i^c) = \underline{a})$ for the latter

The expansion of the above proof from $\underset{\sim}{I}(i,\ j)$ to $\underset{\sim}{E}(i,\ j)$

for QPL, where $\underset{\sim}{E}(i,\ j)$ satisfies the equivalence (99) is also

obvious, since both the implications relations on the r.h.s of

(99) lead to identical results, namely that

$$(\underline{a}\ \underset{\sim}{I}(i,\ j) = \underline{b}) \quad (\underline{b}\ \underset{\sim}{I}(j^c,\ i^c) = \underline{a}) \tag{106a}$$

$$(\underline{a}\ \underset{\sim}{I}(i^c,j^c) = \underline{b}) \quad (\underline{b}\ \underset{\sim}{I}(j,\ i) = \underline{a}) \tag{106b}$$

We shall not elaborate on this further.

(b) Other general matrix operators

We have considered matrix operators of the types $\underset{\sim}{A}(i,\ j)$,

$\underset{\sim}{Q}(i,\ j)$, $\underset{\sim}{I}(i,\ j)$, $\underset{\sim}{J}(i,\ j)$ and $\underset{\sim}{E}(i,\ j)$ so far. These do not, by

any means, exhaust all the possible 3X3 boolean matrices. The

general type of 3X3 matrix appears to be complicated, but any

such general matrix can be expressed as a combination of two

implications, or three implications, as shown below. One example

of two implications was (49) where the equivalence operator

$\underline{\underline{E}}(i, j)$ was defined in terms of two implications. Another example

of such a relation is the following in QSII:

$$\left[\underline{\underline{a}} = (1\ 0\ 0)\right] \implies \left[\underline{\underline{b}} = (1\quad 0)\right] \ ; \ \left[\underline{\underline{a}} = (0\ 0\ 1)\right] \implies \left[\underline{\underline{b}} = (0\quad 1)\right] \qquad (107)$$

Written in standard terminology, these become

$$\forall(\underline{\underline{a}}) \implies b_T \ ; \ \forall(\neg\underline{\underline{a}}) \implies b_F \qquad (108a,b)$$

Obviously, the third possibility namely $\underline{\underline{a}} = (0\ 1\ 0)$, i.e. $\underline{\underline{a}} \equiv \exists$,

but not $\forall$, is left unspecified in (108a,b) and therefore we should

add this in the form of Eq. (108c):

$$\left[\underline{\underline{a}} = (0\ 1\ 0)\right] \implies \left[\underline{\underline{b}} = (1\quad 1)\right] \ , \ \text{or} \ \ \exists(\underline{\underline{a}}) \implies b_D \qquad (108c)$$

The three implications in (108) can be represented by the

vector-matrix relations

$$\langle\underline{\underline{a}}_1| \ Z_1| = \langle b_1| \ , \ \langle a_2| \ Z_2| = \langle b_2| \ , \ \langle a_3| \ Z_3| = \langle b_3| \qquad (109)$$

where

$$|Z_1| = |Z(1, \underline{1})| = \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 1 \end{pmatrix}, \ |Z_2| = |Z(3, \underline{3})| = \begin{pmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{pmatrix},$$

$$|Z_3| = |Z(5, \underline{2})| = \begin{pmatrix} 1 & 1 \\ 1 & 1 \\ 0 & 1 \end{pmatrix} \qquad (110)$$

If we combine these in the form of a QSK equation

$$\underline{\underline{a}} \ \underline{\underline{z}} = \underline{\underline{b}} \ , \quad \text{or} \quad \angle \underline{a} \mid Z \mid = \angle \underline{b} \mid \tag{111a,b}$$

then obviously, the matrix of the operator $\underline{z}$ will be

$$|Z| = |I(1, \underline{1})| \ \otimes \ |I(3, \underline{3})| \ \otimes \ |I(5, \underline{2})| \tag{112a}$$

and the 3X2 matrix $|Z|$ has the form

$$|Z| = \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{pmatrix} \tag{112b}$$

It is readily verified that this matrix has the properties

mentioned above, by substituting it in (111b).

In this way, any general 3X3, or 3X2, matrix is capable

of being written as the product of three implication matrices.

A 2X3 matrix can also be considered similarly as a product of two

SUB-IPL implications.

For example, the $\overset{3 \times 3}{\text{matrix}}$ $|Z|$ of (113) below,

$$|Z| = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \tag{113}$$

can be interpreted, for $\langle a |\, z\, | = \langle b |$, as

$$q_a(1) \longmapsto (0 \quad 0 \quad 1) \equiv \forall(b_p) \; ; \quad q_a(3) \longmapsto (1 \quad 1 \quad 0) \equiv \exists(b_p):$$

$$q_a(5) \longmapsto (1 \quad 0 \quad 0) \equiv \forall(b_p) \qquad\qquad (11\ a,b,c)$$

which, in words take the form

   (a) If a is true for all, then b is true for none ($\overline{\phi}$)

   (b) If a is true for some, then b is true for some
        or for all ($\exists$)   (115)

   (c) If a is true for none, then b is true for all ($\forall$).

If a has a mixed state, the state of b is obtained simply by

summing, using the operator $\cup$, the outputs corresponding to

each of the basic states contained in a. Thus, if $a = \exists \equiv (1\ 1\ 0)$,

then, for the matrix $|z|$ of (113),

$$b = (0\ 0\ 1) \oplus (1\ 1\ 0) = (1\ 1\ 1) \equiv \triangle, \text{ the indefinite state} \qquad (116)$$

If, however, $a = \wedge = (0\ 1\ 1)$, we obtain a definite state for b

as follows:

$$b = (1\ 1\ 0) \oplus (1\ 0\ 0) = (1\ 1\ 0) \equiv \exists \qquad\qquad (117)$$

We leave the further working out of these considerations for a

later report.

Acknowledgement

Parts III, IV and V of this series were written by the author as a CSIR Distinguished Scientist (Retd.) during January to June 1984. The work was supported by an Extramural Grant from CSIR, India, and the author is deeply grateful to Dr.G.S.Sidhu Director-General, CSIR, for having provided him with this grant as a special case. Thanks are due to Mr.M. Suresh for providing excellent secretarial assistance for the preparation of these reports.

List of References

1. Ramachandran, G.M., Vector-Matrix Representation of Boolean Algebras and Application to Extended Predicate Logic (EPL) Part I, Current Science, 1983, 52, 292-302.

2. -do-   Part II, Current Science, 1983, 52, 335-341.

3. -do-   Part III "Algebraic Theory for Categorical Statements in Quantified Predicate Logic (QPL)" (Submitted for publication) (Matphil Reports No. 34).

4. -do-   Part IV "General Theory for Statements of Type-1 in QPL". (Matphil Reports No. 35, Draft 2.)

5. Ramachandran, G.M., Syad-Nyaya System (SNS) — A New Formulation of Sentential Logic and its Isomorphism with Boolean Algebra of Genus 2, Current Science, 1982, 51, 625-636.

6. Suppes, P., Introduction to Logic, Van Nostrand, New York, 1957, p.50.

FIGURES

.114.

TABLES

————

Studies in Mathematical Logic[*]

Part VI – Elimination of Paradoxes and Incompleteness from Logic

(*This is the continuation of the series "Vector-Matrix
Representation of Boolean Algebras and Application to
Extended Predicate Logic (EPL)" – Part I to V.)

G.N. Ramachandran

INSA Albert Einstein Professor
Mathematical Philosophy Group
Indian Institute of Science
Bangalore 560 055

Studies in Mathematical Logic*

Part VI — Elimination of Paradoxes and Incompleteness from Logic

(*This is the continuation of the series "Vector-Matrix
Representation of Boolean Algebras and Application to
Extended Predicate Logic (EPL)" — Parts I to V.)

G.N. Ramachandran
Gita, 5, 10A Main Road
Malleswaram West
Bangalore 560 055

Preface and Summary

This report deals with two important consequences of the logical representation of both sentential logic and predicate logic which we have developed, using Boolean algebras BA-2 and BA-3. These are that two defects necessarily present in the conventional formulations of logic, namely the existence of paradoxes, and the existence of incompleteness in predicate logic, can both be eliminated by taking into account the new "mixed" states that automatically come in when the complete set of Boolean algebraic states in each case is taken into account. Thus, even propositional calculus (PC), having only the two mutually exclusive states T and F., is incomplete, in that the "mixed" state D ($\equiv$ "may be true or may be false") is a necessary consequence of reversing standard connectives like "or" and "if" in PC itself. However, the incompleteness is removed when PC is extended to SNS, isomorphic to BA-2, and D becomes (1 1), with T = (1 0) and F(0 1). In the same way, using BA-3 representation of predicate logic, which has only the four states—

"for all" ($\forall$), "not for all" ($\wedge$), "for none" (ϕ) and

"there exists"($\exists$), it leads to the existence of the state

envisaged in Godel's incompleteness theorem (which we denote by Σ)

by the application of the consistency check operator ($\vee$) to

$\exists$ and $\wedge$ — $\exists \vee \wedge = \Sigma$. Thus $\exists \otimes \wedge = (1 \ 1 \ 0) \otimes (0 \ 1 \ 1) =$

$(0 \ 1 \ 0) = \Sigma$. This state Σ is not envisaged in standard

predicate logic, but is a necessarily present basic state $(0 \ 1 \ 0)$

in BA-3, in which $\forall \equiv (1 \ 0 \ 0)$ and $\phi \equiv (0 \ 0 \ 1)$ are the

other two basic states. A theorem having this state Σ is sometimes

true, but not for all or for none; but this is not an "incompleteness"

in the extended form EPL (of standard predicate logic), isomorphic

to BA-3, which must have Σ as one of the three basic states,

among the $8(= 2^3)$ possible states in it. Thus, EPL is as complete

for predicate logic, as SNS is for sentential logic.

A paradox is defined to be an argument which, for both

input states T and F, leads to the output term being contradictory

(i.e. in state X). The essential patterns for this to happen

have been analysed and shown to be all derivable from one

pattern having the structure of the "Double Statement Paradox,

(DSP). This has the statements $\underline{a} \underset{\sim}{N} = \underline{b}$, $\underline{b} \underset{\sim}{E} = \underline{a}$. If the

circularity is broken at $\underline{a}$, then, starting from $\underline{a} = T$, or $\underline{a} = F$,

we get back only $\underline{a} = X$. <u>Vice versa</u>, if the circularity is

broken at $\underline{b}$, then $\underline{b} = X$ always follows from both the inputs

$\underline{b} = T$ and $\underline{b} = F$. However, if the SNS state D is allowed, then

$\underline{a} = D$ leads back only to $\underline{a} = D$ <u>via</u> $\underline{b} = D$, and similarly for $\underline{b}$.

Similarly, $\underline{a} = X$, or $\underline{b} = X$, as input also leads back to the

same state. Hence, the solution of the DSP is that the term

$\underline{a}$ "may be true or may not be true" (D), or "is impossible"(X).

There is no paradox; instead, we can derive the conclusion that

"the input statement is invalid" or that "the input statement

has the state D". Which of these is true depends on the semantics

of the problem.

By combining the two statements in DSP, we obtain the

equation $(\underline{a} \underset{\sim}{N} = \underline{a}) \equiv (\underline{a} \Longrightarrow \neg \underline{a}, \quad \neg \underline{a} \Longrightarrow \underline{a}) \equiv (\underline{a} \neq \underline{a})$.

This is called the Cretan Liar Paradox (CLP) and the best

example for $\underline{a}$ is: "What I say is false". Here, both $\underline{a}$ = T

and $\underline{a}$ = F lead to $\underline{a}$ = X, on breaking the circularity by writing

$\underline{a1}$ $\underset{\sim}{N}$ = $\underline{a2}$, $\underline{a1}$ $\underset{\sim}{V}$ $\underline{a2}$ = $\underline{a}$, and we have a paradox. However,

for CLP also, both $\underline{a}$ = D and $\underline{a}$ = X lead back to the input

state D, or X, as the case may be. Thus, a statement having the

pattern of CLP (as "This proposition is not true") may be said

to be invalid (X), or may be taken to have the state D; which

one is to be chosen will depend on the semantics of the sentence.

By reversing the second statement in DSP, we get the

"Contradictory Statements Paradox" (CSP) as follows: $\underline{a}$ $\underset{\sim}{N}$ = $\underline{b1}$,

$\underline{a}$ E = $\underline{b2}$, $\underline{b1}$ $\underset{\sim}{V}$ $\underline{b2}$ = $\underline{b}$. Here also, for $\underline{a}$ = T and $\underline{a}$ = F, $\underline{b}$ = X,

so that it is a paradox in PC; but it is not so, for $\underline{a}$ = D leads

to $\underline{b}$ = D (not X) and $\underline{a}$ = X also leads to $\underline{b}$ = X. No example of

this has been found, but several in the literature belong to a

closely related one (CSP-2), which runs as follows: $\underline{a}$ $\underset{\sim}{I}$ = $\underline{b1}$,

$\underline{a}$ $\underset{\sim}{I}$ $\underset{\sim}{N}$ = $\underline{b2}$, $\underline{b1}$ $\underset{\sim}{V}$ $\underline{b2}$ = $\underline{b}$. With the added condition $\underline{a}$ = T, $\underline{b}$ is X

and a contradiction follows, as in CSP. Examples of these

include Russell's paradox ($\underline{a} \equiv \ ^{"}\mathcal{R}$ is the set of all exclusive

sets"; $\underline{b} \equiv \mathcal{R} \subset \mathcal{R}$); and Cantor's paradox ($\underline{a} = \ ^{"}\mathcal{C}$ is the set

of all sets"; $\underline{b} \equiv \ ^{"}\mathcal{C}$ is the largest set"). It is our contention

that the CSP-2 is not a paradox, for $\underline{a}$ = F does not lead to a

contradiction, and only the assumption $\underline{a}$ = T leads to $\underline{b}$ = X,

so that we can deduce from it that $\underline{a}$ is F, from the rules of

classical propositional calculus. We thus deduce that there is

no set $\mathcal{R}$, which is at the same time "<u>exclusive</u>" and the "set of

<u>all</u> such sets". Similarly, there is no $\mathcal{C}$ which is both a "set,

<u>having a power set</u> $\mathcal{P}$ ($\mathcal{C}$) larger than itself" and "the <u>set of</u>

<u>all sets</u>". In the literature, both these examples are listed

as paradoxes having the pattern of CLP, and it is our contention

that they are not paradoxes even in classical propositional c

calculus, but arguments from which valid conclusions can be

derived by the <u>reductio ad impossible</u> method .

Thus, paradoxical arguments, which have no solution in PC, have the solution in the states D, or X, of SNS. The choice is X in most cases, saying that the premise of the argument is invalid; but there are situations in which the state D has semantic value, and does not merely state the indefiniteness of the available knowledge. As examples of the relevance of D as the only possible state of a term are given some properties of infinity (which will be extended further in a later report).

Finally, it is shown that, just as SNS (BA-2) and EPL (BA-3) are complete, so is any multivalued logic isomorphic to BA-n. So also, there can be no paradoxes in all logics isomorphic to BA-n, including SNS and EPL. Since PC has necessarily to be extended to SNS, when the paradoxes in it are removed, we thus prove quite generally that there cannot be any incompleteness or paradoxes in any logical system, provided the states in it are extended to be isomorphic to the appropriate Boolean algebra BA-n.

This report in the first draft has been prepared for

comment and discussion. Sections 1,2 and 3 are therefore written

as outlines, to be suitably modified for publication . Sections

4,5,6 and 7 are fairly complete and contain all the new material.

The author would invite critical comments on the contents and

presentation of this report.

———

<p style="text-align: center;">CONTENTS</p>

<u>Elimination of Paradoxes and Incompleteness from Logic</u>

<u>Introduction:</u>

1./ Logic is the science of reason and therefore the

formulation of the rules in logic cannot be such that they are

sometimes invalid, leading to paradoxes, or incomplete, leading

to possibilities not envisaged in the original rules. It is

our contention that the basic formalism and the set of rules

that are to be applied can be so made as to avoid both of the

above two difficulties — namely existence of paradoxes and

occurrence of incompleteness. This is done by mathematical

representation of logical states and their inter-relationships

in terms of the vector-matrix formalism for Boolean algebra

and logic that we have developed.

<u>Boolean-algebraic representation of logic:</u>

2./The connection between logic and set theory is a well

established one. So also is the isomorphism between the theory

of sets and Boolean algebra. Thus Boolean algebra BA-n containing

n basic elements, (in which all the states can be represented

by a Boolean vector with n elements $(a_1, a_2, ..., a_n)$), is

isomorphous with the algebra of all the subsets of a set

generated by n independent subsets in which no pair of subsets

have a common element. It has been shown in our earlier studies

that BA-2 is the appropriate representation for propositional

calculus (PC) which has two independent states "true"(T) and

"false"(F). Similarly BA-3 has been shown to be the appropriate

representation for quantified predicate logic (QPL) in which the

three independent non-overlapping states are "for all" ($\forall$),

"for some" ($\lesssim$) and "for none" (ϕ). The formalism that

arises out of this isomorphic representation of logic in terms

of the Boolean algebras BA-2 and BA-3 appears to be very powerful,

and their consequences, particularly in relation to manipulation

of arguments and working out their consequences, have been discussed

in a series of papers [1 to 6] .

"Mixed" states and their relevance to the main thesis of this

3./The most interesting consequence of this isomorphic   report:

representation by BA-2 and BA-3 is that   new "mixed" states

can be obtained by the superposition of the "basic" states in

both BA-2 and BA-3. In the former case, we get two new states

"doubtful" $(D \equiv T \oplus F)$ and "contradictory" $(X = T \otimes F)$, leading

to four different possible states in the extension of PC. We have

named this the "Syad-Nyaya-System(SNS)" from the Sanskrit words

Syad(= may be) and Nyaya (= logic). In the same way, in BA-3,

in addition to the four well-accepted quantified states

$\forall$, $\exists$, ϕ, Λ (not for all), four more states are introduced —

namely $\lessgtr$ (for some), $\ominus$ (for all or none), Δ (indefinite, for all,

for none, or for some), $\emptyset$ (impossible). This extended predicate

logic (EPL) contains $2^3 (= 8)$ possible states arising from the

mixture of two or more of the three basic states of BA-3 (for

details see $[1,2,3]$).

The important point to be remembered is that the BA-2

representation of PC automatically leads us, from two to the four

possible states T,F,D,X of SNS, so that the extension from the

usual BA-1 algebra of PC to the resulting BA-2 algebra of SNS

is inescapable. (See Section 4). So also, the representation

of the four common states of QPL, _via_ the three basic states of

BA-3, leads necessarily to the eight possible states mentioned

above for EPL. Consequently this is the proper set of states

which should be taken into account in discussing and working out

arguments in predicate logic.

In Section 4, we shall give a very brief outline of the

new properties of PC and QPL, in their extended forms, namely

SNS and EPL, obtained by using the Boolean algebraic representations

BA-2 and BA-3. We shall then show that both sentential logic

and predicate logic are complete under the usual operations

employed in standard logical arguments. In Section 5, we shall

give a general account of paradoxes as they are discussed in

standard books and articles on logic, but employing our formalism

and nomenclature. We shall show that they can be classified into

three types; and then demonstrate how the paradoxical situations

which arise in these examples, (namely that the equations lead to

contradiction for both inputs T and F), can be put in the form of

normal logical statements which is valid in the extended form of

PC, namely SNS, so that the equations are satisfied by either of

the new states D or X. In Section 6, examples of paradoxes listed

in the literature are examined and the resolution of these are

described. In Section 7, we show more generally that there can be

no paradoxes in logic and no incompleteness in any particular

formulation of multivalued logic. We thus show, in effect, that

provided we take into account the right algebraic formulation for

logic, then the mathematical condition, that the algebra is complete

and does not have contradictions, also leads to the result that

the homologous logical formalism is also free of inconsistencies,

paradoxes and incompleteness.

4. (a) Propositional calculus and SNS logic: From the example

of the conclusion to be derived from the two different types of

evidence, namely eye-witness and alibi, regarding the guilt or

(see Ref. [1]),
absence of guilt of the acused in a murder case / we can show that

there are four possibilities for guilt ($\underline{g}$): Guilty (g_T),

not guilty (g_F), may be or may not be guilty (g_D), and, the two

arises
evidences are contradictory (g_X). The last/ from the fact that

one of the two, namely eye-witness or alibi must be wrong

(See Ref [1]). The vector-matrix formulation of this in terms

of the two basic states T = (1 0) and F = (0 1) of SNS is

obvious, and we obtain D = (1 0) $\oplus$ (0 1) = (1 1) and

X = (1 0) $\otimes$ (0 1) = (0 0). (1 1) means that the answer

to the question, "can it be true?" is "yes," and also the answer

to the question, "can it be false?" is "yes". Similarly X = (0 0)

indicates that the answer to the question, "can it be true?" is

"no", and the answer to the question, "can it be false?" is also "no".

Therefore, there is no logical answer to the conclusion of the

statement which has the state (0 0). Consequently, either the

premises, or the construction of the argument, is faulty.

Good examples in which the new states D and X arise are

the following:

$$\underline{a} \, \underline{I} = \underline{b}, \; \underline{a} = F \longmapsto \underline{b} = \text{"T or F"} = D \tag{1}$$

Similarly,

$$\underline{a} \underset{\sim}{A} \underline{b} = T, \quad \underline{a} = F \longmapsto \underline{b} = X \tag{2}$$

The former one is one of the commonest statements that occur

in logic. We have the result that $/\underline{a} \Rightarrow \underline{b}/$ if is valid, and if $\underline{a}$ is true

then $\underline{b}$ is necessarily true. However, if $\underline{a}$ is false, we can

say nothing about $\underline{b}$—namely whether it is true or false. Therefore

its state in SNS is "T or F" = (1 0) $\oplus$ (0 1) = (1 1).

Using the formalism developed in Refs [1] and [2]

this eqn. takes the form

$$(0 \quad 1) \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} = (1 \quad 1) \quad \text{for} \quad \langle a \mid I \mid = \langle b \mid \tag{3}$$

For more details see the appropriate sections of Ref. [1] or [2].

In the same manner, taking Eq.(2) we/given that are $\underline{a}$ & $\underline{b}$ is

true, which necessarily requires that <u>both</u> $\underline{a}$ and $\underline{b}$ have the

logical state T. Therefore, if we have the condition that

$\underline{a} = F$ as the second equation in the l.h.s of (2), then there is

a contradiction, in that we have $\underline{a} = T$ as the conclusion of the

first statement and $\underline{a}$ = F as the condition of the second

statement, which are mutually contradictory. As mentioned in

Parts I and II, the vidya operator $\underset{\sim}{V}$ has been designed to check

for such contradictions and its algebra is as follows:

$$\underline{a}_1 \underset{\sim}{V} \underline{a}_2 = \underline{a} \longmapsto a_{1\alpha} \otimes a_{2\alpha} = a_\alpha \, , \quad a_{1\beta} \otimes a_{2\beta} = a_\beta \qquad (4)$$

It is readily verified that if $\underline{a}_1 = T = (1 \quad 0)$, $\underline{a}_2 = F = (0 \quad 1)$,

then $a_\alpha = 0$, $a_\beta = 0$ leading to $\langle a| = (0 \quad 0)$.

Since one whole set of paradoxes discussed in the literature

have to do with the occurrence of such contradictory states for

$\underline{a}_1$ and $\underline{a}_2$, we shall discuss here itself the conditions under which

Eq.(4) can lead to identical states for $\underline{a}_1$ and $\underline{a}_2$, when these two,

(namely $\underline{a}_1$ and $\underline{a}_2$), are negations of each other. The discussion

below is based mainly on Boolean algebra and the logical consequences

are pointed out thereafter. The negation operator is represented

by the matrix $|N|$ (as in 5a) with the properties, as in (5b) and (5c):

$$|N| = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} ; \quad (1 \quad 0) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = (0 \quad 1) \, ; \quad (0 \quad 1) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = (1 \quad 0) \quad (5a,b,c)$$

so that

$$a_T \underset{\sim}{N} = a_F \quad , \quad a_F \underset{\sim}{N} = a_T \qquad (5d,e)$$

For both these basic states T and F for $\underset{=}{a}$, therefore, we have the

result that $\underset{=}{a}$ and $\underset{=}{a}$ N are mutually contradictory, i.e.

$$\underset{=}{a} \underset{\sim}{V} (\underset{=}{a} N) = (0 \quad 0) = X \qquad (6)$$

However, the Boolean algebra of SNS has, in addition, two other

mixed states D and X, and for these we have the following

consequences, as in (7) and (8):

$$a_D \underset{\sim}{N} = (1 \quad 1) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = (1 \quad 1) = a_D \qquad (7)$$

$$a_X \underset{\sim}{N} = (0 \quad 0) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = (0 \quad 0) = a_X \qquad (8)$$

Hence, for the mixed state D, for $\underset{=}{a}$, Eqn.(6), written in the form

(9a, b)

$$\underset{=}{a}_1 = \underset{=}{a}_1 \quad , \quad \underset{=}{a}_2 = \underset{=}{a}_1 \underset{\sim}{N} \qquad (9a,b)$$

does not lead to X, when $\underset{=}{a}_1$ = D. Instead, we obtain the result

in (9c),

$$\underset{=}{a} = \underset{=}{a}_1 = \underset{=}{a}_2 \quad , \quad \text{for } \underset{=}{a}_1 \underset{\sim}{V} \underset{=}{a}_2 = \underset{=}{a} \quad , \quad \text{if } \underset{=}{a}_1 = D \qquad (9c)$$

The logical consequence of this is very simple. If we are

given that a statement a_1 leads to a negated form of it as a_2,

then a_1 and a_2 can both be simultaneously valid (true) provided

a_1 is not one of the two basic states T or F, but is the mixed

state D. To put it in words, if something is D, i.e. "it may be

true or may be false", then the negation of this has the property that

"it may be false or it may be true", which is equivalent to the

state D once again. In other words, if the statement is such

that it is impossible to assert the absolute truth of it or the

absolute falsity of it, then its negation being equivalent to

itself does not lead to a contradiction.

It will be noticed that what has been said above has nothing

paradoxical about it. We have merely considered the consequences

of equating a statement in propositional calculus with its negation

and tried to find out if there are any possible logical states

for the statement for which this equation does not lead to a

contradiction. Then, we find that if a = D, then, by (7), a N

is also D so that $\underline{\underline{a}} = \underline{\underline{a}} \underline{N}$.

This property that $\underline{\underline{a}}_1 = \underline{\underline{a}}_2$, as in (9a, b) is also true

for a_X, i.e. for $\underline{\underline{a}} = X$, as is shown in (8). However, now

$$\underline{\underline{a}}_1 = \underline{\underline{a}}_2 = \underline{\underline{a}} = X, \text{ where } \underline{\underline{a}} = \underline{\underline{a}}_1 \underset{X}{V} \underline{\underline{a}}_2 \qquad (10a,b)$$

The logical consequence of this is that, if something is X, i.e.

"it has the property of being impossible" (i.e $\langle a| = (0 \ 0))$, then

its negation $\underline{\underline{a}}' = \underline{\underline{a}} \underline{N}$ is also (0 0) and is impossible. It is

a moot   point if this can be taken to mean that $\underline{\underline{a}}' \equiv \underline{\underline{a}}$, but

algebraically (8) shows that $\langle a_X| N| = \langle a_X|$. We shall show in

Section 6 that the states D and X, thus shown to satisfy the

equation $\langle a| N| = \langle a|$, can be made use of for getting over a

whole range of paradoxes. Anticipating the discussion in Section 6,

we shall call these to be of the Cretan Liar Paradox (CLP) type.

(b) Absence of closure for PC and extension to SNS: Clearly, if

we operate only with the two states of BA-1, whose algebra is

isomorphic to that of propositional calculus (as discussed in

standard books on logic), then the equation $\underline{\underline{a}} = \underline{\underline{a}} \underline{N}$ has no solution

in BA-1, or the Cretan liar paradox is readily a "paradox",

and a defect of the logical system of PC. However, we have

shown that the inclusion of "reverse" relations in PC lead to

the four states T, F, D, X, of SNS, isomorphic to the four

elements of BA-2, namely (0 0), (0 1), (1 1) and (0 0).

To be specific, in PC we ask the reverses of relations like —

$$\text{"}\underline{a} \text{ or } \underline{b} \text{ is T ; } \underline{a} \text{ is T ; What is the state of } \underline{b}\text{?"} \tag{11a}$$

and

$$\text{"}\underline{a} \text{ and } \underline{b} \text{ is T, } \underline{a} \text{ is F ; What is the state of } \underline{b}\text{?"} \tag{11b}$$

and we get the answers,

$$\underline{b} = D = (1 \quad 1) \text{ for } (11a) \tag{12a}$$

$$\underline{b} = X = (0 \quad 0) \text{ for } (11b) \tag{12b}$$

This extension of PC to SNS for sentential logic has not been done

for the purpose of dealing with paradoxes, but to make the algebra

"closed". We find that the inclusion of reverse relations in BA-1

leads to its extension to BA-2 unequivocally, as in (11) and (12).

As we will show in Section 6, it also leads to the elimination of

the CRP type of paradoxes, including new paradoxes, of the types which

we shall formulate.

We shall now ask whether the extended SNS sentential logic

is also complete or whether it has to be extended still further,

as PC with two states had to be extended to four states by asking

questions as in (11) and (12). The answer to this is that, if

only the relations of the type envisaged in standard propositional

calculus as discussed in the literature are asked, then SNB and

its associated algebra BA-2, are sufficient to represent the

logic of all statements in propositional calculus. This is

very briefly shown by the following considerations. Thus, there

are unary and binary relations in PC, and these are represented

by equations of the type,

$$\langle a| \; Z| \; = \; \langle b| \tag{13a}$$

and

$$\langle a| \; Z \; |b\rangle = c_\alpha \; , \quad \langle a \; | \; Z^c|b\rangle = c_\beta \; , \quad \langle c| \; = (c_\alpha \; , \; c_\beta \;) \tag{13b}$$

When these are reversed, we obtain from (13a) the relation (13c):

$$\langle b| \; Z^t| \; = \; \langle a| \tag{13c}$$

in which $|Z^t|$ is also one of the 10 (or 16) matrices that are

possible for $|Z|$, so that no new relations arise from the reversing of

unary relations. On reversing binary relations, as shown in

Section 3(b),

Ref[2], | we have in effect,

$$\langle a \,|\, Z \,| = \langle b \,| \;, \quad \text{if} \quad \langle c \,| = T$$

$$\langle a \,|\, Z^C \,| = \langle b \,| \;, \quad \text{if} \quad \langle c \,| = F$$

$$\langle a \,|\, D \,| = \langle b \,| \;, \quad \text{if} \quad \langle c \,| = D$$

$$\langle a \,|\, X \,| = \langle b \,| \;, \quad \text{if} \quad \langle c \,| = X$$

(14a-d)

and once again the operative logical equations are of the unary

type which are contained in (13a). From matrix connectives, applied

of relations,

in the unary and binary form/nothing new comes out by reversing

them. Since all possible logical relations using the standard

(of PC)

connectives "not", "and", "or"/ can be represented by 2X2 Boolean

matrices in BA-2, we have the result that such connectives lead

to closure for PC.

This is, however, not the complete picture, for in BA-2

we can also have binary relations involving the connectives

$\underset{\sim}{U}$ and $\underset{\sim}{V}$ and we shall examine below their reversal and show that

this leads to states more complex than those envisaged by BA-2 even

in the extended form of PC, namely SNS.

(c) Absence of closure for SNS and extension to EFL: As

mentioned above, reversal of matrix connectives, namely those

obtained from "not", "and", "or", leads only to relations

expressible once again by 2X2 Boolean matrix connectives in SNS.

However, as mentioned in Ref [1], there are two more connectives,

namely "upon" ($\underset{\approx}{\underline{U}}$) and "vidya" ($\underset{\approx}{\underline{V}}$), which lead to binary relations

between SNS logical systems. Their properties have been

extensively dealt with in Refs [1, 2 and 3] . However, this has

been done only $\overset{for}{\sout{at}}$ forward operators in the form of relations

of the type $\underline{\underline{a1}} \ \underset{\approx}{\underline{U}} \ \underline{\underline{a2}} = \underline{\underline{a}}$, $\underline{\underline{a1}} \ \underset{\approx}{\underline{V}} \ \underline{\underline{a2}} = \underline{\underline{a}}$. If we assume that

there are only two states T and F, then $\underset{\approx}{\underline{U}}$ and $\underset{\approx}{\underline{V}}$ have the same

properties as the matrix connectives $\underset{\sim}{O}$ and $\underset{\sim}{A}$ respectively,

and there is no meaning in introducing two new operators $\underset{\approx}{\underline{U}}$ and $\underset{\approx}{\underline{V}}$

in the algebra of logic containing only truth values T and F

(as in PC). This fact is well-known. However, the truth table

for $\underset{\approx}{\underline{U}}$ is quite different from that of $\underset{\sim}{O}$, and similarly that of

$\underset{\sim}{V}$ is quite different from that of $\underset{\sim}{A}$ in SNS logic. (See

Table 4 of [1] , and also Eqns (14) and (15) of the same

reference.) For ready reference we give below in Table 1 the

truth table for the vidya operator $\underset{\sim}{V}$ in the forward direction,

namely for the relation,

$$\underline{a} \underset{\sim}{\underline{V}} \underline{b} = \underline{c} \tag{15}$$

Table 1. SNS truth table for the vidya operator $\underset{\sim}{V}$. (See page 18)

Now if we ask the reverse of (15) — namely, given the SNS states

of $\underline{c}$ and $\underline{b}$, what is the logical state of $\underline{a}$? — then we get

very strange results. Just as such a question asked for the

reversal of the "and" and "or" relations in PC (BA-1) led to

states that are not expressible in BA-1, but only in BA-2, here

also we find, as the result of the reverse relation

$$\underset{\sim}{\underline{V}} \underline{a} = \underline{b} \tag{16}$$

states that are not expressible in BA-2. For this purpose, we

have listed below in Table 2 the states of $\underline{b}$ corresponding

to the combinations $(\underline{c}, \underline{a})$, as deduced from the data given

in Table 1.

Table 2. Reverse relations obtained from Table 1

We shall indicate the derivation of the various lines in

Table 2 as below. Taking Sl.No. 1, if $\underline{c}$ is T and $\underline{a}$ is T, we

consider the horizontal row 1 against entry T in Table 1 and

look up the corresponding entries for $\underline{b}$, whenever $\underline{c}$ = T.

We thus obtain the two possibilities, T and D. Thus, we get the

possibilities that if $\underline{c}$ = T and $\underline{a}$ = T in the relation $\underline{a} \underset{\sim}{V} \underline{b} = \underline{c}$,

then $\underline{b}$ can be T or $\underline{b}$ can be D. However, this is not the complete

answer. It should be noted that $\underline{b}$ cannot be F, since the corresponding

entry for $(\underline{a}, \underline{b})$ = (T, F) is X in the first row of Table 1.

Thus, D has to be qualified by saying that it is not F. This is

what is written against Sl.No. 1 for $\underline{b}$ in Table 2. The row against

Sl.No. 5 follows in exactly the same manner. We have thus the

new state "D, but not T", in addition to "D, but not F" found in Sl.No.1

Table 1. <u>SNS truth table for the vidya operator</u> $\underset{\sim}{V}$

$$\underline{a} \; \underset{\sim}{V} \; \underline{b} \; = \; \underline{c}$$

$\underline{a}$	$\underline{\underline{b}}$	T	F	D	X
T		T	X	T	X
F		X	F	F	X
D		T	F	D	X
X		X	X	X	X

Table 2. <u>Reverse relations obtained from Table 1</u>

$$\underline{c} \; \overset{\leftarrow}{\underset{\sim}{V}} \; \underline{a} \; = \; \underline{b}$$

Sl. No.	$\underline{c}$	$\underline{a}$	$\underline{\underline{b}}$
1	T	T	T ;"D, but not F"
2	T	F	Impossible
3	T	D	"Always T"
4	F	T	Impossible
5	F	F	F ;"D, but not T"
6	F	D	"Always F"
7	D	T,F	Impossible
8	D	D	"D, but not T, and not F"

If we take Sl.No. 2 namely c = T, a = F, we look at the second

rwo in Table 1 against a = F when we find that there is no entry

c = T. Consequently this combination of c and a is impossible,

and cannot lead to any logical state for b, except the "impossible"

state. (Anticipating later discussion, this "impossible" state

is (0   0) in BA-2 and (0   0   0) in BA-3. We have explained

the property of this state in BA-3 in Refs [3-6] and utilized

it for various purposes.) A similar explanation holds for the

entry of [an] "impossible" state in [each of] Sl. Nos. 4 and 7.

        Considering Sl.No.3, c = T, a = D, the third row [of Table 1] shows

that there is only one such entry for c and this corresponds to

b = T. Hence b = F and b = D are eliminated, and we indicate

it by writing for b "Always T" against Sl.No.3. We shall explain

below the difference between [a] simple T and always T. A similar

explanation holds for "Always F" in row No. 6. We have only

the last row 8 to be considered. In this case, we wish to find out

the state of $\underline{b}$ corresponding to $\underline{c}$ = D and $\underline{a}$ = D and looking

at row No. 3 in Table 1, we find that $\underline{b}$ = D. However, this is

not the whole answer. In SNS, D corresponds "T or F", but in

the outcome of the combinations of $\underline{c}$ and $\underline{a}$ in row 8, we cannot

have the possibilities "T" or "F" for $\underline{b}$, ~~for~~ *since* these lead to T or F for

$\underline{c}$ also and not D as *w* ~~we~~ required. Hence it is that the outcome

of ~~&~~ *(the combination in Row 8)* for $\underline{b}$ is given as "D, but not T, and not F".

We shall now show that the strange new states for $\underline{b}$ outlined

in Table 2 are expressible by a Boolean vector-matrix representation

in BA-3. We shall not derive the structure of the Boolean algebra

which is isomorphic to the states contained in Table 2; instead

we shall mention first that such an algebra is BA-3, ~~and~~ state

the isomorphism between its three basic states and three out of

the various states mentioned in the r.h.s of Table 2, and then

show that this Boolean algebra represents all the different

states for $\underline{b}$ that is listed in Table 2. Since we already know

from the Ref $\left[2\text{-}5\right]$ that an isomorphism exists between BA-3 and

EPL, we shall use the notation of the latter to denote the

corresponding states in Table 2. It turns out that this is

indeed most appropriate and the corresponding states, obtained

by the reversal of the vidya operator in SNS, and those that are

found in the extension of the four states of predicate logic,

have a close logical interrelationship.

Thus, the three basic states (1 0 0), (0 1 0), (0 0 1)

of BA-3 will represent the following three states from among

those listed in Table 2.

$$\text{"Always T"} \longmapsto (1 \quad 0 \quad 0) \equiv \forall \quad \text{(say)} \tag{17a}$$

$$\text{"D, but not T, and not F"} \longmapsto (0 \quad 1 \quad 0) \equiv \Sigma$$

$$(\equiv \text{"Sometimes T"}), \text{ say} \tag{17b}$$

$$\text{"Always F"} \longmapsto (0 \quad 0 \quad 1) \equiv \Phi \quad (\equiv \text{"Never T"}), \text{ say} \tag{17c}$$

In this, the second basic state "D, but not T and not F" has the

property that it includes all possibilities where it is sometimes

T and sometimes F, but that it excludes the states "Always T", and

"Always F". In this sense, Σ is different from the mixed state

D (= (1   1)) in SNS logic. Thus, D $\otimes$ T = T and D $\otimes$ F = F, although D itself is a mixed state which is sometimes T and sometimes F. However, in the extended BA-3 representation as given in (17) the second state, (0   1   0), also represents a doubtful state, but it $\underline{excludes}$ the pure states T and F. In consequence $\Sigma \otimes \bigvee$ = (0   0   0) and $\Sigma \otimes \Phi$ = (0   0   0). We mention this here to indicate that the extension from BA-2 to BA-3 produces certain changes in the connotation and representation of the corresponding logical states in SNS and EPL respectively. (The ideas of basic states and mixed states, as employed in the above Boolean algebraic representation, have quite a lot to do with the properties of pure states and mixed states in the vector space representation of states in quantum mechanics).

Having defined the three basic states of BA-3 by (17a), (17b) and (17c), we can associate three more of the states for $\underline{b}$ given in Table 2 with their Boolean algebraic counterparts as follows;

"D, but not F" $\longmapsto$ (0 1 0) $\oplus$ (1 0 0) $\eqcirc$ (1 1 0) $\eqcirc$ $\exists$ (say) (18a)

"D, but not T" $\longmapsto$ (0 1 0) $\oplus$ (0 0 1) $\eqcirc$ (0 1 1) $\eqcirc$ $\bigwedge$ (say) (18b)

" Impossible " $\longmapsto$ (0 0 0) $\eqcirc$ $\emptyset$ (say) (18c)

In making these associations, we notice that the D which occurs

in these is the SNS D and includes the possibilities of T and F.

Hence "D but not F" = $\forall \oplus \Sigma$ = (1 1 0) and "D, but not T" =

$\Phi \oplus \Sigma$ = (0 1 1). Effectively, we erase out the limiting state

T or F which is stated to be not present, and thus get two more EPL

states analogous to well-known states in quantified predicate logic,

as used in standard literature.

On examining (17a-c) and (18a-c), it will be seen that we have

obtained the analogues of the four possible states as used in

standard first order predicate logic — "for all" ($\forall$), "there

exists" ($\exists$), "for none" (Φ) and "not for all" ($\bigwedge$). Hence

we can say that effectively we have generated predicate logic and

its four logical states from the reverse relations obtained using

the vidya operator and using different states for $\underline{c}$ and $\underline{a}$ in this

relation $c \overset{\leftarrow}{\underset{\sim}{V}} a = b$. Incidentally, there are two more states

that are obtained by checking through the list of new states

obtained from the above reverse relation, namely $\Sigma = (0\ \ 1\ \ 0)$ which

corresponds to EPL state "some", and $\emptyset = (0\ \ 0\ \ 0)$ corresponds

to EPL state "impossible". These two states are not considered

in standard literatu‹e on predicate logic. In fact, as will be

seen in the Sections 5 and 6, the state Σ, excluding V and Φ, 4A, occurrence of the

is considered to be an indication of the incompleteness of first

order predicate logic, (as in Gödel's incompleteness theorem). However, Obviously it is an extra state obtainable

by manipulation with the four QPL states, for example,

$$\exists \otimes \wedge = (1\ \ 1\ \ 0) \ \otimes \ (0\ \ 1\ \ 1) \ = \ (0\ \ 1\ \ 0) \tag{19}$$

We have indicated this in Ref [3] (p. 340) and it is certainly

true that this state is outside the range of states acceptable

for logical terms in standard predicate logic. However, as

mentioned earlier, the correct way of looking at these states is

by their isomorphism with the corresponding states of BA-3 and

we find that BA-3 has this state $\sum = (0 \ 1 \ 0)$, as one of the

basic states of the algebra. Hence, there is no incompleteness

if the proper extended set of states of predicate logic are

taken into account. In that sense, we shall complete the set of

six states for Table 2 — in (17) and (18) and obtain two more

so as to complete the set of eight states which the BA-3 representation

associated with it has in its list. Thus we define,

"D, or T, or F" = $(0 \ 1 \ 0) \oplus (1 \ 0 \ 0) \oplus (0 \ 0 \ 1) = (1 \ 1 \ 1) \equiv \Delta$ (say)

and
$$(20a)$$

"T or F, but not D" = "Always T" or "Always F" =

$(1 \ 0 \ 0) \oplus (0 \ 0 \ 1) = (1 \ 0 \ 1) \equiv \ominus$ (say) $\qquad$ (20b)

This way of arriving at the eight states of BA-3 and their

logical correspondences is very instructive, in that we are able

to see the full scope of the meaning of these eight states in

predicate logic. The really new state $\sum = (0 \ 1 \ 0)$ comes out as

"D, but not T and not F", a state which comes only in a complicated

way from the four states of classical predicate logic as in Eq.(19).

We are able to see straightaway the existence of <u>three</u> primitive

states in the logic and a re-examination of the equations in

(17) to (20) will show that they have a coherent structure and

form a complete set of $2^3 = 8$ states that correspond to BA-3,

which, therefore, is the right set to be adopted for dealing with

predicate logic. Thus, SNS (BA-2) on extension leads to EPL

(BA-3), just as PC(BA-1) on extension leads to SNS(BA-2). Also,

SNS is complete (closed) with respect to matrix operators used

in unary forward, unary reverse, binary forward and binary reverse

relations. It is also closed with respect to Boolean operators

$\underset{\sim}{\text{U}}$ and $\underset{\sim}{\text{V}}$ applied in binary <u>forward</u> relations; but it has to be

extended to a Boolean algebra of higher genus when these are

reversed. Therefore, if we say that sentential logic (which

requires the SNS form of algebra for its full representation)

is closed, we mean only what has been stated above — i.e. it

is closed with respect to matrix operators of all types and only

for Boolean operators applied in the forward direction. Let us

consider the corresponding closure properties of EPL which will

be done in the next subsection 4(d).

(d) Gödel's incompleteness theorem does not hold for EPL, but
   only for QPL: We have seen above that SNS is closed in the

classical sense i.e. with respect to matrix operators as mentioned

there. It is very easy to see that EPL (BA-3) is also closed

to the same extent, i.e. for matrix and Boolean operators applied

in exactly the same way (see the general theory for matrix

connectives in the theory of relations in Ref.[2] sections $2^{\ell}3$).

What we see from that general theory is that the eight states of

EPL form a closed set when matrix connectives are applied to them

for unary forward, unary reverse, binary forward and binary reverse

relations. Also this set is closed with respect to binary forward

Boolean operators $\mathbb{U}$ and $\mathbb{V}$ as in $a \oplus b = \mathbb{C}$ and $a \otimes b = \mathbb{C}$.

If a and b are one of the eight states of EPL, then $\mathbb{C}$ is also

one of these. Hence predicate logic, in its extended form EPL, is

logic

as much a closed system, as sentential is considered to be so.

in EPL,

(We are not considering the reversal of <u>Boolean</u> operators as

it is too complicated; but the existence of this possibility

is to be remembered while considering completeness of the

extended predicate logic).

As mentioned in the headings of this subsection, the

completeness of EPL does not mean that the smaller set of four

states $\forall$, $\exists$, Φ, $\wedge$ form a closed set. In fact, by applying

the Boolean operators of BA-3, namely "upon" ($\cup$) and "vidya"

($\vee$), we obtain states other than these four as in (21a-d).

$$\forall \cup \Phi \equiv (1 \quad 0 \quad 0) \oplus (0 \quad 0 \quad 1) = (1 \quad 0 \quad 1) = \ominus \qquad (21a)$$

$$\forall \vee \Phi \equiv (1 \quad 0 \quad 0) \otimes (0 \quad 0 \quad 1) = (0 \quad 0 \quad 0) = \emptyset \qquad (21b)$$

$$\exists \cup \wedge \equiv (1 \quad 1 \quad 0) \oplus (0 \quad 1 \quad 1) = (1 \quad 1 \quad 1) = \Delta \qquad (21c)$$

$$\exists \vee \wedge \equiv (1 \quad 1 \quad 0) \otimes (0 \quad 1 \quad 1) = (0 \quad 1 \quad 0) = \Sigma \qquad (21d)$$

Hence if suitable sentences are written in QPL using only the

four standard states of predicate logic as in the literature,

but using the properties of $\bigcup$ and $\bigvee$, then the resultant

conclusion may have the BA-3 states corresponding to one of

the four additional states given in (21a-d). In particular,

it should be mentioned that the combined information from two

such QPL statements in the QPL states $\beth$ and Λ can lead <u>via</u>

the vidya operator $\bigvee$ to the state Σ . We have seen that Σ

corresponds to the property of $\overset{a}{/}$ statement $\underset{\wedge}{/}$ being "true for some x,
$\overset{\mathcal{D}(x)}{}$

but not always true or never true". This statement is the

famous G$\overset{"}{o}$del's incompleteness theorem, which we $\underset{\wedge}{/}$ proved to be
$\overset{have}{}$

possible, in one step in Eq (21d). What we say in effect is that

if we have only the QPL states available, then by using the

logic of the vidya operator as $\underset{\wedge}{/}$ connective, we can get out of them
$\overset{the}{}$

the G$\overset{"}{o}$del's state Σ . At the same time, we also say that the

occurrence of Σ is not an indication of incompleteness of

predicate logic, for this state Σ occurs in the extended form EPL

of QPL, and, together with seven other states, forms a closed

system for predicate logic. This closed system has the Boolean

algebraic structure of BA-3 and it should be noted that some

of the states of BA-3 are absolutely necessary for properly

representing the properties of QPL. Hence the extension to

EPL is not artificially made, but is a natural consequence of

obtaining the whole BA-3 set of states to represent the properties

of predicate logic. For ready reference, we may mention that

these four/states are $\odot$ ($\equiv$ All or none), $\emptyset$ ($\equiv$ impossible),

additional

$\triangle$ ($\equiv$ universal doubt), Σ ($\equiv$ some, but not all and not none).

We shall give examples of the theory mentioned in this

subsection when dealing with paradoxes and incompleteness in

the next sections 5, 6 and 7.

5. Aetiology of Paradoxes

In this section, we shall first consider in general what

an
are the broad properties of/argument that lead to a paradox.

they
Most generally / may be stated as follows. We carry out the

steps listed for the argument and finally arrive at two truth

values a̲1̲ and a̲2̲ for the conclusion a̲ of the argument, which,

however, have the property that

$$\underline{a1} \; \underset{\sim}{V} \; \underline{a2} \; = \; X \; = \; \text{"impossible"} \tag{22}$$

Also, the steps of the argument, and the truth values of the

terms that are input into the argument are all either unquestionable,

or, for all possible states of the inputs, the result (22) follows.

Then, the argument is said to be a paradox. The result in Eq.(22)

can occur, if there occur the equations:

$$\underline{a1} \; = \; \underline{s} \; , \; \underline{a2} \; = \; \neg\underline{s} \; = \; \underline{s} \; \underset{\sim}{N} \tag{23}$$

and further a̲1̲ and a̲2̲ are restricted to the possible truth

values T and F. Put in words, a paradoxical statement always

leads to the conclusion that something is true and that it is

false at the same time. Then if true and false are the only

states that are permissible (as in propositional calculus), then

a̲1̲ V̲ a̲2̲ will lead to the impossible state X = (0   0). (The

extension of these criteria for a paradoxical statement in

predicate calculus is discussed in Section 7.)

Depending upon how $\underline{a}1$ and $\underline{a}2$ in (22) are arrived at, there

is a whole range of possibilities of writing paradoxical arguments.

We shall, however, consider, in detail, only three structural

types that cover practically all examples — namely Double

Statement Paradox (DSP), Cretan Liar Paradox (CLP) and

Contradictory Statement Paradox (CSP).

We shall give here   the essence of the argument,

with a symbolic presentation of the essence of the ideas. Actual

examples are given in the next section 6.

(a) Double statement paradox (DSP): The essential structure of

this type of argument can be given by the two equations

$$\underline{a} \underset{\sim}{N} = \underline{b} \quad ; \quad \underline{b} \underset{\sim}{E} = \underline{a} \qquad\qquad (24a,b)$$

If we call the input to (24a) as $\underline{a}1$ and the output of (24b)

as $\underline{a}2$, then it is readily seen that

For both $\underline{a}1 = T$ and $\underline{a}1 = F$, $\underline{a}1 \underset{\sim}{V} \underline{a}2 = X$ \qquad (24c)

The graph of this process, which leads to a contradiction,

irrespective of whether we start with $\underline{a}$ = T, or $\underline{a}$ = F, is shown

in Fig 1(a and b).

Fig.1. Graph of the essential logic of the Double Statement Paradox
       (a) By breaking the circularity at $\underline{a}$
       (b) By breaking the circularity at $\underline{b}$

In Fig.1(a),   we have broken the circularity of the argument

in (24) at the term $\underline{a}$ (following the procedure mentioned in

Part $\overline{V}$, Section 4(c)), and obtained $\underline{a1}$ and $\underline{a2}$, which are to be

compared by the vidya operator to give the resultant $\underline{a}$. Then the

results given below in (25) follow.

$$\underline{a1} = T \longmapsto \underline{b} = F \longmapsto \underline{a2} = F \longmapsto \underline{a} = X \qquad (25a)$$

$$\underline{a1} = F \longmapsto \underline{b} = T \longmapsto \underline{a2} = F \longmapsto \underline{a} = X \qquad (25b)$$

As mentioned above, both the states T and F introduced for $\underline{a1}$

lead to the contradictory state as output for $\underline{a}$, showing that

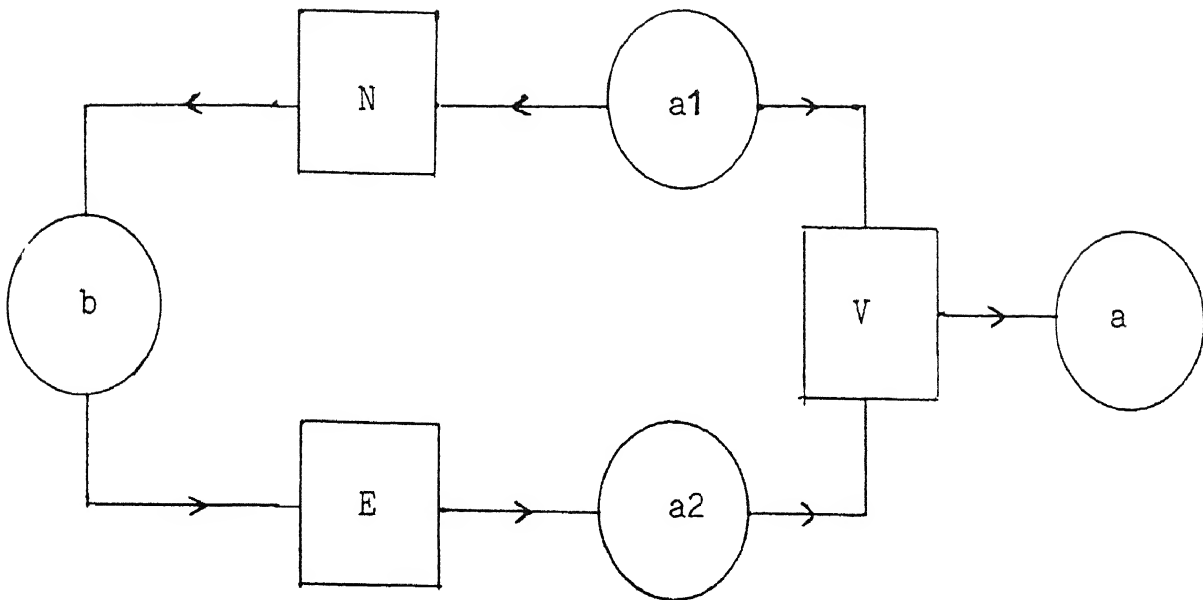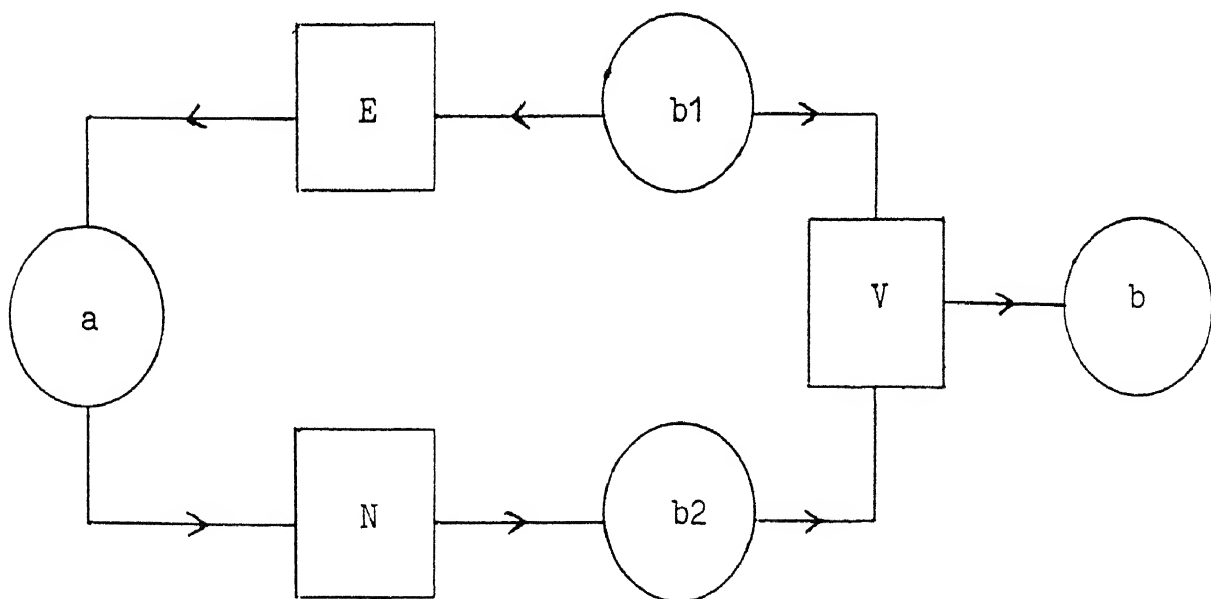the argument is invalid, both for the inputs T and F. Hence it

Fig. 1(a)

Fig. 1(b)

Fig.1. Graph of the essential logic of the Double Statement Paradox
(a) By breaking the circularity at a
(b) By breaking the circularity at b

forms a paradox, since no state in propositional calculus for

the input leads to a valid result.

We may also break the circularity of the argument in (24)

at the term $\underline{b}$ and obtain Fig. 1(b). For this, we obtain,

analogous to (25a,b), the results

$$\underline{\underline{b1}} = T \longmapsto \underline{a} = T \longmapsto \underline{\underline{b2}} = F \longmapsto \underline{b} = X \qquad (26a)$$

$$\underline{\underline{b1}} = F \longmapsto \underline{a} = F \longmapsto \underline{\underline{b2}} = T \longmapsto \underline{b} = X \qquad (26b)$$

Thus, we see that, even if we start with $\underline{b}$, the output of the

argument is X, for both the states T and F of $\underline{b}$, so that it is

a paradox.

Eqs. (24a,b) are typical of all <u>circular arguments</u> that are

paradoxes. We have a path from $\underline{a}$ to $\underline{b}$ and another back from

$\underline{b}$ to $\underline{a}$, and, effectively, one has the unary connective $\underset{\sim}{N}$ connecting

$\underline{a}$ with $\underline{b}$, and the other has the unary connective $\underset{\sim}{E}$. (See below

for an even shorter paradoxical statement obtained by combining

the two equations (24a) and (24b). This yields the CLP).

In fact, the DSP is the most crucial paradoxical argument

to be considered, for the other two, namely CSP and CLP, can

both be worked out by suitably manipulating the two equations

in it. We shall give a very simple example of DSP in Section 6

below.

It is obvious that the state $\underline{a1}$ = D leads back to $\underline{a2}$ = D,

so that $\underline{a1} \underset{\sim}{V} \underline{a2}$ = $\underline{a}$ gives the same state as the input state $\underline{a1}$.

(This is in SNS). This also happens for $\underline{a1}$ = X. Hence the

solution to the double statement paradox, (as stated in (24))

is that the entities denoted by the input $\underline{a1}$ can have the

possibilities,

"It may be true or not true"; or "It does not exist".

when the output $\underline{a2}$ is the same as the input $\underline{a1}$, and the paradox

is eliminated.

(b) Contradictory statements paradox (CSP): The statements in (24a,b,c

can also be written equivalently as in (26a,b,c), in which we

obtain two outputs $\underline{b1}$ and $\underline{b2}$, starting from $\underline{a}$ as input (whose

graph is shown in Fig. 2).

$$\underline{a} \underset{\sim}{N} = \underline{b1} \; ; \; \underline{a} \underset{\sim}{E} = \underline{b2} \; ; \; \underline{b1} \underset{\sim}{V} \underline{b2} = \underline{b} \qquad (26a,b,c)$$

On inputting a = T and F, we obtain,

$$\underline{a} = T \longmapsto \underline{b1} = F \; , \; \underline{b2} = T, \text{ so that } \underline{b} = X \qquad (27a)$$

and $\quad \underline{a} = F \longmapsto \underline{b1} = T \; , \; \underline{b2} = F, \text{ so that } \underline{b} = X \qquad (27b)$

Fig. 2. Logical graph of the contradictory statements in (a) the
CSP paradox as given in Eqs(26a,b,c); (b) the CSP-2
paradox in (30a,b,c,d).

Hence, $\underline{b}$ becomes "impossible" (X) for both the inputs T and F

for $\underline{a}$, so that the argument symbolized in (26) (which is

equivalent to DSP), is again a paradox. We shall call this as

the Contradictory Statements Paradox (CSP), since the two

statements in (26a) and (26b) are mutually contradictory. It

is generic of all examples where, starting from an input $\underline{a}$, we

follow two paths, and arrive at mutually contradictory results

$\underline{b1}$ and $\underline{b2}$ for $\underline{b}$, and this is so, irrespective of whether the input

$\underline{a}$ has the state T or F. We shall discuss this further also
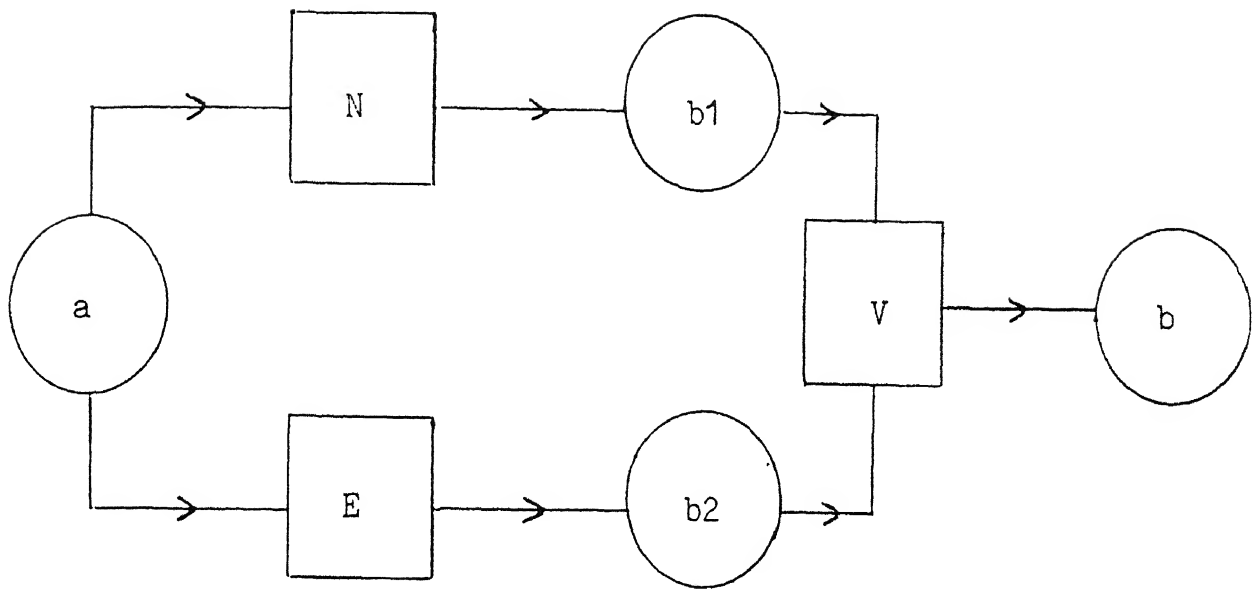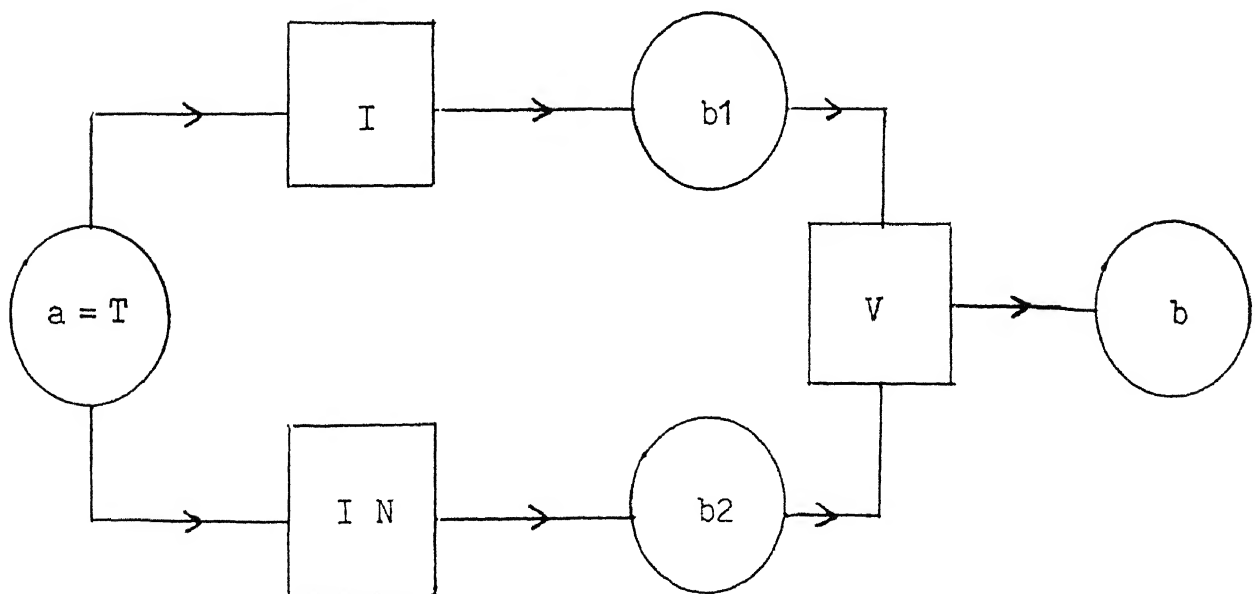
in Section 6.

Fig.2(a)

Fig.2(b)

Fig.2. Logical graph of the contradictory statement in
(a) the CSP paradox as given in Eqs. (26a,b,c);
(b) the CSP-2, as in (30a,b,c,d).

(a)   (b)

Fig.3   Logical graph of the Cretan Liar Paradox;

(a)   Circular argument as in Eqn. (27).

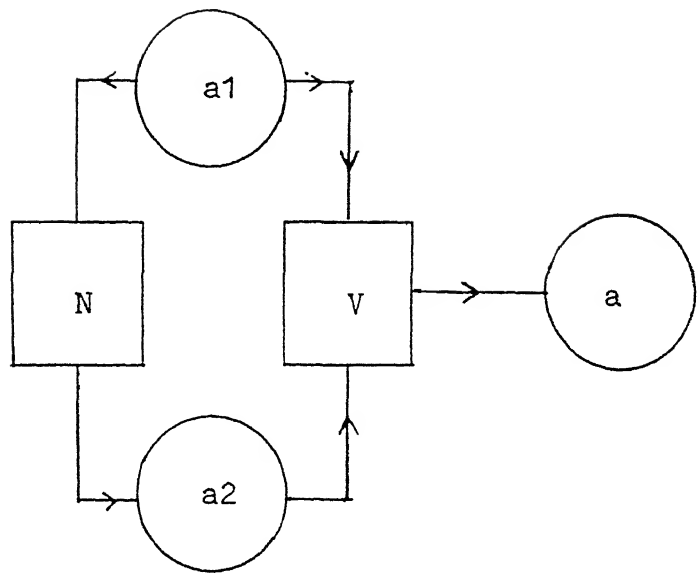(b)   With circularity broken and introducing the vidya operator.

Thus, $\underline{a}$ can be neither T nor F, if $\underline{a} \underset{\sim}{N} = \underline{a}$ of Eq. (26) is

to be satisfied. It is given the name "Cretan liar" paradox (CLP),

since this equation covers the logic of the well-known paradox

in Greek logic with this name, in which the Cretan says, in

effect, "What I say is not true". Is that statement itself true

or false? Here $\underline{a}$ stands for the statement, and the contents

of the statement give the equation $\underline{a} \underset{\sim}{N} = \underline{a}$. Then, if we start

with $\underline{a}$ = T, an endless succession of truth values $\underline{a}$ = F,

$\underline{a}$ = T, $\underline{a}$ = F, ..., is obtained, by successively applying the

equation, so that $\underline{a}$ can be "neither T nor F", if it is to be

only one of T or F (which is a paradox in PC). As already

mentioned earlier, $\underline{a}$ = D and $\underline{a}$ = X are possible solutions of

$\underline{a} N = \underline{a}$ in SNS, and we shall indicate their reasonableness and

relevance to the CLP in Section 6.

6. <u>Examples of paradoxes and the principles governing / elimination</u>
their

In this section, we shall take the three typical forms of

paradoxical arguments given in the last section in a symbolical

language and give examples from practical logic for each of them.

While doing this, we shall also indicate how the resolution of

the paradoxes as mentioned earlier, namely by taking into account

the SNS states D and X, is carried out. It is believed that these

examples will make clear how other examples of similar paradoxes

can be manipulated so as to eliminate the paradoxical nature of

the arguments in them.

(a) <u>Double statements paradox:</u> The best example of illustrating

this, in the form of Eqns (24a, b), is as follows:

$$\text{The next statement is false} \quad (29a)$$

$$\text{The previous statement is true} \quad (29b)$$

If we take $\underline{a}$ to indicate the truth value of the first statement

and $\underline{b}$ to indicate the truth value of the second statement, then

it is obvious that the two are related by the Eqns (24a,b).

Then,   neither of the states T or F for $\underline{a}$ or $\underline{b}$, leads back

to itself. Instead, in all cases, it leads back to the opposite

state, and Eqn.(24c), namely $\underline{a1} \underset{\sim}{V} \underline{a2} = X$, is the result.

Thus, in both cases, the argun..... is self-contradictory and is

therefore invalid irrespective of whether $\underline{a}$ is T or F, or $\underline{b}$ is T or F.

Since, in propositional calculus, there are no other input states

available, we get the result that the argument composed of the

two sentences (29a) and (29b) is a paradox and does not have

any solution.

Does this mean that the argument is necessarily "self-

contradictory", because this is the first conclusion that one

would arrive at from the conditions $\underline{a1} \underset{\sim}{V} \underline{a2} = X$ and $\underline{b1} \underset{\sim}{V} \underline{b2} = X$.

It is not necessarily so, because we can show that the state D as

input for $\underline{a}$ leads to D for $\underline{b}$, and back again to D for $\underline{a}$, leading

to no contradiction. Hence a non self-contradictory solution to

the two Eqns (29a and b) is the state D for $\underline{a}$ and $\underline{b}$, which means

that neither/statements can be assumed to be either true at
of the

all times or false at all times. Its truth or falsity cannot be

asserted, but the best that we can do is to say that it "may be

true" or "may be false".

This solution appears to be trivial and that it has no

information content about a or b, because when we know nothing

about a statement a, we say that it is in the state D. For

example, we can readily show that the equations

$c \underset{\sim}{O} d = T$ and $c = T$ lead only to $a = D$, in which case

we can say nothing definite about the state $\underset{\sim}{d}$. However, we shall
of d

show in Section 6(d) that, under certain conditions, the state D

for the truth value of a logical statement can have a profoundly

important value. Since the appreciation of this will need a

better acquaintance with our methods of dealing with paradoxes,

we shall discuss this only in Section 6(d) after we have considered

examples of all the three types of paradoxes mentioned in Section 5.

CSP and CSP-2

(b) Contradictory statements paradoxes/ The paradox CSP, whose

symbolic representation is given in Eqns (26a,b,c) starts from

the same statement $\underline{a}$ and arrives at two logical states $\underline{\underline{b1}}$ and $\underline{\underline{b2}}$

for the statement $\underline{b}$ via two different paths and it is found

that $\underline{\underline{b1}}$ and $\underline{\underline{b2}}$ are self-contradictory, irrespective of whether

$\underline{a}$ is true or $\underline{a}$ is false. In order to make the discussion

sufficiently general and to consider the examples available in

the literature of mathematics, particularly in set theory, we

shall slightly modify this and give it another form. This can

be written symbolically as *CSP-2, below:*

$$\underline{a} \; \underset{\sim}{I} = \underline{\underline{b1}}, \quad \underline{a} \; \underset{\sim}{I} \underset{\sim}{N} = \underline{\underline{b2}}-, \quad \underline{a} = T; \quad \underline{b} = \underline{\underline{b1}} \underset{\sim}{V} \underline{\underline{b2}} = X \qquad (30a,b,c,d)$$

In this, the implications $\underset{\sim}{I}$ and $\underset{\sim}{I} \underset{\sim}{N}$ are used, rather than the

equivalence operators $\underset{\sim}{E}$ and $\underset{\sim}{N}$. However,   the starting point $\underline{a}$

is given to be true without question and in that case we get a

self-contradiction. It may be questioned as to how two mutually

contradictory results $\underline{\underline{b1}}$ and $\underline{\underline{b2}}$ can be arrived at from the same

assumption — namely the truth of $\underline{a}$. Before giving examples,

we shall indicate that this/happens because the statement $\underline{a}$

only

contains two conditions in it, one of which leads to $\underline{b1}$ and

the other to $\underline{b2}$, and it is quite possible that these two

consequences $\underline{b1}$ and $\underline{b2}$ are mutually contradictory. Eqn.(30)

only gives the logical structure of this type of paradox.

The genesis of the two mutually contradictory conclusions is a

matter of semantics and as will be seen from the three

examples in basic mathematics which are given below, such a

situation is quite possible. We shall first take the Russell's

paradox, which may be stated as follows:

> "An exclusive set is one that does not contain
> itself as one of its numbers. If $\mathcal{R}$ is the   (31)
> set of $\underline{all}$ exclusive sets, does it contain
> itself or not?".

Thus we have two statements,

$$\underline{a} = \text{"}\mathcal{R} \text{ is the set of all exclusive sets"} \qquad (32a)$$

$$\underline{b} = \mathcal{R} \subset \mathcal{R} \qquad (32b)$$

The paradox arises by assuming the two properties namely,

" $\mathcal{R}$ is a set of <u>all</u> sets" and " $\mathcal{R}$ is an <u>exclusive</u> set" *to be true*

The first leads to the condition that $\mathcal{R} \subset \mathcal{R}$ so that $\underline{\underline{a}} \Rightarrow \underline{\underline{b}}$

and $\underline{\underline{b1}}$ = T. The second one, however, leads to the result that

$\mathcal{R} \not\subset \mathcal{R}$, or $\underline{\underline{a}} \Rightarrow \neg \underline{\underline{b}}$ and $\underline{\underline{b2}}$ = F. Hence we have the result

(30d) namely $\underline{\underline{b}} = \underline{\underline{b1}} \underset{\sim}{V} \underline{\underline{b2}}$ = X , or a contradiction.

Thus, the assumption that $\underline{\underline{a}}$ is true, namely that there is

a set $\mathcal{R}$ which consists of all exclusive sets, leads to the

conclusion that $\mathcal{R}$ is both contained in $\mathcal{R}$ and not contained in $\mathcal{R}$.

This is Russell's paradox.

The answer /is that the assumption made at the start, namely

$\lfloor$ *to the question in (31)* $\rfloor$

that $\underline{\underline{a}}$ = T, is invalid, and that $\underline{\underline{a}}$ should be F; or that "there

exists /$\mathcal{R}$ which is the set of all exclusive sets" is an invalid

an

statement.

In fact, such a resolution of / that the obviously permissible

$\lfloor$ the paradox, namely $\rfloor$

statement which we start with is an impossible one, occurs in an

even simpler example, namely the <u>Burali-Forti Paradox</u>: This can

be stated as follows:

"If $\mathcal{N}$ is the set of all integers from 1 to ∞ is

    its ordinal number N the largest integer?"   (33)

In this, the statements $\underline{\underline{a}}$ and $\underline{\underline{b}}$ are

$\underline{\underline{a}} \cong$ "There exists N, which is the ordinal number of $\mathcal{N}$,

    which is the set of all integers from 1 to ∞ ."   (34a)

$\underline{\underline{b}} \cong$ "N is the largest integer."   (34b)

Here also there are two conditions/ $\underline{\underline{a}}$ ^in^ , which lead to mutually

contradictory consequences for $\underline{\underline{b}}$. Thus, /that ^the fact^ N is the ordinal

number of $\mathcal{N}$ yields the result that N is larger than every

ordinal number, or $\underline{\underline{b}}$ = T. On the other hand, the condition/ ^that^ N

is an integer demands that there is another integer greater than

itself, since every integer has a successor; this means that

$\underline{\underline{b}}$ = F. Hence we obtain the contradiction in the style of CSP-2.

    The solution, as in the case of Russell's paradox is that

the statement (33) is itself invalid; i.e. we cannot talk of the

"ordinal number of the set of all integers". As is well-known,

in Peano's axiomatic formulation of the theory of numbers, it

is stated that every integer (natural number) has a successor

and that there is a first integer; but no mention is made about

the last integer. The last statement is quite correct, because

the assumption of the existence of a last integer N for the

set of integers 1 to ∞ leads to the contradiction mentioned above.

The third example — namely Cantor's paradox — is even

shorter. It goes as follows:

"If $\mathcal{C}$ is the set of all sets, is it the largest set?"   (35)

Here, we use the fact that, from every set $\mathcal{S}$, we can construct

its power set $\mathcal{P}(\mathcal{S})$ (i.e. the set of all subsets of $\mathcal{S}$), which

is larger than itself. Hence, $\mathcal{P}(\mathcal{C})$ is a set larger than $\mathcal{C}$,

and the answer to (35) is "No". On the other hand, since $\mathcal{C}$ is

said to contain all sets, answer to (35) is "yes". The two are

mutually contradictory.

In our notation, the statements $\underline{a}$ and $\underline{b}$ are

$$\underline{a} \equiv \text{ " } \mathscr{C} \text{ is the set of all sets"} \tag{36a}$$

$$\underline{b} \equiv \text{ " } \mathscr{C} \text{ is the largest set"} \tag{36b}$$

and we obtain that $\underline{a} \Rightarrow \underline{b}$ and $\underline{a} \Rightarrow \neg \underline{b}$ from the two

properties "all" and "set" of $\mathscr{C}$. Consequently, if $\underline{a}$ is true,

there is a contradiction, leading to the conclusion that $\underline{a}$ is

false, or that "there is no set of all sets".

Thus, we have seen three examples of CSP-2. The author

has not been able to trace any example of CSP as such, where,

for both $\underline{a}$ = T and for $\underline{a}$ = F, the conclusions are of the form

$\underline{a} \Rightarrow \underline{b}$ and $\underline{a} \Rightarrow \neg \underline{b}$, which is what is demanded for $\underline{a} \equiv \underline{b}$

and $\underline{a} \not\equiv \underline{b}$ at the same time. This is surprising, for CSP is

equivalent in its logic to DSP, for which there are examples.

However, we know that some theorems of the form $\underline{a} \Rightarrow \underline{b}$ can be

proved only in the form $\neg \underline{b} \Rightarrow \neg \underline{a}$ in a <u>reductio ad impossible</u>

proof, but it may not be possible to formulate the direct proof.

It should be noted that, in all the three examples given

in this section, $\underline{a}$ = T $\longmapsto$ $\underline{b}$ = X, and we can then deduce that

$\underline{a}$ = F even using BA-1 theory of propositional calculus, without

any recourse to SNS states. There is no "paradox" at all in

all these examples, for $\underline{a}$ = F does not lead to a contradiction.

The contradiction, in fact, arises from <u>two different assumptions</u>

in $\underline{a}$, which we may call $\underline{p}$ and $\underline{q}$, which lead to

$$\underline{p} \Longrightarrow \underline{b1} \ , \quad \underline{q} \Longrightarrow \underline{b2} = \neg \underline{b1}, \quad \underline{b1} \underset{\sim}{V} \underline{b2} = X \qquad (37a,b,c)$$

so that, $\underline{p}$ = T and $\underline{q}$ = T leads to a contradiction X. Hence,

we can deduce that

$$\underline{p} \ \& \ \underline{q} \ = \ F \tag{38}$$

i.e. $\underline{p}$ = T demands $\underline{q}$ = T and $\underline{q}$ = T demands $\underline{p}$ = F .

For example, in Russell's paradox, there is no "<u>exclusive</u> set",

which is at the same time the "set of <u>all</u> such sets". One or

other is not true.

However, in the literature, these are shown as paradoxes

of CLP type, <u>viz</u> $\underline{a} \equiv \neg \underline{a}$ (see [7], p 128 for example). The

argument for Russell's paradox is given in the form

$$(\mathcal{R} \subset \mathcal{R}) \Rightarrow (\mathcal{R} \not\subset \mathcal{R}) \quad \text{and} \quad (\mathcal{R} \not\subset \mathcal{R}) \Rightarrow (\mathcal{R} \subset \mathcal{R}) \qquad (39a,b)$$

so that

$$(\mathcal{R} \subset \mathcal{R}) \equiv (\mathcal{R} \not\subset \mathcal{R}), \text{ a contradiction} \qquad (39c)$$

Denoting $(\mathcal{R} \subset \mathcal{R})$ by $\underline{b}$, as in (32b), it is to be noted, that $^{\lfloor however,\rfloor}$

it is not the $\subset$ in $\mathcal{R} \subset \mathcal{R}$ that leads to $\underline{b} \Rightarrow \neg \underline{b}$ as in

(39a), but it is the "exclusive" in $\underline{a}$ of (32a) that leads to

$\underline{a} \Rightarrow \neg \underline{b}$. Similarly, it is not the $\not\subset$ in $\mathcal{R} \not\subset \mathcal{R}$ that leads

to $\neg \underline{b} \Rightarrow \underline{b}$ as in (39b), but it is the "all" in $\underline{a}$ of (32a)

that leads to $\underline{a} \Rightarrow \underline{b}$. Thus, Russell's paradox, which is

clearly a CLP paradox, if it is stated in the form $\underline{b} \Rightarrow \neg \underline{b}$

and $\neg \underline{b} \Rightarrow \underline{b}$, is <u>not a paradox at all</u> when it is formulated

in the form of the CSP-2 equations, but <u>only a contradiction,</u>

leading back, from $\underline{a} = T$ and $\underline{b} = X$, to $\underline{a} = F$, a perfectly

admissible deduction in PC.

The situation is similar for the Burali-Forti paradox

and Cantor's paradox. In fact, quite generally, CSP-2 is not

a paradox, but only indicates an inner contradiction in the

input statement $\underline{a}$, of the form $\underline{p} \, \& \, \underline{q}$. If $\underline{b} = X$, then it

follows straightaway that either $\underline{p}$, or $\underline{q}$, is F; __both__ properties

$\underline{p}$ and $\underline{q}$ cannot be present in the input statement. The author

believes that this point of view must be carefully considered.

(c) Cretan liar paradox: An example of this has already been

given in Section 5(c) of the form "what I say is not true".

If this stands for $\underline{a}$, then $\underline{a} \Rightarrow \neg \underline{a}$ [and $\neg \underline{a} \Rightarrow \underline{a}$] leading to the

paradox. Any number of examples can be set up which has this

form, or the equivalent form with two sentences in the double

statement paradox. In these cases, the question is as to what

is the way of resolving the paradox. In the case of the Cretan liar,

the statement $\underline{a}$ can have the SNS states D or X and then

the input and the output of the equation $\underline{a} \, \underset{\sim}{N} = \underline{a}$ are the same,

and there is no difficulty. In simple language, the former

(namely the state D) means that we cannot say whether what the

Cretan says is in fact true or false; a distinction between the

two states T and F is not possible. This is one solution.

Another solution is to say that the statement is invalid and

impossible. Such a statement cannot be included in a logical

argument, for it is neither true nor not true.

The discrimination whether the first solution (namely that

$\underline{a}$ = D) or the second solution (namely $\underline{a}$ = X) should be considered

as the actual solution of the equation $\underline{a} \underset{\sim}{N} = \underline{a}$ depends upon

the particular problem, and would depend on semantic considerations,

rather than considerations of pure logic. Thus, if $\underline{a}$ is the

statement "This proposition is <u>not true</u>", then, the state D adds

nothing to the knowledge in that argument. On the other hand,

it can be considered invalid, and rejected from the argument,

by the consideration that it leads to the contradictory state X

for $\underline{a}$.

(d) The state D having semantic value-properties of infinity

As mentioned in Section 6(a), there may be situations

where the doubtful state D($\equiv$ T or F) for the truth value of the

solution of the paradoxical statement $\underline{a} \Rightarrow \neg \underline{a}$ may

contribute to the understanding of the semantic consequences

of this statement. Thus, if we talk of the integer "infinity"

(∞), then the question whether it is odd or even cannot be

answered definitely either way, and the answer to the question:

"Is ∞ odd?" is that it may be so, or it may not be so. This

is because we have the well-known equation (40) satisfied by

∞ (given, for instance, in [8] , p. 86):

$$\infty = \infty + 1 \tag{40}$$

Because of this equation (40) which ∞ obeys, we obtain that

if ∞ is odd then it is even, and if it is even then it is odd.

This has the structure of the CLP, namely $\underline{a} \Rightarrow \neg \underline{a}$. Therefore,

from purely logical considerations, as discussed in Section 5 and 6,

we can have either of   two possible answers to the question:

"Is ∞ odd or even?"   namely

    (a)   The question about the odd or even nature

           of infinity is inadmissible (state X),   (41a)

    (b)   Infinity can be either odd or even and it is

           not possible to decide between these two

           possibilities (state D).   (41b)

Which of these is the real answer depends upon mathematical

theory, and not on logic. The author believes that the latter

is a more reasonable statement for the even-odd nature of

infinity, as deduced from the equation $\infty = \infty + 1$. In fact,

several more properties of ∞ can be seen to have the D state,

if this state is included along with the states T and F in

sentential logic.

For example, for any pair of finite numbers x and y, either

of the relations $x > y$ or $x \leqslant y$ holds; but one excludes the

other. Hence, if we denote the statement $x > y$ by $\underline{t}$, then

$\underline{t}$ is either T or F, for finite numbers. However, it follows

that $\underline{t}$ may be "T or F"$(\equiv D)$ if $y = \infty$ and $x = \infty + 1$, so that

the answer to the question "Is $x > y$?" is that it may be "yes"

and may be "no". Hence, questions, for which a definite answer

having one of the states T or F is possible for finite numbers,

may have only an answer in the state D when the numbers become

infinite. A study of the considerations regarding the need and

relevance of the SNS state D in dealing with such properties of

∞ will be taken up for detailed study in a later report. Some

new results are obtained regarding cardinal numbers of infinite

sets.

7. Absence of Paradoxes and Incompleteness in Multi-valued Logic

(a) Impossibility of incompleteness due to closure of Boolean algebras.

Just as we have shown that BA-2 and BA-3 are closed with respect to matrix connectives (either unary or binary; and in the forward or reverse form) and the Boolean connectives $\oplus$ and $\otimes$ in the forward direction, it is obvious, from the general theory of BA-n in Part I [2] , that this is the case for all BA-n generated by n basic states. Hence, closure is assured for all n-valued logics, having n independent states ($n \geqslant 2$), leading to a set of 2^n possible states for that logic. Thus, the application of the theory in the earlier sections to BA-n, assures the absence of incompleteness (as is commonly understood) in any multi-valued logic.

(b) Non-closure of a new type leading to higher genus Boolean algebras

However, reversal of Boolean connective relations in BA-n can lead to a number of new states not contained in BA-n. Thus,

in BA-3, the reversal of the equation in (42a) leads to (42b):

$$(\mathcal{Q} \otimes \exists = \exists) \equiv (\exists \overleftarrow{\mathbb{V}} \exists = \mathcal{Q}) \tag{42a,b}$$

This has as solution " $\exists$ or $\triangle$, but not ϕ ", which is not

contained in BA-3. If we take BA-7, having for its seven basic

states the seven non-impossible states of EPL (BA-3), then this

will be contained in it. Thus, we need BA-$\underline{m}$ with $\underline{m} = (2^7-1)$ for

representing all possible states generated by solutions of

equations of the type (40b) in EPL.

This process is recursive, and, as mentioned in [1] and [3],

there is an essential incompleteness in the formulation of

mathematical logic. Thus, a logic, based on BA-$\underline{n}$, necessarily

leads to one based on BA-$\underline{m}$, with $\underline{m} = 2^n-1$, and so on ad infinitum.

However, we have not been able to find examples of arguments

that require more than EPL, isomorphic to BA-3.

(c) Absence of paradoxes in SNS and EPL and in all logic with n basic states (n $\geq$ 2)

The impossibility of incompleteness in logics isomorphic

to BA-n (n $\geq$ 2) is also indicative of the result that no

paradoxes can be formulated in them. Thus, considering SNS, we

have already seen that the typical pair of paradoxical statements

as in DSP, namely $\underline{a} \underset{\sim}{N} = \underline{b}$, $\underline{b} \underset{\sim}{E} = \underline{a}$ (or the CLP $\underline{a} \underset{\sim}{N} = \underline{a}$), has the

permissible state D as a solution for $\underline{a}$. This is possible because

$T \oplus T^C = T \oplus F = D$ is one of the admissible states of SNS.

In exactly the same way, if we have the following equations

in EPL —

$$\underline{a} \; \underset{\sim}{E^C} (i, \; i^C) = \underline{b} \; ; \; \underline{b} \; \underset{\sim}{E} = \underline{a} \qquad (43a,b)$$

leading to

$$\underline{a} \; \underset{\sim}{E^C} (i, \; i^C) = \underline{a} \qquad (44)$$

we will find that, both for (43) and for (44), if $\underline{a1}$ is the

input and $\underline{a2}$ the output state for $\underline{a}$, then

For $\underline{a1} = q_i$, $\underline{a2} = q_i^C$; $\underline{a1} \bigvee \underline{a2} = \emptyset$ \qquad (45)

so that the two complementary states q_i and q_i^c lead to a

contradiction, similar to what happens with T and F in the DSP

and the CLP of PC. However, it is readily seen that

$(q_i \oplus q_i^c) = \Delta$ is a permissible solution of (43), or (44).

Since Δ is included in EPL itself, there is no paradox produced

by (45a,b).

Even more generally, we can say that, because EPL is closed

as described in Section 7(a) above, no set of relations formulated

in EPL can lead to a state not contained in it. Hence, no

paradox will be generated by an argument in EPL (and in SNS).

The same line of argument will give the still more general result

that no logic isomorphic to the Boolean algebra BA-n (which is

closed under the conditions mentioned above) can lead to a paradox.

The only exceptions are those obtained by the reversal of

relations having Boolean connectives, when there is absence of

closure and a higher genus Boolean algebra is required to describe

and represent the result of the argument.

Thus, in full generality, we can say that mathematical

logic, employing Boolean algebraic vectors and matrices, is

a complete story, having no inconsistencies or paradoxes.

References

1. Ramachandran, G.N., Syad-Nyaya System (SNS) — A New Formulation of Sentential Logic and its Isomorphism with Boolean Algebra of Genus 2, Current Science, 1982, 51, 625-636.

2. Ramachandran, G.N., Vector-Matrix Representation of Boolean Algebras and Application to Extended Predicate Logic (EPL) Part I, Current Science, 1983, 52, 292-302.

3. -do- Part II, Current Science, 1983, 52, 335-341.

4. -do- Part III "Algebraic Theory for Categorical Statements in Quantified Predicate Logic (QPL)" (Submitted for publication) (Matphil Reports No. 34).

5. -do- Part IV "General Theory for Statements of Type-1 in QPL". (Matphil Reports No. 35, Draft 2.)

6. -do- Part V "Theory for/general argument in logic including SNS, QPL and QSN statements" (Matphil Reports No. 36, Draft 2).

7. Stoll, R. R., "Set Theory and Logic", W.H. Freeman and Company, New York, 1963.

8. Russell, B., "Introduction to Mathematical Philosophy", Simon and Schuster, New York, 1982.

TABLES Page No.

FIGURES

Molecular Biophysics and Biomedical Research*

(Acceptance Talk at the award ceremony of the
Rameshwardas Birla Award for medical and related
fields on 28th January 1985)

G.N. Ramachandran

INSA Albert Einstein Professor
Mathematical Philosophy Group
Indian Institute of Science
Bangalore 560 012

Molecular Biophysics and Biomedical Research*

(Acceptance Talk at the award ceremony of the
Rameshwardas Birla Award for medical and related
fields on 28th January 1985)

G.N. Ramachandran

INSA Albert Einstein Professor
Mathematical Philosophy Group
Indian Institute of Science
Bangalore 560 012

Molecular Biophysics and Biomedical Research*

G.N. Ramachandran
INSA Albert Einstein Professor
Mathematical Philosophy Group
Indian Institute of Science
Bangalore 560 012

I would like to say, first of all, how deeply touched

I was when I learnt that the Rameshwardas Birla Award for

Medicine has been given to me for 1984-85. It is a great honour

to be chosen for this Award, but it is particularly so because

this is the first time that the Committee has selected a person

who is not working on practical aspects of medical science, but

only in basic fields related to biology and medicine, namely

molecular biophysics and molecular biology.

I shall not try to give a complete outline of the

achievements in the two laboratories devoted to molecular

biophysics which I have established in the University of Madras,

and in the Indian Institute of Science, respectively. However,

I shall touch upon some aspects of these studies in which I was

personally connected quite closely. One of the earliest among

these was the enunciation of the triple helical structure of

---

collagen, in 1954, very soon after the structure of alpha helix

was discovered from theory by Pauling in 1951, and the DNA

double helix by Watson and Crick in 1953. The most essential

feature of the chemistry of collagen which led to the discovery

of the triple helix is shown in Slide 1[*], namely its amino acid

composition which consists of one third glycine (the smallest of

all amino acids) and approximately another one third of the

two amino acids proline and hydroxyproline, with rigid side

chains. These features had not been specifically incorporated

in the design of several structures made previously by very

eminent scientists in the early 1950s. The structure which we

proposed is shown from the side in Slide 2. The three helices

run parallel to one another and are interconnected by hydrogen

bonds. The architecture is made clearer by looking down the

triple helix from the top down the central axis and the backbone

of the three helices is shown in Slide 3. In this, each helix

has a three-fold screw axis and the three of them are also

related by a three-fold screw axis interconnecting them.

However, the x-ray diffraction pattern of collagen, shown in

---

[*]The slides will be shown in the lecture, but the diagrams
are not included in the written report.

Slide 4, indicated that the number of units per turn is not

3 but 3.3. This was adopted for the triple helix and then

the pattern of the winding of the three helices about a common

central axis was discovered to be as shown in Slide 5.

Essentially, the structure of collagen could be said to have

been solved when this pattern of the backbones in the three

individual helices was worked out. These studies were made

during 1954-55 and the essential correctness of the structure

is shown by an optical diffractometer picture, of a drawing of

the collagen structure, which is shown in Slide 6. The occurrence

of a strong meridional reflection on the 10th layer and

off-meridional reflections in the 3rd and 7th and the weak

ones on the 4th, completely agreed with the x-ray pattern

shown earlier and they justify an approximate value of 3.3

for the number of residues per turn. Afterwards, it was a

question of fitting various pieces of information regarding the

protein into the structure, and the ball and spokes model

shown in Slide 7 (which was the best structure available in

1968) was obtained in our laboratory. (It should be mentioned

that although the occurrence of the triple helix with 3 residues

per turn was first worked out by us in 1954, the supercoiled
triple helix with 3.3 residues per turn was arrived at
simultaneously, both in Madras, and in London and Cambridge in
1955).

The spurt of activity in biophysics and molecular biophysics
that came out after the discovery of the essential helical
nature of many biopolymers and also the discovery of the crystal
structures of globular proteins which came at about the same
time, led to the recognition that biological function is
completely explicable from the molecular structure and chemical
interactions of the component biochemical entities in a living
system. Consequently, one could make experiments in the
laboratory on aspects of various biological systems and try to
find out the exact mechanism of the reactions that take place
in normal biological activity and in pathological conditions.
Many studies of this type in biochemistry and molecular biology
were facilitated to a great extent by the understanding of the
chemical structure and molecular architecture of biomolecules
obtained by using theoretical techniques as well as various
methods of biophysical chemistry.

In the evolution of the theoretical techniques of molecular biophysics, workers in our laboratory in Madras had contributed to a material extent. The first consistent explanation of the principles governing the conformation, or three dimensional structure, of a protein molecule was theoretically worked out in Madras in 1963, using a pair of peptide units as the model system. The picture of this is shown in Fig.8 and two dihedral angles (rotation angles ϕ and ψ) are marked in this diagram. It may be shown theoretically that the pair of angles (ϕ, ψ) can occur only for certain combinations of these and we worked out which of these are allowed and which of these are disallowed, by employing contact criteria of rigid, rubber-like spherical atoms. The resultant diagram first published in the Journal of Molecular Biology, 1963, has come to be known now-a-days as the 'Ramachandran Diagram' and is shown in Slide 9. This slide is taken from a well-known text book on Biochemistry by Lehninger which was published as early as 1975 and is used widely in graduate and post-graduate teaching in USA. It is understood that now-a-days this diagram is taught even to high-school students in biochemistry.

Very soon after this diagram came into being, the

corresponding energy diagram was worked out in several

laboratories including ours, and a typical one is shown in

Slide 10. It shows regions of (ϕ, ψ) which are most likely

to occur, and regions which are forbidden. It is very reassuring

to see that when a large number of protein structures are

examined, the diagram is fully substantiated, as may be seen

from Slide 11 which is taken from a recent book on protein

structure published from Germany.

Soon after the enunciation of the (ϕ, ψ)-rotation angles

in proteins, the essential rotation angles that are necessary

for the specification of the three dimensional structures of

nucleic acids and polysaccharides were also developed in

Madras (during 1965-70), although they were also worked out

in other laboratories during the same period. The diagram

showing the dihedral angles required for a nucleotide chain,

worked out in Madras, is shown in Slide 12 and the corresponding

diagrams for two saccharide units in α-glucose and β-glucose

are also shown in Slides 13 and 14. The essential difference between the stable configurations of the latter two is very clear, namely that the polymer of α-glucose (amylose) is going round and round while the chain is rigid and straight in the polymer (cellulose) of β-glucose. Amylose is nothing but starch and cellulose is the essential component of wood. So the difference in the properties of food and wood is to be attributed to this molecular mechanism occurring for the linking of the saccharide units that go to build up the molecular chains of these. It should be mentioned that V. Sasisekharan was associated with me in working out the dihedral angles of the nucleotide chain, and similarly, V.S.R. Rao for polysaccharide chains, in our laboratory in Madras. Both of them have gone ahead and built up schools of research of their own in these two respective subjects, during the last twenty years mainly in Bangalore.

We shall not discuss the various steps by which the branch of molecular biophysics has interacted with molecular biology in its development. However, during the last few years,

scientists in the latter field have even succeeded in imitating Nature in the production of new living organisms. Biotechnology and bioengineering are highly specialized experimental fields covering aspects of biology that were almost unthought of even twenty years ago. The confidence with which scientists working in these fields go towards new directions is not to a small extent the consequence of the great spurt in the knowledge of biomolecules created by biophysical chemistry and the theoretical and physical methods that were developed since the 1950's and 60's.

As a consequence, we could imitate chemicals that produce specific actions in living systems and produce them in the laboratory for being given to persons who lack in such chemicals, due to genetic reasons, or otherwise. But this is only a small part of the great achievements of modern biochemical research. Now-a-days, we can even improve upon Nature. By studying in minute detail the biochemistry and molecular structure and action of the component chemicals in a particular tissue or organism and trying to give this a good theoretical explanation

from ideas of physical chemistry and chemical physics, it

has become possible to design drugs that are even more effective

than the chemicals that have been chosen by Nature to carry out

the particular function. Thus, molecular biophysics has played

a subtle but important part in the explosive developments in

molecular biology during the last decade.

A good example is the explanation of the mode of action

of the well-known antibiotic penicillin. This drug acts on

the bacterial cell wall whose molecular structure is shown in

Slide 15. It consists of a series of parallel polysaccharide

chains which are linked together by short peptide chains in

which the sequence D-Ala-D-Ala occurs. The structure of

penicillin is shown in Slide 16 and, looked at in a careful

manner, it has a sequence of L-Val-D-Cys. The chemists were

always puzzled by the fact that the D—D sequence is imitated

by the L—D sequence of the peptide chain, which is chemically

the cause for the inhibition of the synthesis of the peptide

chains required to build the bacterial cell wall. In a study

made in Bangalore by Prof. V.S.R. Rao and colleagues, they could

show very clearly that the theoretically allowed conformations

of D-Ala-D-Ala which is free and of L-Val-D-Cys in the

beta-lactam configuration in penicillin are, in fact, very

closely similar to one another. The best conformation of each

of these has been pictured and shown side by side in Slide 17

and a close similarity in the locations of the biochemically

important groups in the two molecules is very clearly seen.

Thus we have, for the first time, a purely physicochemical

explanation of the mode of action of penicillin. Starting

from this, the workers in Bangalore have shown why certain

drugs of the penicillin group are more effective than others,

and so on, and studies of this type should help very much in

the design of the new drugs of the penicillin type which could

have the required properties.

We shall give one more example concerning the biochemistry

of vitamin C, where pure theory, combined with ideas from

different fields of sciences, have led to results of relevance

to medicine. Our studies on collagen are closely related to

this and led to the proof of Pauling's theory that vitamin C

increases resistance to diseases. This discovery arose out

of pure curiosity on my part in trying to understand the

molecular structure of a component known as complement C1q
                                                          the
of/immune response system, in comparison with the triple-helical

structure of the individual molecules of collagen. Collagen

fibres require hydroxyproline for the stability of its triple

helical structure, and hydroxyproline is not taken as such

from the pool of amino acids and incorporated in the molecular

chain of collagen that is being built up; rather proline is

incorporated at first and then, where it is needed, the proline

side groups are hydroxylated by an enzyme which produces

hydroxyproline. The way in which hydroxyproline stabilizes

the interaction between two neighbouring helices in the triple

helix is shown in Slide 18. It will be seen that a water molecule

occurs in the hydrogen-bonds linking an NH group of one chain

and the $C=O$ group of a neighbouring chain and the same water

molecule also links with the OH group of hydroxyproline, thus

forming a tight interconnecting link between the two

neighbouring helices to make the triple helix very stable.

Detailed study of the chemical process of hydroxylation

made by several persons, including, in particular, Prockop in

USA, indicated that the enzyme requires as co-factor ascorbic

acid (vitamin C) for the mechanism to be carried out. At

about the same period, namely the late 1960's and the early 1970's,

the molecular structure of complement C1q had also been

investigated and elaborated in the laboratory of Porter in

Oxford, U.K. One of the features that came out of these studies

was that complement has plenty of hydroxyproline, and also

contains one third glycine as in collagen, so that it should

possess a triple-helical structure similar to collagen. From

electron microscope studies, Porter and collaborators could

in fact show the existence of the triple helix in its molecular

architecture. The two features — triple helix and occurrence

of hydroxyproline —previously known to occur only for collagen,

was thus deleniated in complement C1q. Discussing about such

medico-chemical inter-relationships with Prof. George Pickering,

former Dean of Medicine in Oxford, I was rudely shaken when

he insisted that the suggestion of Linus Pauling that vitamin C

is not only needed for good health, but that it also improves

immunity to diseases, was all "Bosh". In a flash the association

of the words Pickering-Oxford-Porter-complement led to the ideas

of hydroxyproline-collagen triple helix-Prockop-vitamin C, and

I could see, from these that vitamin C (ascorbic acid) is as

much required for the synthesis of the collagen-like filaments

of C1q as for collagen itself. Thus, the exact possibility

whereby vitamin C improves immunity became clear by analogy —

complement requires ascorbic acid for production, and Pauling's

suggestion is absolutely sound and theoretically explicable.

The logical interconnections between these ideas which

were known to everybody (including me) for a number of years,

is shown in Slide 19. However, the remarkable conclusion

mentioned above had to be brought out by the stimulus of

personal discussions. I believe that it was the interaction

with senior scientists in various fields, which was made possible

for me, by being provided with the very prestigeous Fogarty

International Fellowship by US National Institutes of Health

(which gave me a whole year of freedom in 1977 to talk with

learned scientists all over the United States) that led

to the conclusions shown in the slide.

I have many suggestions for the creation of a proper

atmosphere in the institutions of higher learning in our

country which will really provide interdisciplinary exchange

of thoughts. Due to lack of time, I shall only mention one,

namely that, just as biochemistry flowered as a discipline

for study in Indian Universities and institutions in the first

half of this century, biophysics, which really grew very fast

in the second half, must also be included as a separate discipline

in every teaching and research institution dealing with

bio-sciences at the master's degree level and in higher stages

of learning. Somehow, the progress in establishing full-fledged

courses in biophysics has been slow in development, and it is

to be hoped that rapid progress in this direction will come

in the next few years.

———